Adquisición de Imágenes biológicas y biomédicas, un enfoque matemático

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motivation: medical diagnosis, imaging the human body

- focus: medical **diagnosis** is an inverse problem *par excellence*, and one of the oldest that has confronted humanity: we are faced with the problem of determining the cause of the symptoms of an illness.
- we are often asked for **images** of some kind: **x-rays, ultrasounds, resonances, scanners, electrocardiograms**, etc.
- behind each of these images there is an **inverse problem** and involve revolutionary physics, mathematics and technology to build the electronic devices that make a reality to see images of our **inner world**.

illness → symptoms

body composition and living tissues...

- the human body is more than **50% water**. The rest of their mass are **living tissues**, softer and harder: fat, muscles, bones.

	water	fat (storage)	fat (essential)	muscle	bone	remainder
women	**52**%	15%	12%	36%	12%	25%
men	**57**%	12%	3%	45%	15%	25%

Behnke's model of reference man and woman 1978

- waves or particles can **travel through** the body: **sound or elastic waves, electricity, radiation, light**.
- we can register their effect to see the inner body→ **inverse problem**

wave propagation

photon transport/scattering

First part (photon transport):

- 1.- X-rays, CT, nuclear medicine, PET, SPECT:
  - *currect research topic: simultaneous source and attenuation in SPECT*
- 2.- Fluorescence molecular tomography FMT :
  - *currect research topic: mathematics of light-sheet microscopy*

Second part (wave phenomena):

- 3.- Magnetic resonance MRI:
  - *current research topic: non-invasive pressure and blood flow estimation*
- 4.- Ultrasound, elasticity imaging, MRE:
  - *current research topic: bone porosity estimation by ultrasound*

First part (photon transport)

1.- X-rays, CT, nuclear medicine, PET, SPECT

currect research topic: simultaneous source-attenuation identification in SPECT

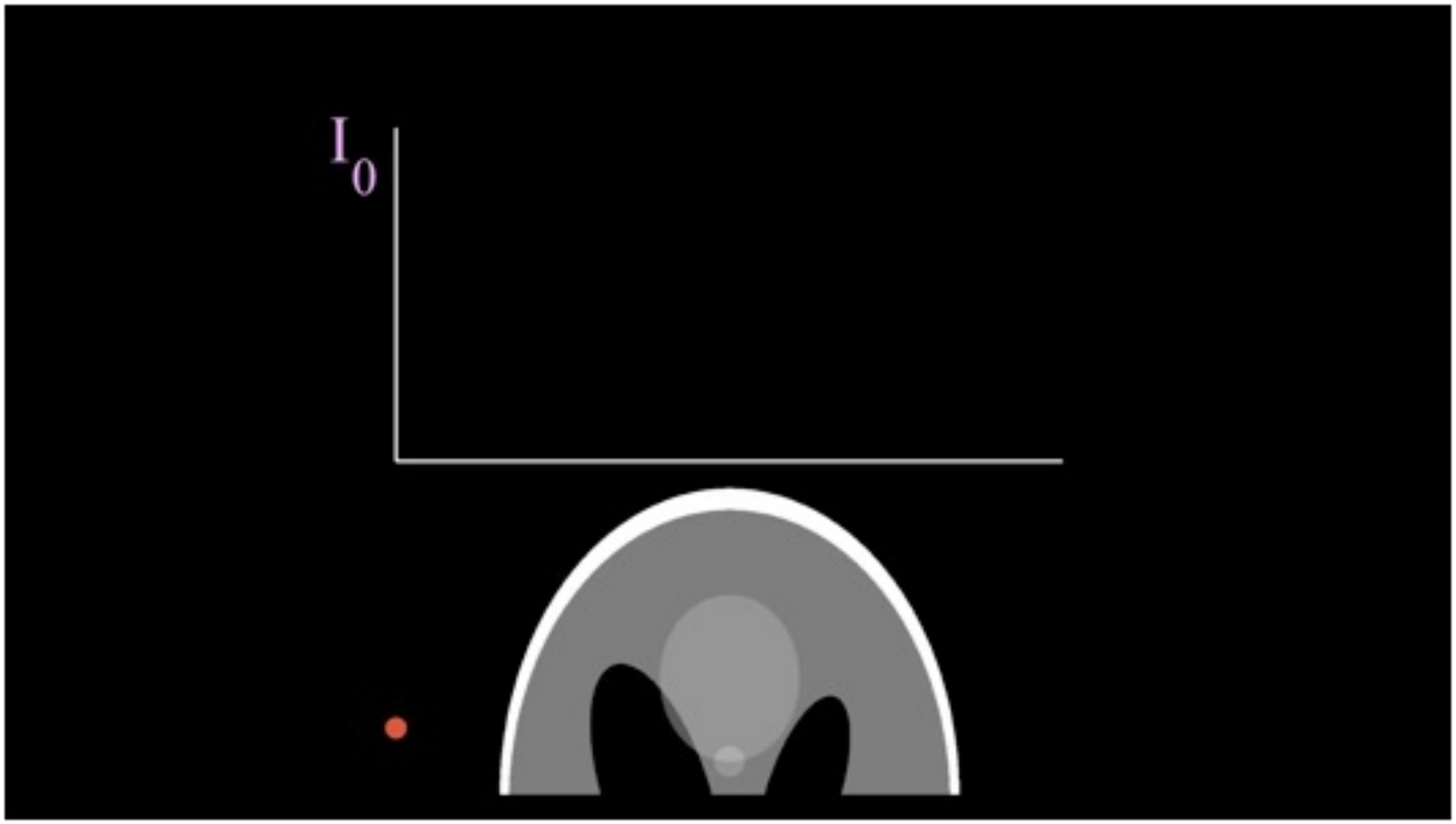
first the basis... X-rays... the scanner

- Wilhelm Röntgen, a German engineer, obtained the Nobel in 1901 for detecting and producing **X-rays** for the first time.
- X-rays: invisible high energy electromagnetic waves that cross the body almost as if it were transparent, **attenuated** principally by bones.
- In 1917, Johann Radon, an Austrian mathematician, developed the mathematical basis for combining X-rays at different angles to obtain **three-dimensional images**.
- The British engineer Godfrey Hounsfield build the **first scanner** in 1971 and obtained the Nobel for this in 1979.
- Before that, in 1953, diffraction X-ray images were used by the crystallographer Rosalind Franklin to discover the form of the **ADN double helix** (1962 Nobel)

X-ray attenuation

credit: Samuli Siltanen

the measurent $\log(I_0/I_1)$ is the integral of the attenuation along the line



The Shepp-Logan phantom: Larry Shepp and Benjamin F. Logan for their 1974 paper *The Fourier Reconstruction of a Head Section*

Beer's law

(monochromatic, X-ray beam no refraction or diffraction)

Intensity decay...

$$\Delta I = I_{out} - I_{in}$$

is proportional to attenuation, thickness and intensity...

$$\Delta I = -a(x) I \Delta x$$

The infinitesimal change...

$$\frac{dI}{dx} = -a(x) I$$

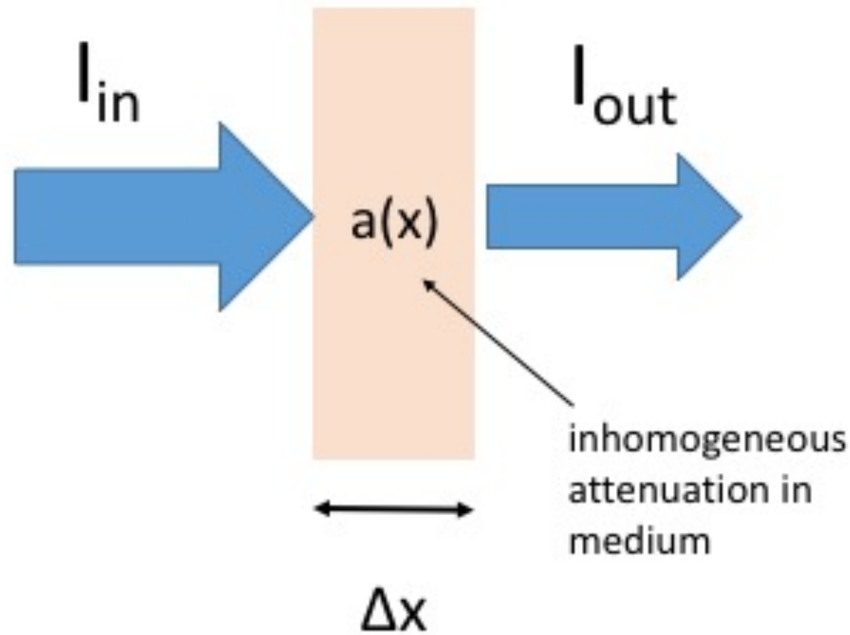
gives by integration on lines the solution:

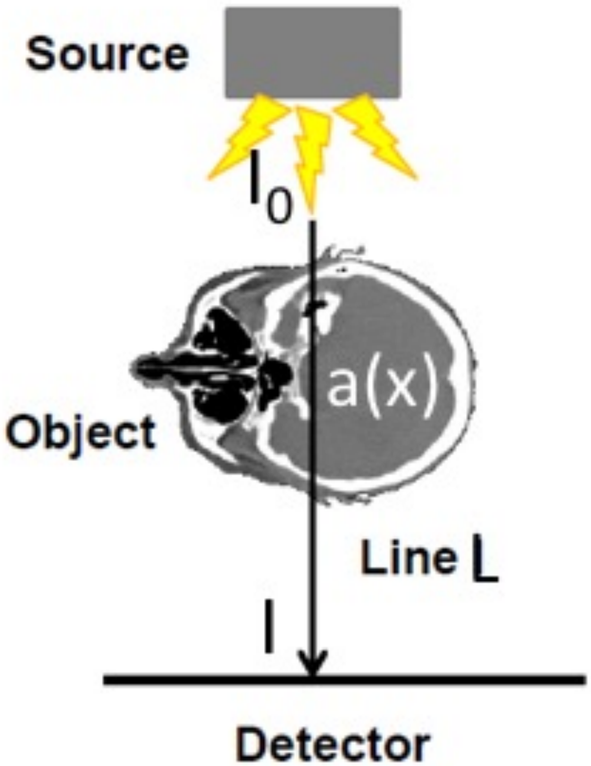
$$I = I_0 \exp\left(- \int_L a \right)$$

$$L : \text{line from } x_0 \text{ to } x$$

so the sum over lines can be measured

$$\int_L a = - \ln \frac{I}{I_0}$$





attenuation integral

$$\int_L a = -\ln \frac{I}{I_0}$$

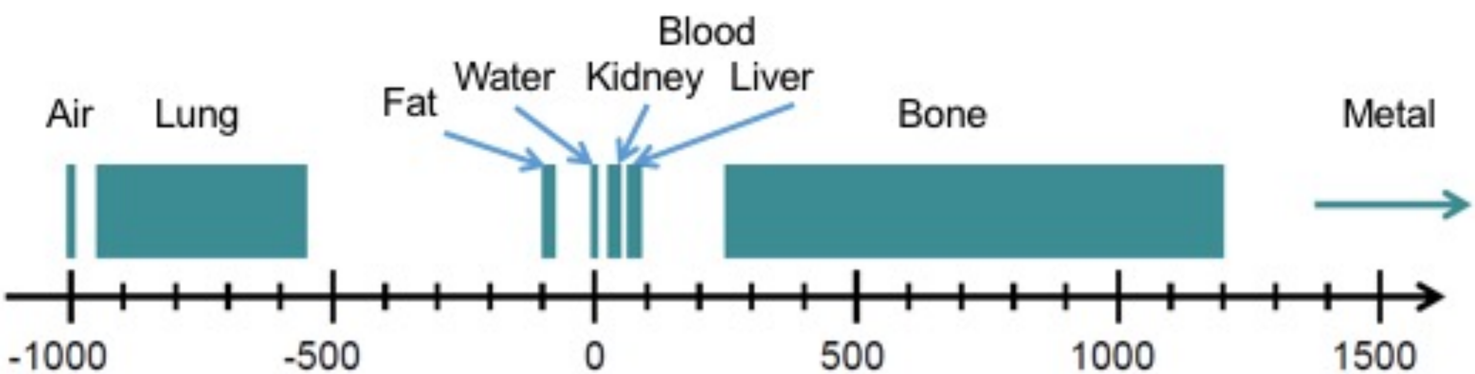
- Photons with energies 50-120 keV are emitted by an X-Ray source
- Interaction with biological tissue absorbs & scatters some of the photons (photo-electric and Compton effect)

credit: Dr. Castañeda

Typical X-ray data: Hounsfield Units (HU)

CT-scanner callibration Air=-1000, Water=0

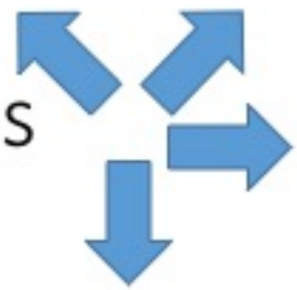
$$HU = \frac{\mu - \mu_{Water}}{\mu_{Water}} \cdot 1000$$



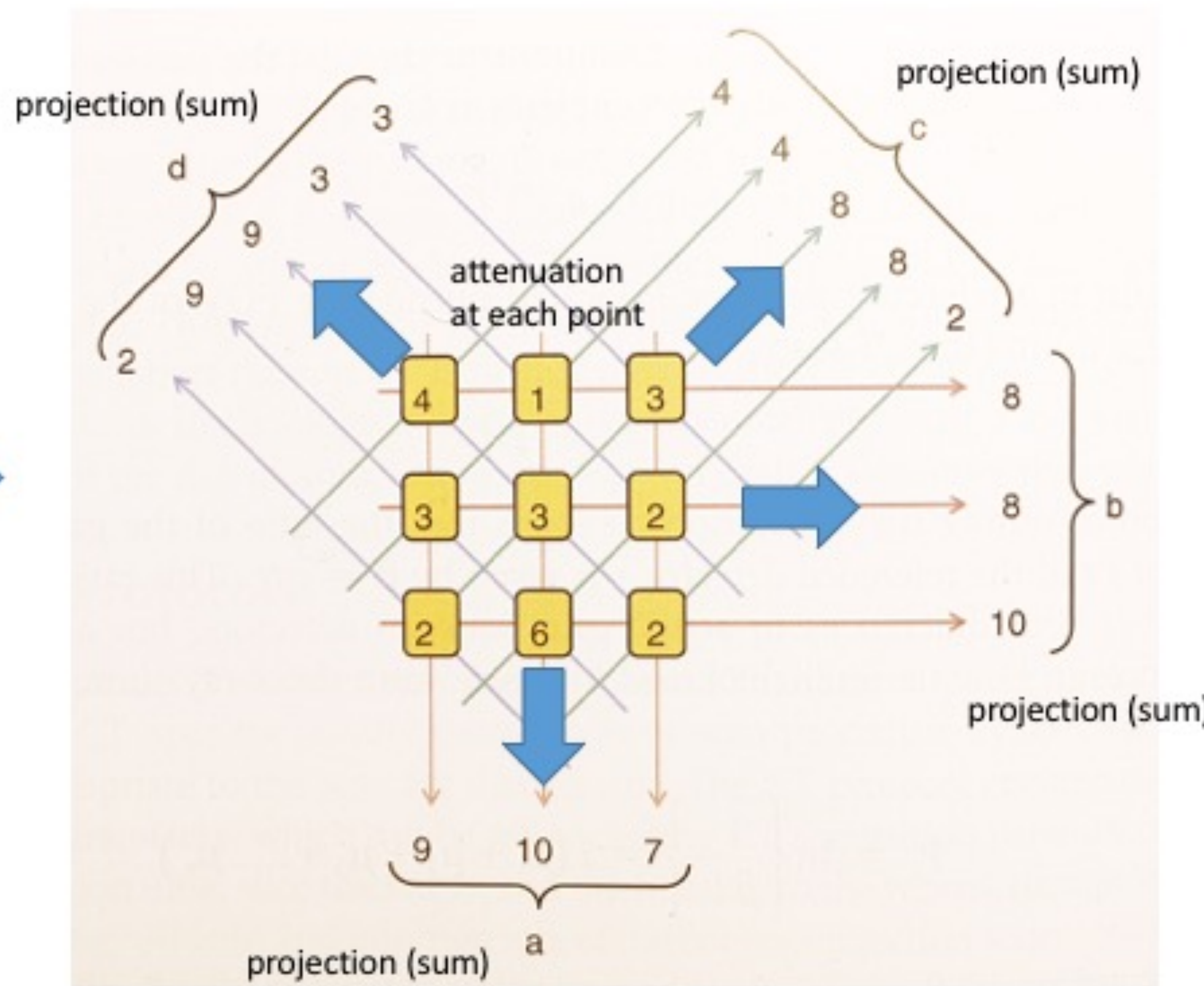
credit: Dr. Castañeda

Principle of Computed Tomography:

Projections

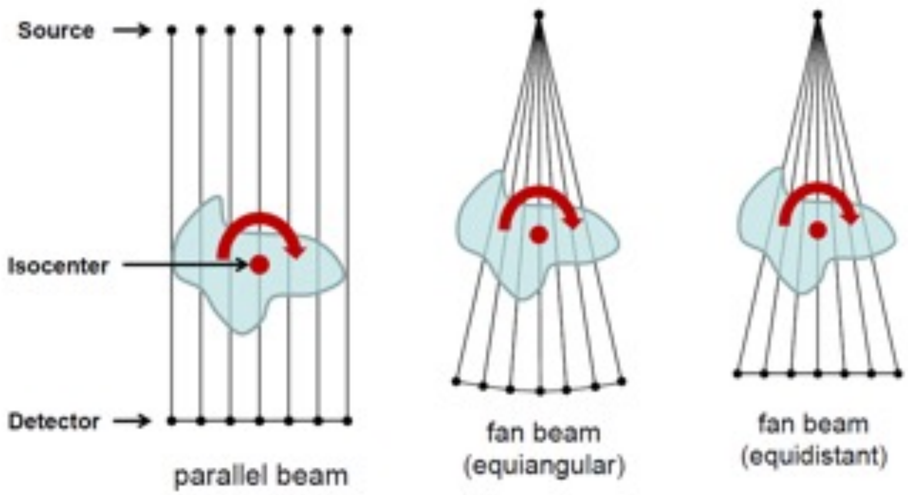


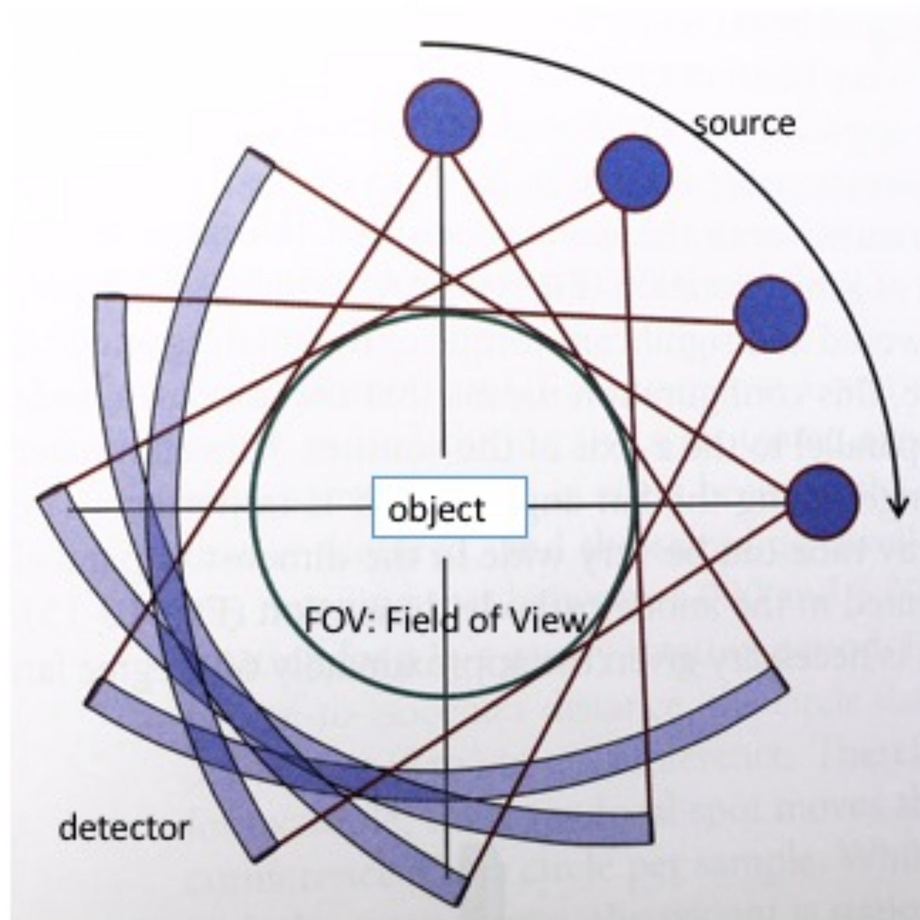
How to recover the attenuation from sum of lines?



credits figure: ref 2

CT-scanner: fan/parallel beam





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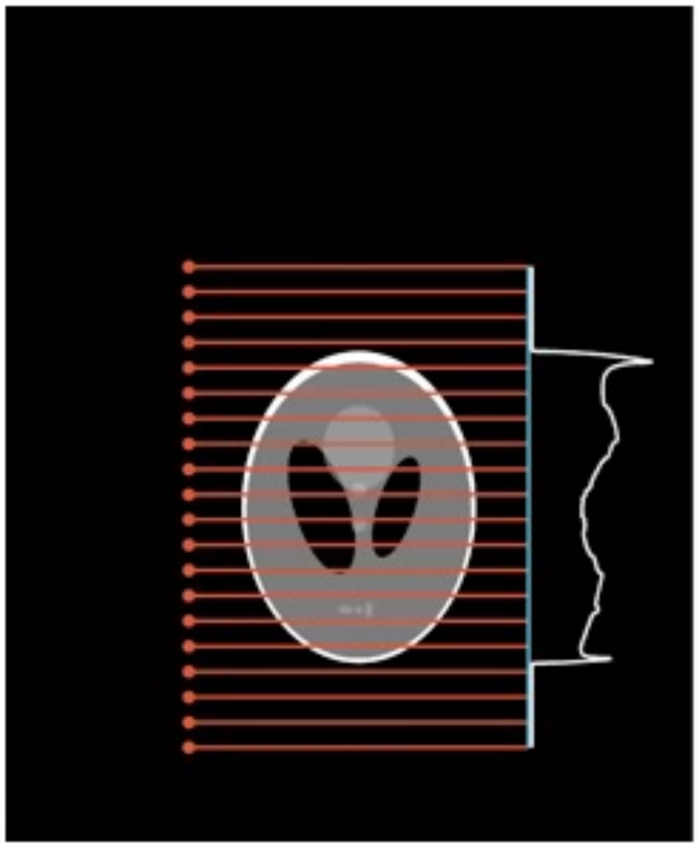
CT - scanner

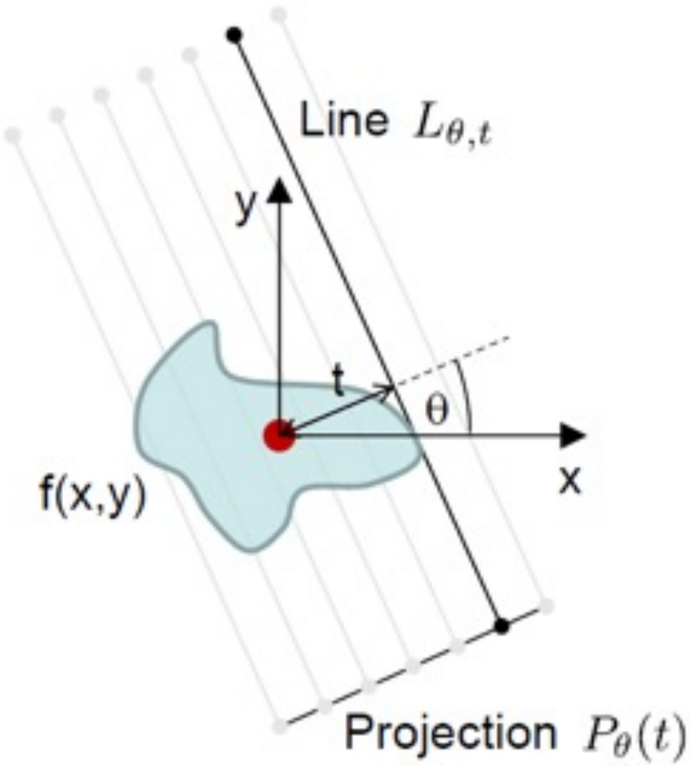
image? → sinogram (Radon transform)

Mathematical tool: the inverse Radon transform



credit: Samuli Siltanen

The Radon transform

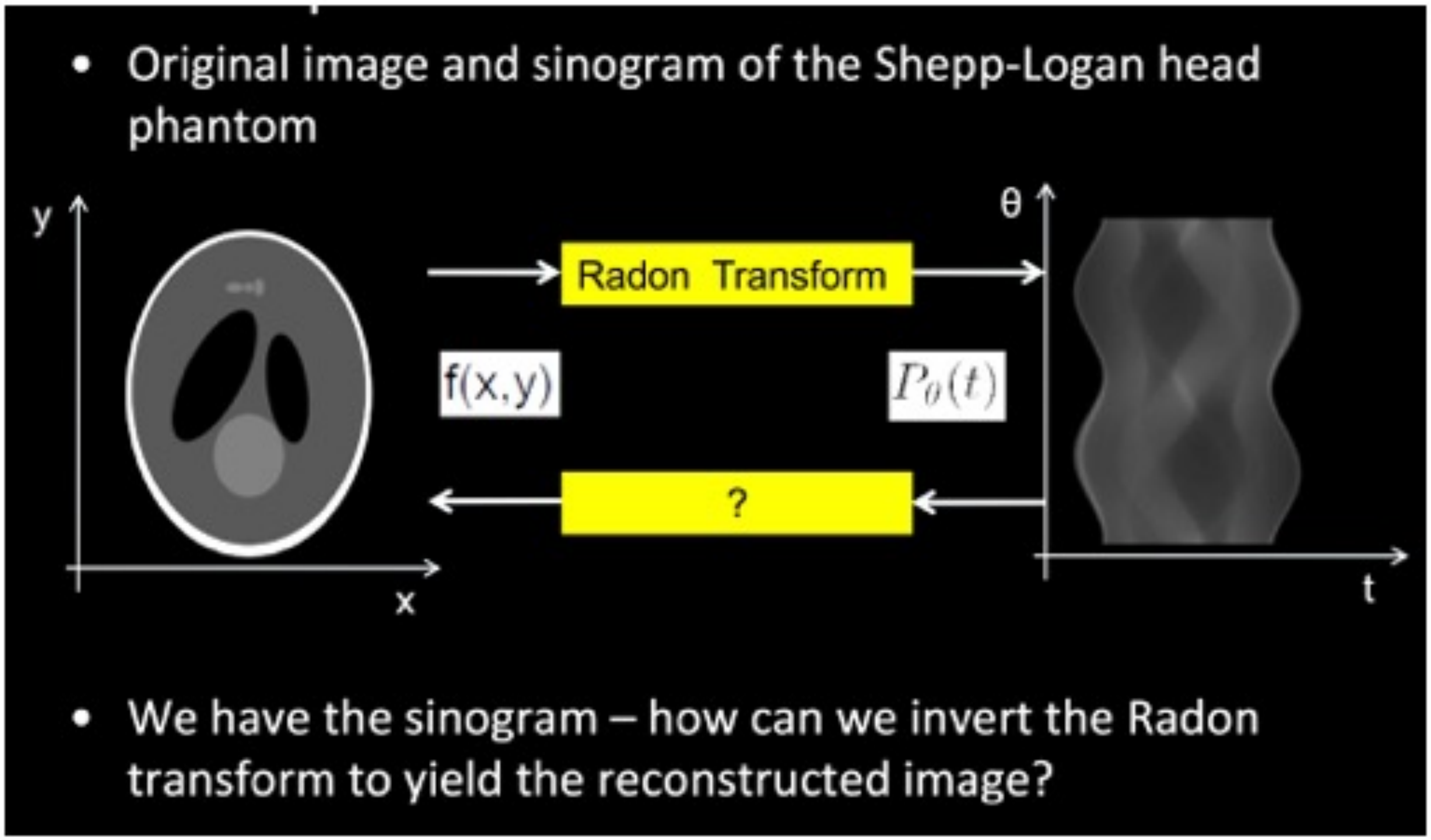


Integral of a function over lines

$$P_\theta(t) = (R \circ f)(\theta, t) = \int_{L_{\theta,t}} f(\vec{x})\, ds$$

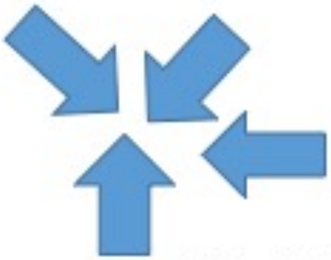
credit: Dr. Castañeda

The inverse problem (Johann Radon, 1917)



credit: Dr. Castañeda

Backprojections



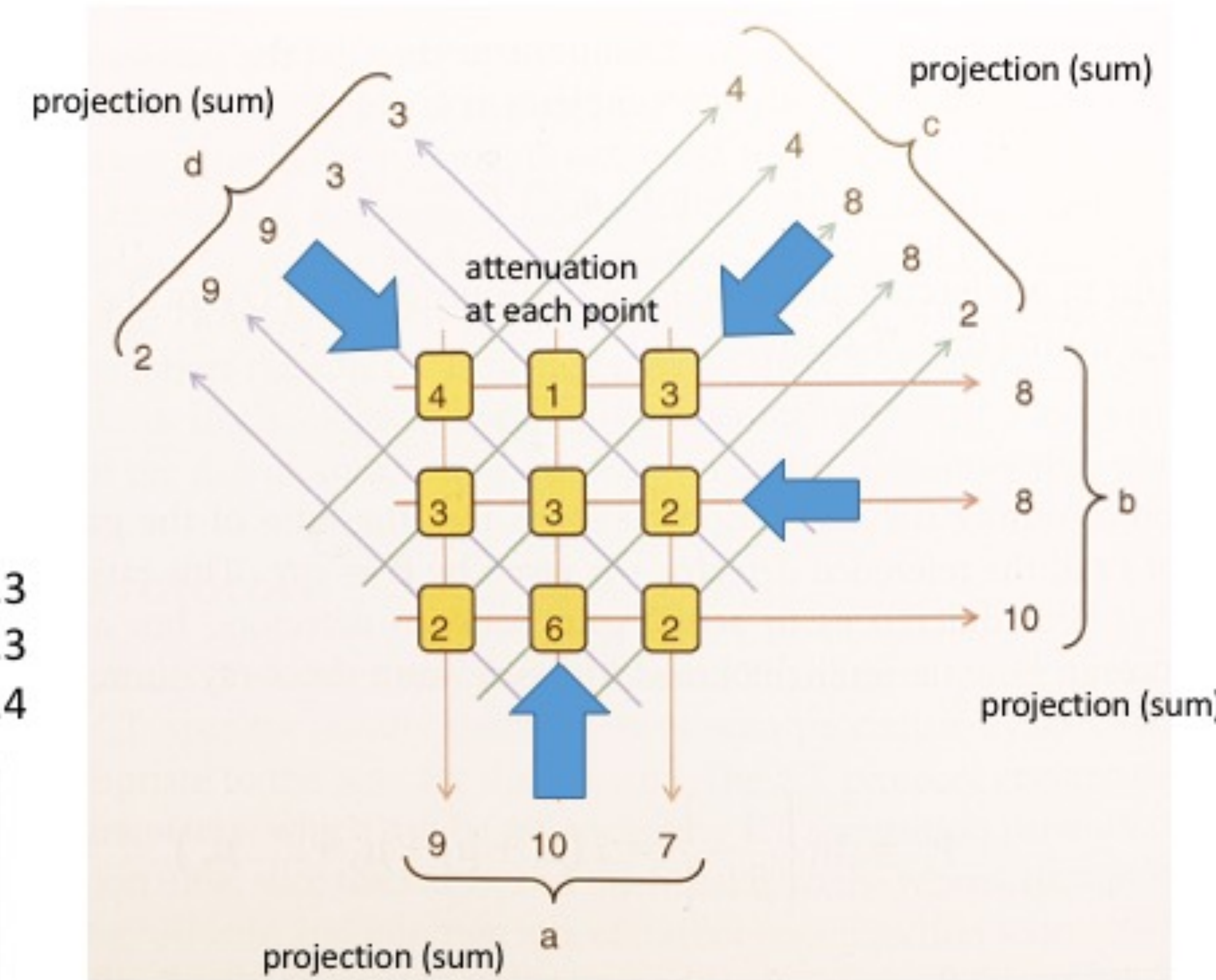
30	25	26
30	35	26
29	37	28

*9/26/4

2.6	2.2	2.3
2.6	3.0	2.3
2.5	3.2	2.4

Filtered Backprojection

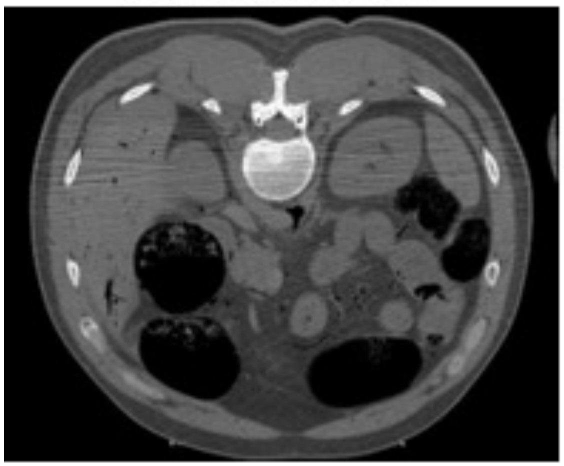
Unfiltered Backprojection



credits figure: ref 2

Backprojection:
effect of an increasing
number of projections

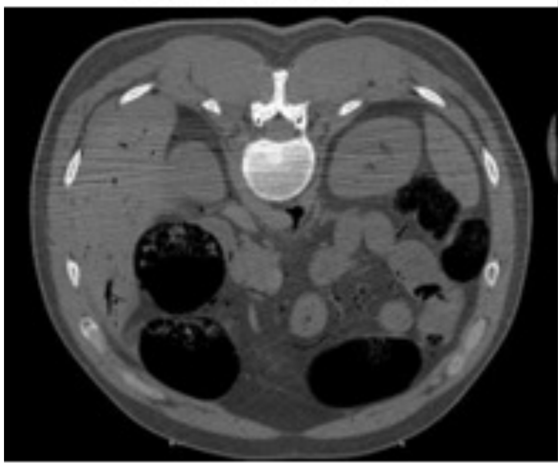
original (axial view)

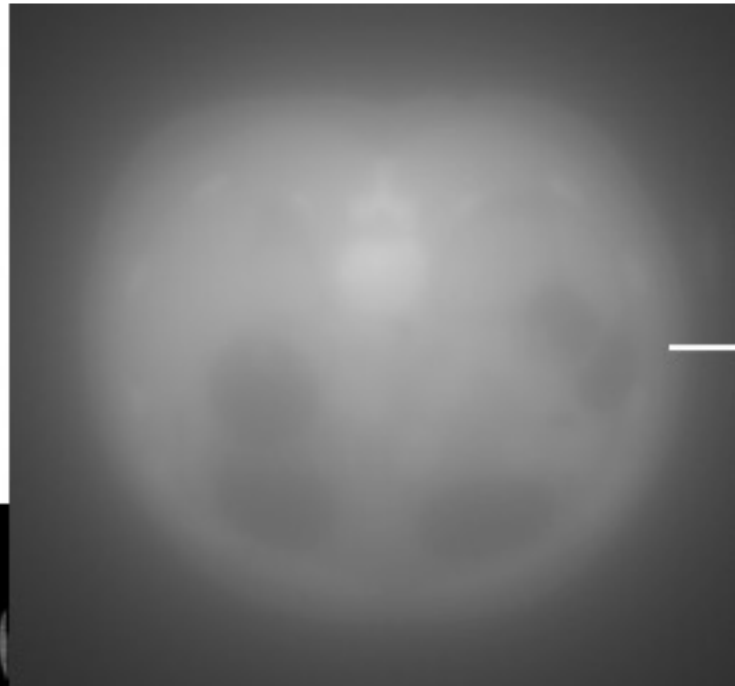


magic!

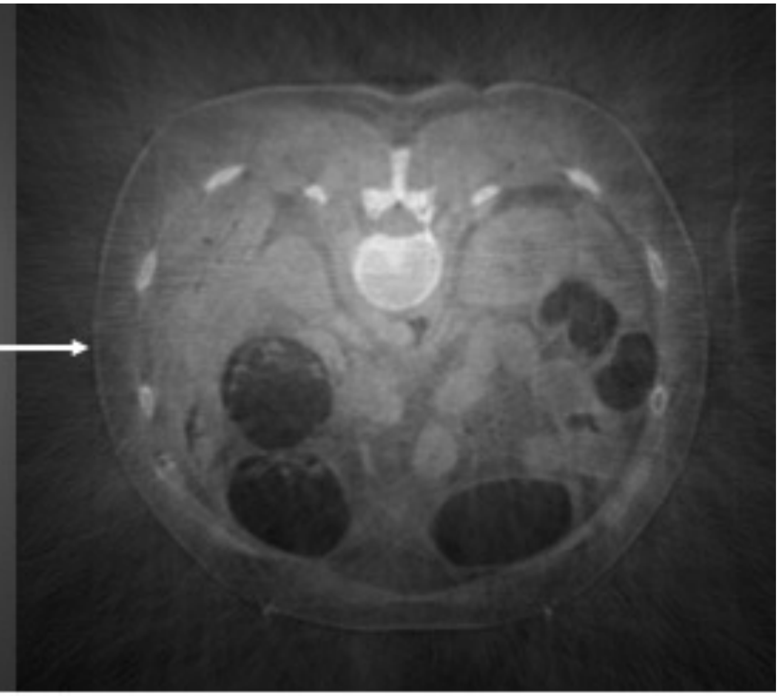
Deconvolution Filter

original (axial view)





backprojection

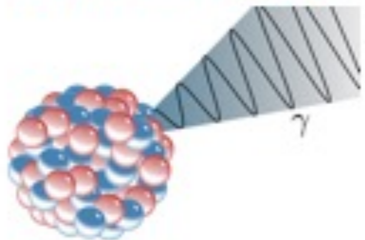


filtered backprojection

Source: http://www.perlproductions.at/index.php?choice=referenz&lang=en&id=15

2nd... gamma rays

- Paul Villard discovered the **gamma rays** in 1900 in his laboratory of rue d'Ulm in Paris.



- 3 years later Marie and Pierre Curie, together with Henri Becquerel, would receive the Nobel Prize for the discovery of **radioactivity**.
- Nowadays, in **nuclear medicine**, radio-pharmaceuticals are used to emit gamma rays into the body.
- The idea is to mark certain key molecules that reflect the activity in the study of diseases such as **Alzheimer's or epilepsy,** or evidence of the metabolism when evaluating an **oncological** treatment.

Nuclear Medicine

- Radioisotop

  What do we want:

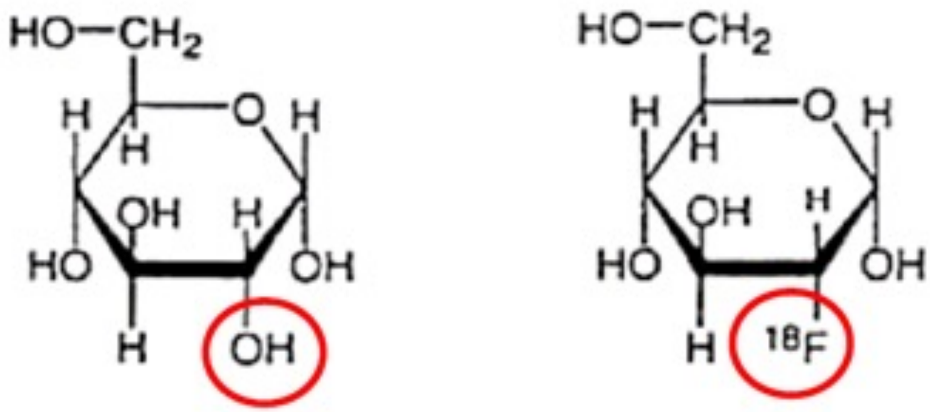
  - not too short and not too long half life time
  - only γ radiation (α & β would increase patient dose without gain for diagnosis)

Nuclide	Half life time	Decay	Energy
^{99m}Tc	6 h	γ	140 keV
^{201}Tl	73 h	γ	70 keV
^{123}I	13 h	γ	159 keV
^{18}F	110 min	$e^+, \gamma\gamma$	511 keV
^{11}C	20 min	$e^+, \gamma\gamma$	511 keV
^{13}N	10 min	$e^+, \gamma\gamma$	511 keV

credit: Dr. Castañeda

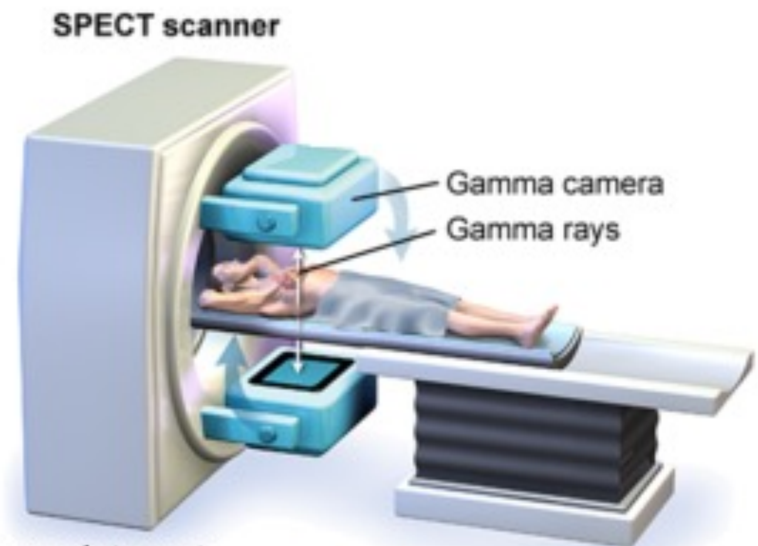
Nuclear Medicine

- **Radiopharmaceuticals**
  - The radioisotope has to be connected (labelling) to a pharmaceutical based on the organ specific question.
  - Radiopharmaceuticals should not disturb the process under investigation.
  - e.g. FDG (18F-Fluorodesoxyglucose) to analyze the Glucose metabolism

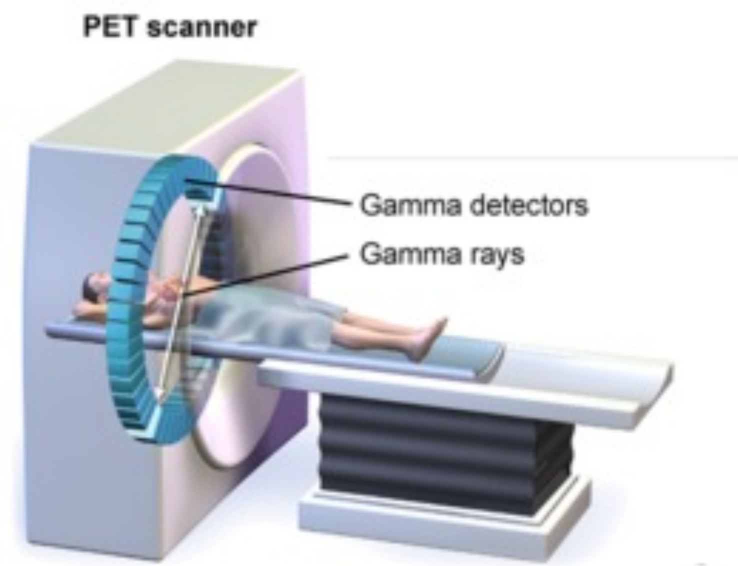


credit: Dr. Castañeda

Nuclear medicine

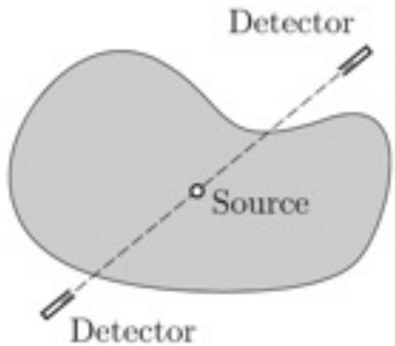


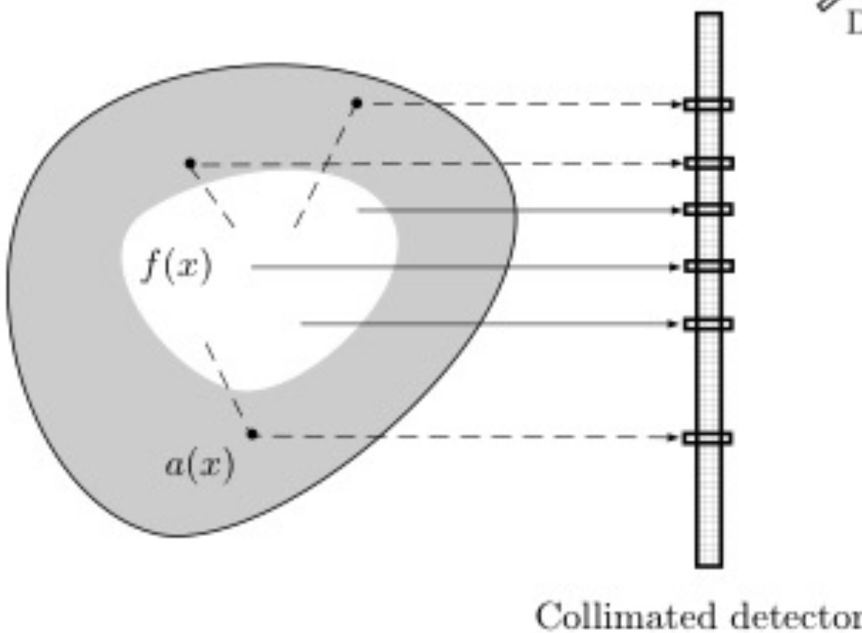
source: internet



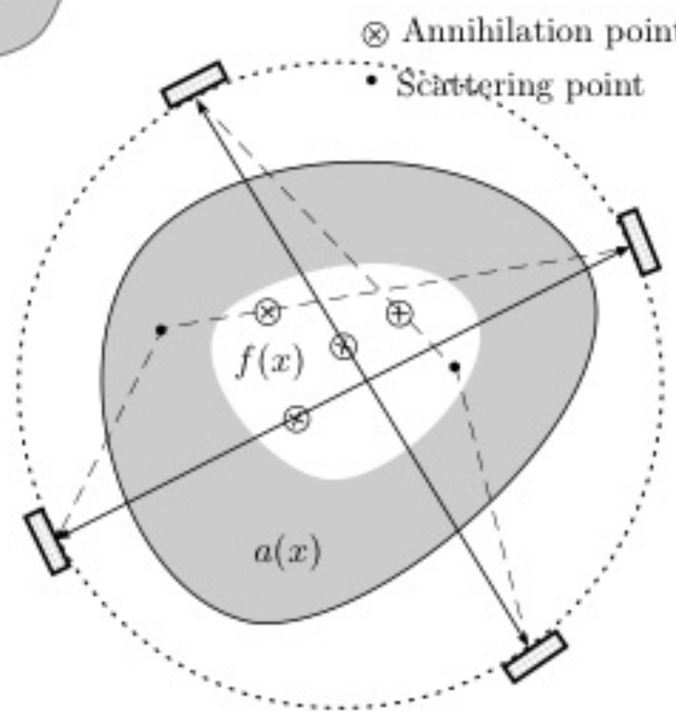
- monophotonic (SPECT) and biphotonic (PET) emission tomography the most extensively used in nuclear medicine

SPECT and PET





SPECT: Single photon emission computed tomography

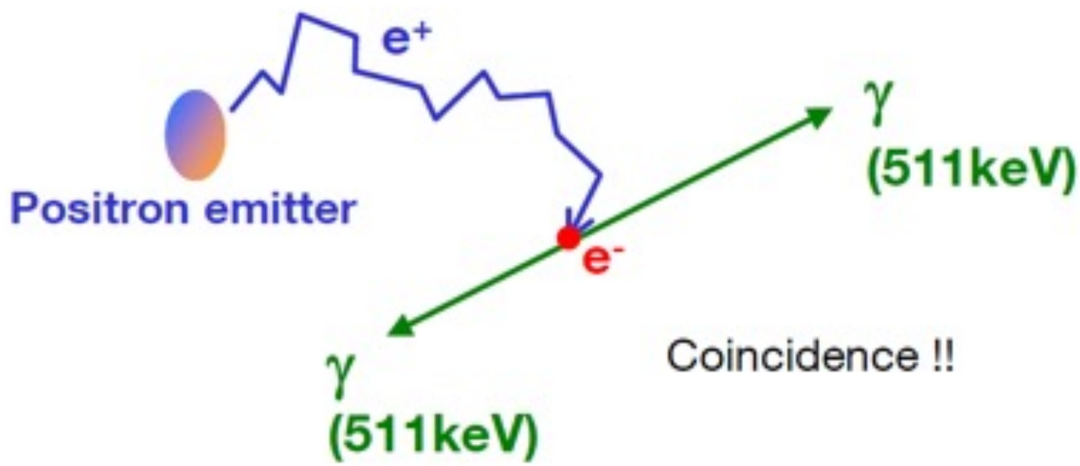


PET: Positron emission tomography

Nuclear Medicine

- **Positron Emission Tomography – PET**
  - Positron decay: $^{18}F \rightarrow {}^{18}O + e^{+} + \nu_e$

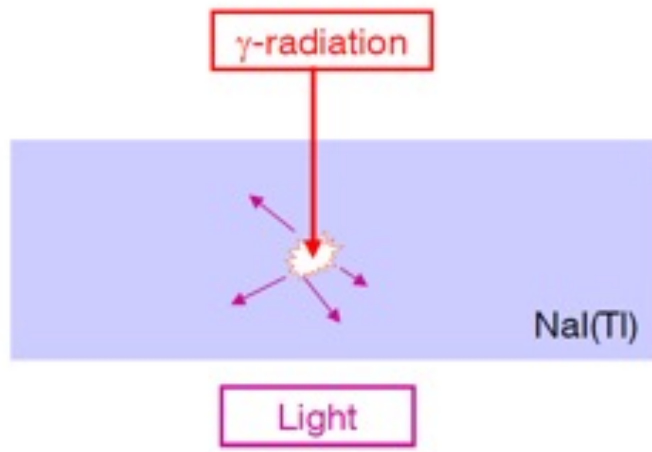
    $p \rightarrow n + e^{+} + \nu_e$



credit: Dr. Castañeda

Nuclear Medicine

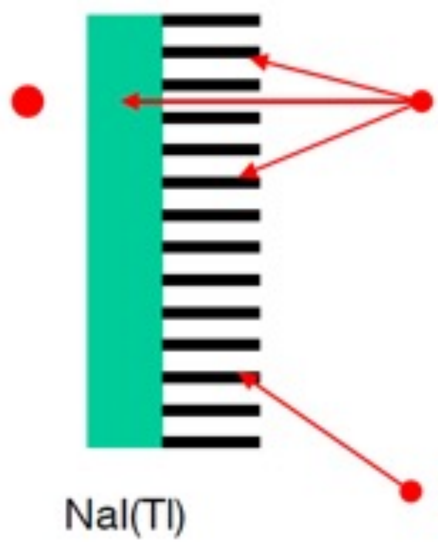
- **Radiation-Detection**
  - Scintillators, e.g. NaI(Tl) – Thallium doped Sodium Iodide



credit: Dr. Castañeda

Nuclear Medicine

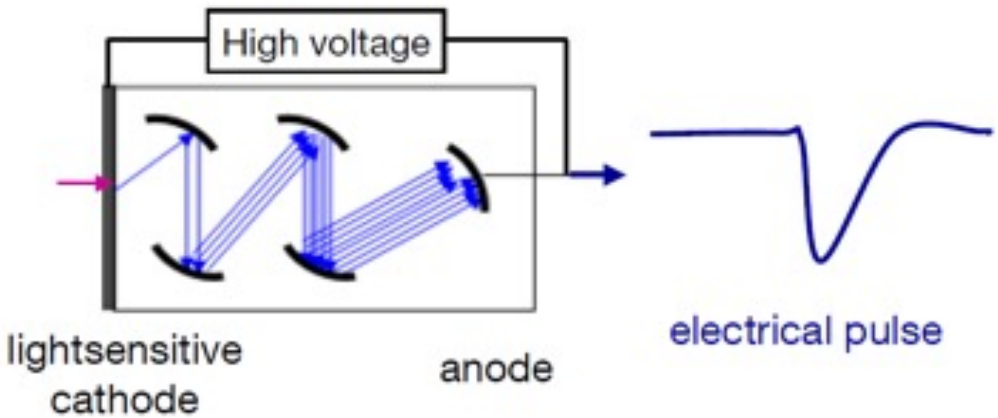
- Gamma camera



credit: Dr. Castañeda

Nuclear Medicine

- Photo multiplier
  - multiply the signal produced by incident
  - light by as much as 100 million times



γ-energy ~ light ~ pulse height

credit: Dr. Castañeda

Example: SPECT scan of the brain

- primarily used to view how blood flows through arteries, veins and capillars;
- can detect reduced blood flow (more sensitive than MRI or CT);
- tracer stays in blood stream rather than being absorbed by tissues;
- cheaper and more readily available than higher resolution PET scans.

(source: www.mayfieldclinic.com)

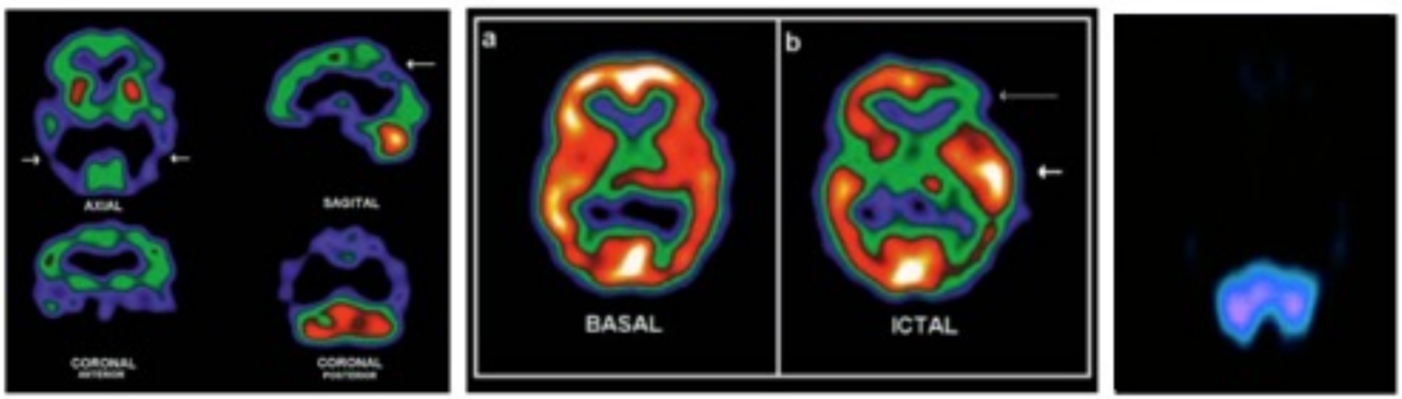
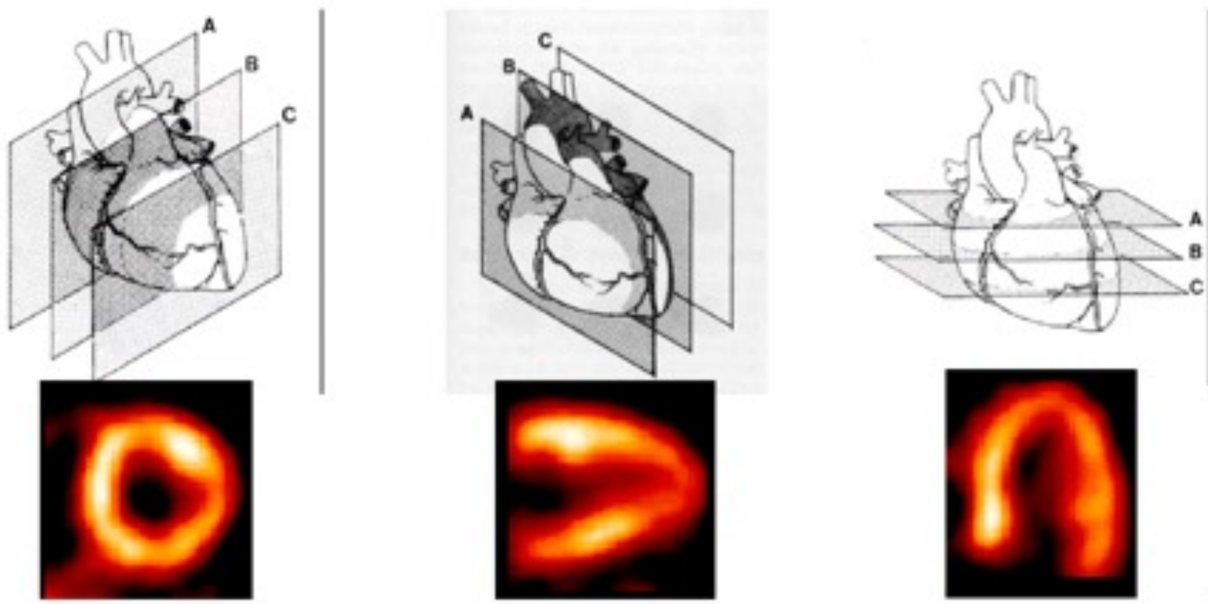


Figure: Left: Alzheimer desease patient with low blood perfusion (arrows);
Right: different blood flow before and during an epileptic seizure (arrows).
(source: Quintana, J.-C., Neuropsiquiatría: PET y SPECT . Rev. chil. radiol. 2002)

Nuclear Medicine

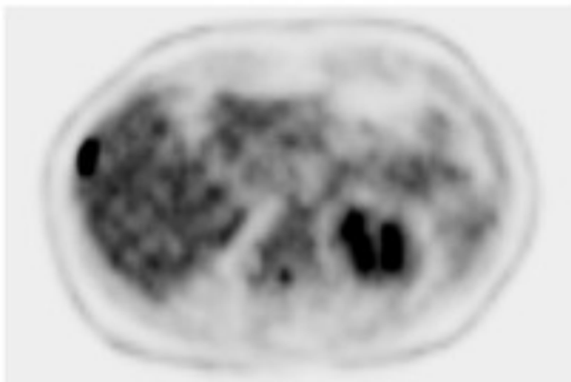
- **Single Photon Emission Tomography - SPECT**
  - Myocard SPECT – ^{99m}Tc MIBI (methoxyisobutylisonitrile) myocardial perfusion in cardial rest and stress for diagnosis and staging (prognostic factor) of coronar disease

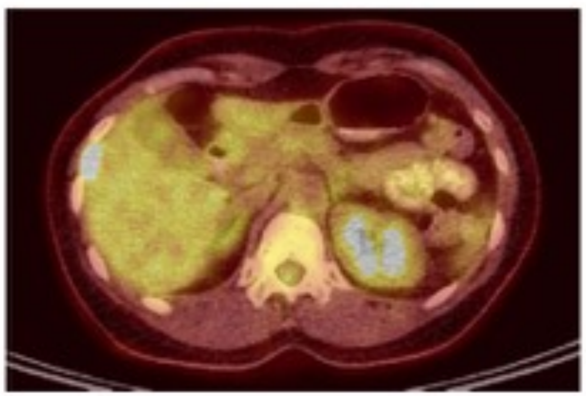


credit: Dr. Castañeda

Nuclear Medicine

- **Multimodality – PET/CT**





credit: Dr. Castañeda

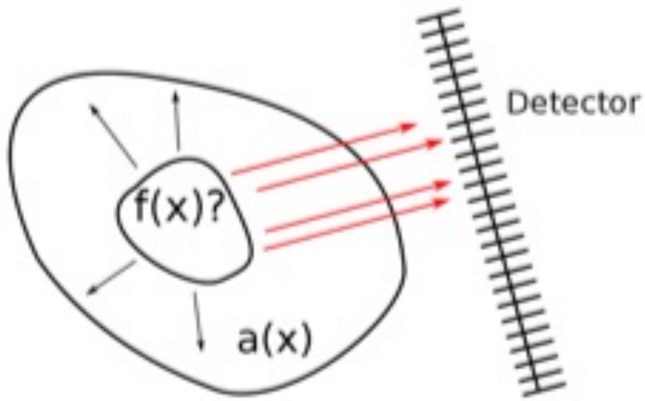
Definition 1 (Attenuated Radon Transform)

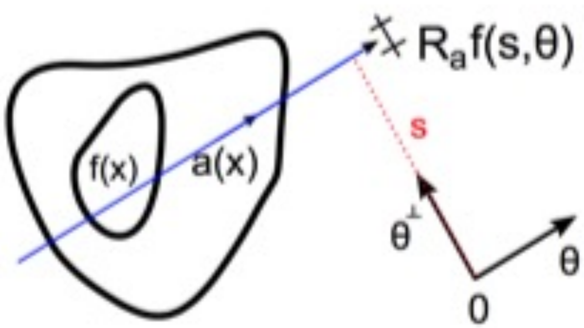
For $a, f \in L^1(\mathbb{R}^2)$, we define the Attenuated Radon Transform as

$$R_a f(s,\theta) = \int_{\mathbb{R}} f(s\theta^\perp + t\theta) e^{-\int_0^\infty a(s\theta^\perp + t\theta + \tau\theta)d\tau} dt \quad s \in \mathbb{R}, \theta \in S^1$$

attenuation

where $\theta^\perp = (-\theta_2, \theta_1)$ for $\theta = (\theta_1, \theta_2)$. (same notation if f depends on (x, θ))





SPECT Inverse Problem. Reconstruction problem.

From the knowledge of $R_a f(s,\theta)$, for all $s \in \mathbb{R}^+, \theta \in S^1$, we want to recover $f(x), \forall x \in \mathbb{R}^2$.

Inversion of the Attenuated Radon transform

Theorem 1 (Inversion of the Attenuated Radon transform, Arbuzov, Bukhgeim and Kazantsev 1997, Novikov 2001)

Let $a, f \in \mathcal{C}^1(\mathbb{R}^2)$, a known. The following inverse formula holds pointwise

$$f(y) = \frac{1}{4\pi} \text{ Re div} \int_{S^1} \theta e^{B\,a(y,\theta^\perp)} (e^{-h} H e^h R_a f)(y \cdot \theta, \theta) d\theta,$$

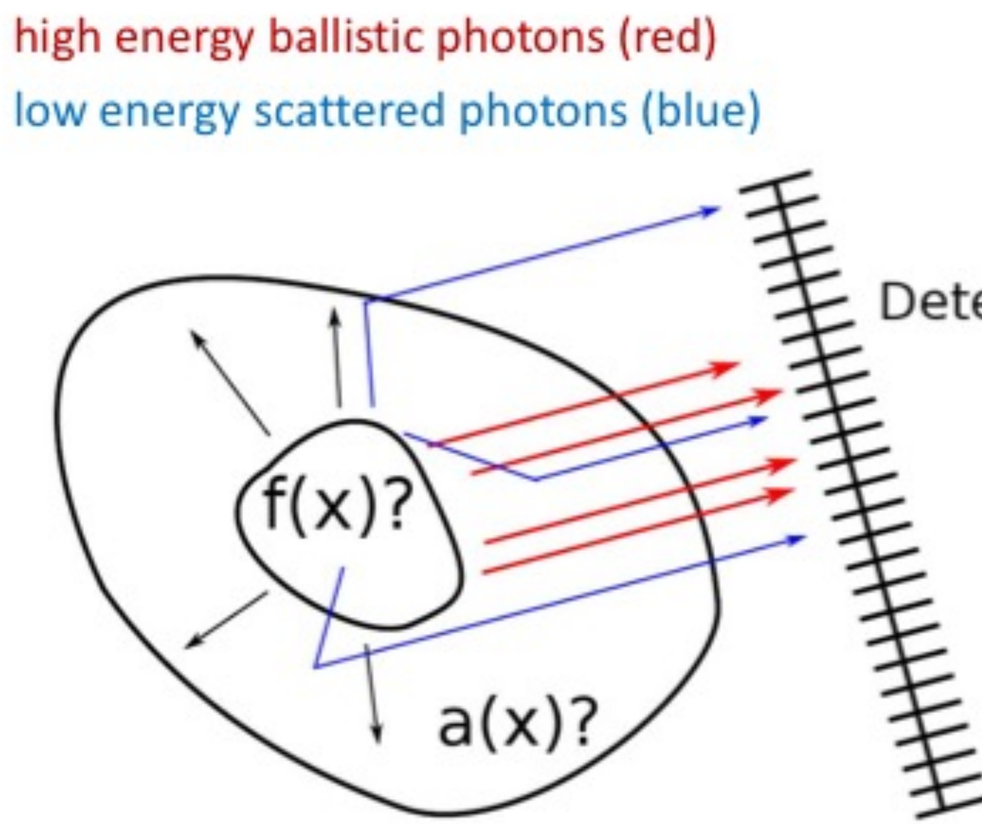
where $h(s,\theta) = \frac{1}{2}(I + iH)Ra(s,\theta)$, H is the Hilbert transform in the space variable s and $B\,a(y,\theta) = \int_0^\infty a(y + t\theta)dt$ is the beam transform.

Theorem 2 (Natterer 2001)

*Novikov's inversion formula can be implemented numerically with a controlled approximation error (*Filtered Backprojection algorithm *for the AtRT).*

The identification problem:

simultaneous reconstruction of source and attenuation



- *Simultaneous source and attenuation reconstruction in SPECT using ballistic and single scattering data.*

M. Courdurier, F. Monard, A. Osses and F. Romero. Inverse Problems, 2015

The Identification Problem

On the reconstruction of f and a from the knowledge of the $R_a f$.

non-uniqueness of the pair f, a:

J. Boman 1993, D.C. Solmon 1995, G. Bal and A. Jollivet 2011, P. Stefanov 2013.

for particular cases of source functions:

Natterer 1981, 1983, J. Boman 1993, G. Bal and A. Jollivet 2011, A. L. Bukhgeim 2011.

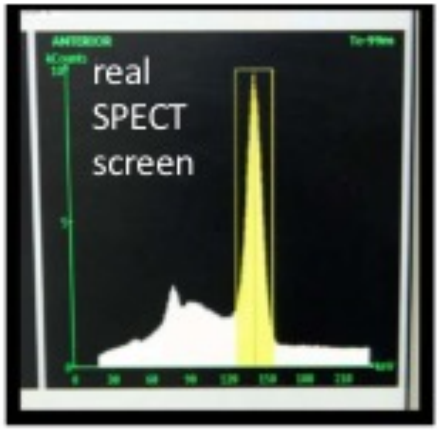
linearized problem, localized uniqueness and stability results:

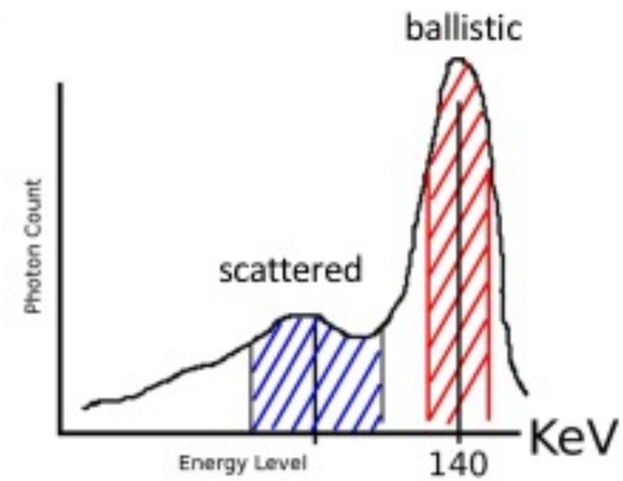
G. Bal and A. Jollivet 2011, P. Stefanov 2013.

numerical aspects and reconstructions:

Y. Censor, D. E. Gustafson, A. Lent and H. Tuy, 1979, S. H. Manglos and T. M. Young 1993, R. Ramlau and R. Clackdoyle. 1998, V. Dicken. 1999, A. V. Bronnikov. 1999, 2000, D. Gourion and D. Noll. 2002, H. Zaidi and B. Hasegawa 2003, S. Luo, J. Qian and P. Stefanov 2014.

Main clinical idea: two measurements, two unknowns...





Energy distribution at one detector
(courtesy PUC-Hospital, Chile)

ballistic photons

$$\mathcal{A}_0(x,\phi) := \int_{-\infty}^{\infty} f(x+t\phi)e^{-\int_t^{\infty} a(x+s\phi)ds}dt$$

$$\mathcal{A}_1(x,\phi) := C\int_{-\infty}^{\infty} a(x+t\phi)M_{\varphi}[a,f](x+t\phi,\phi)e^{-\int_t^{\infty} a(x+s\phi)ds}dt$$

scattered photons (once)

ballistic photons (scattered)

$$M_{\varphi}[a,f](x,\phi) := \int_{S^1} \varphi(\phi\cdot\phi')u_0(x,\phi')d\phi'$$

scattering kernel

Radiative Transfer Equation (RTE)

RTE: isotropic media and almost angularly uniform scattering

$$\phi \cdot \nabla_x u(x,\phi) + a(x)u(x,\phi) = f(x) + k(x)\int_{S^1} \varphi(\phi\cdot\phi')u(x,\phi')d\phi'$$

$$\lim_{t\to\infty} u(x - t\phi, \phi) = 0,$$

- $u(x,\phi) : \mathbb{R}^2 \times S^1 \to \mathbb{R}$, intensity of photon in the spatial position $x \in \mathbb{R}^2$ traveling in direction $\phi \in S^1$.
- $\varphi = 1 + \delta\varphi : [-1,1] \to \mathbb{R}$ almost uniform angular scattering kernel.
- $a(x) : \mathbb{R}^2 \to \mathbb{R}$, total attenuation map (unknown).
- $f(x) : \mathbb{R}^2 \to \mathbb{R}$, radioactive source distribution (unknown).
- $k(x) : \mathbb{R}^2 \to \mathbb{R}$, spatial component of the scattering kernel (unknown).

Radiative Transfer Equation (RTE) (more assumptions)

Assuming separable scattering kernel plus φ known (e.g. Klein-Nishina formula), $(k(x,\phi',\phi) = k(x)\varphi(\phi'\cdot\phi))$,

$$\phi \cdot \nabla_x u(x,\phi) + a(x)u(x,\phi) = f(x) + k(x)\int_{S^1} \varphi(\phi'\cdot\phi)u(x,\phi')d\phi'$$

- $a(x) : \mathbb{R}^2 \to \mathbb{R}$, total attenuation map.
- $k(x) : \mathbb{R}^2 \to \mathbb{R}$, spatial part of the scattering kernel.

we assume $\varphi(\cdot) = 1+$small (quite far from true for K-N at 140 KeV).

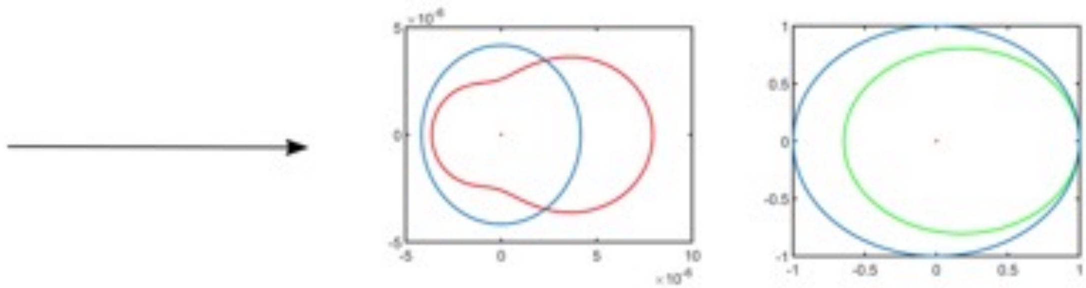
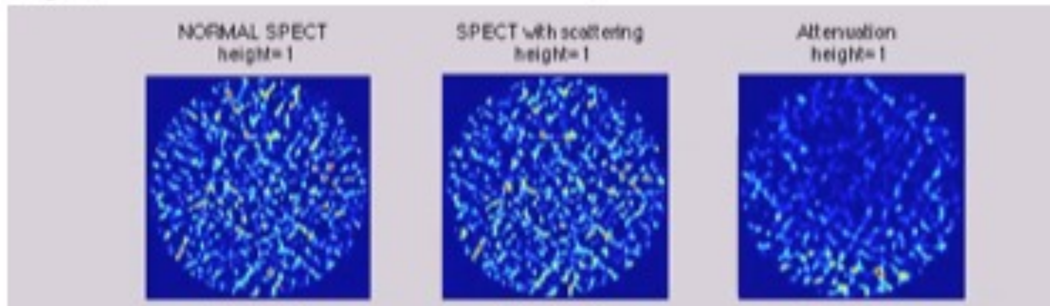


Figure: Klein-Nishina formula: a) (Red) Distribution of direction after scattering. b) (Green) Ratio of outgoing/incoming energy after scattering.

This leads to **theoretical results of uniqueness and stability**

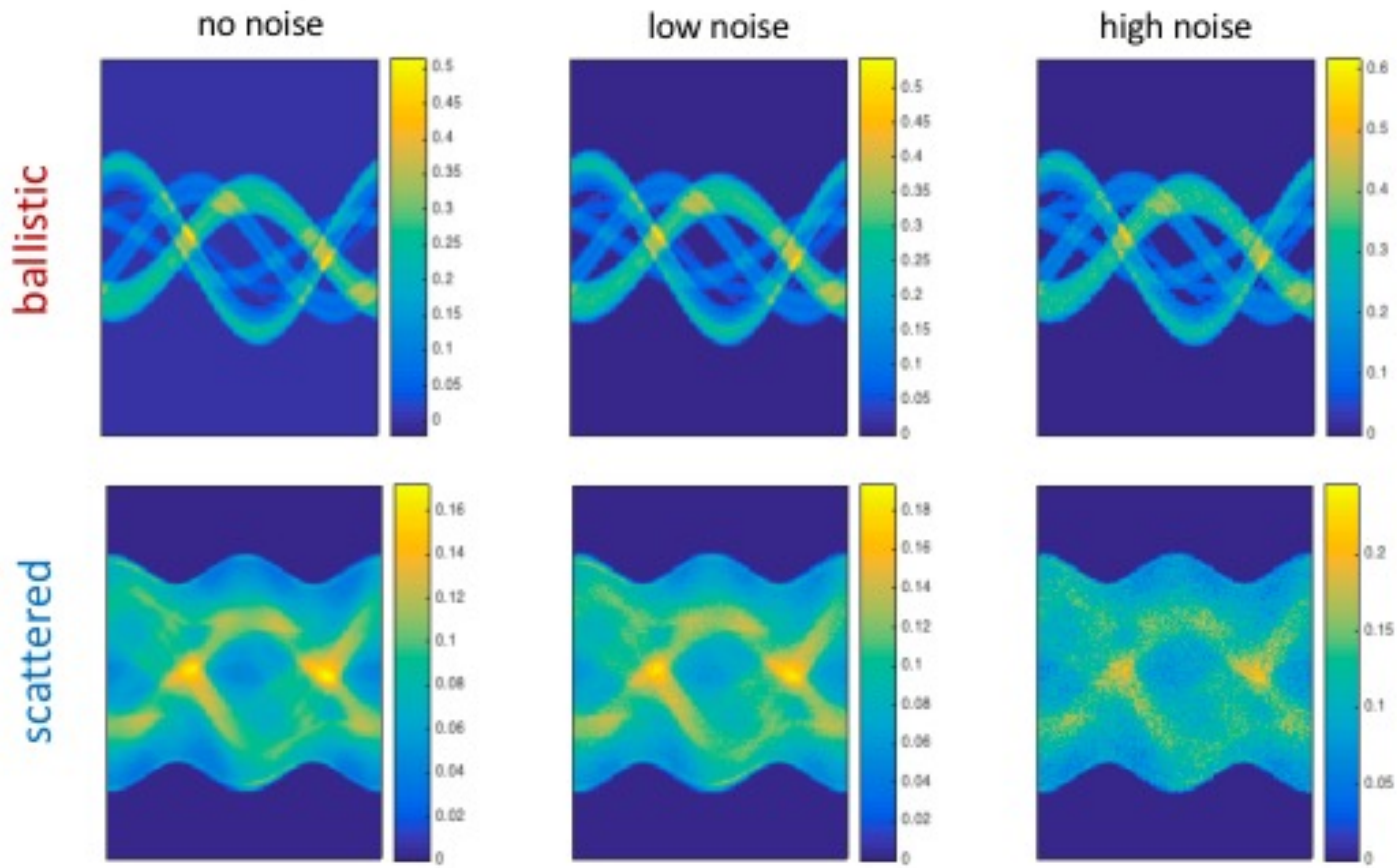
Now... for the reconstruction... **numerical methods**:

- case of isotropic scattering (inversion formula)
  - Neumann series
  - Fixed point
  - Newton method

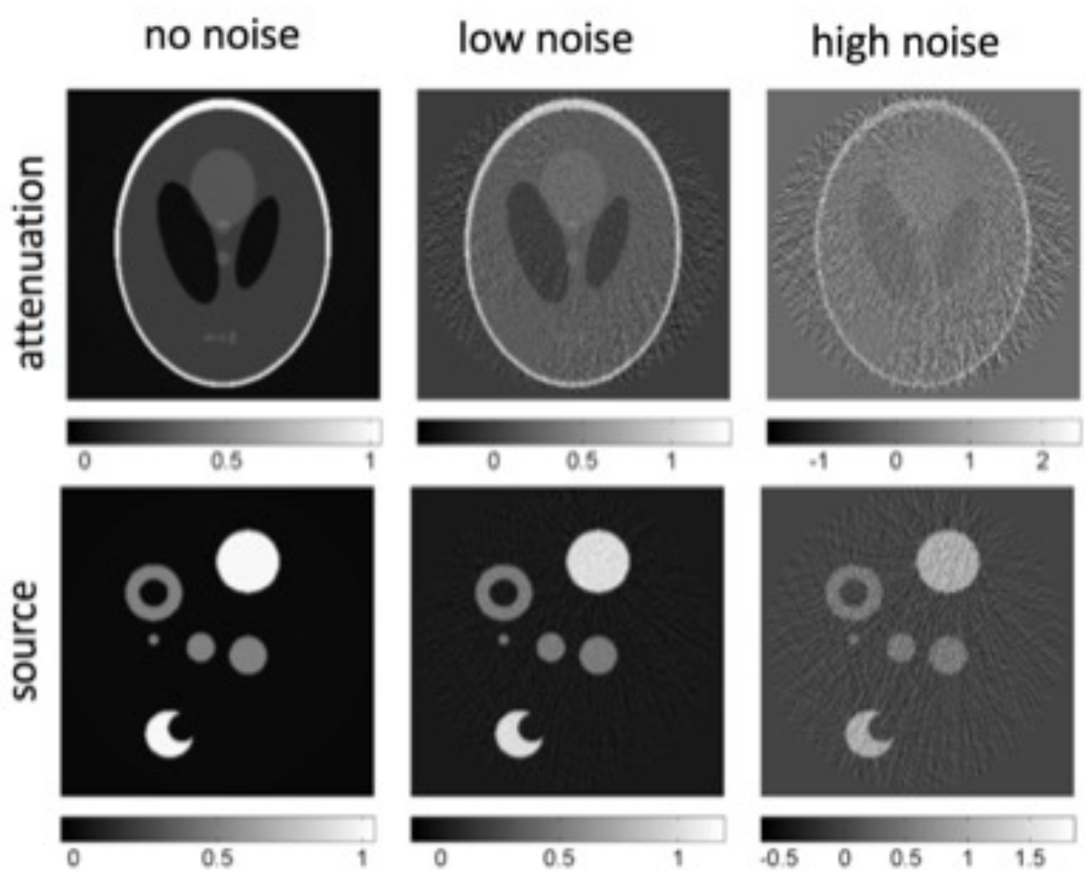


- case of near isotropic scattering (near Klein-Nishina)
  - Algebraic reconstruction

SPECT measurements

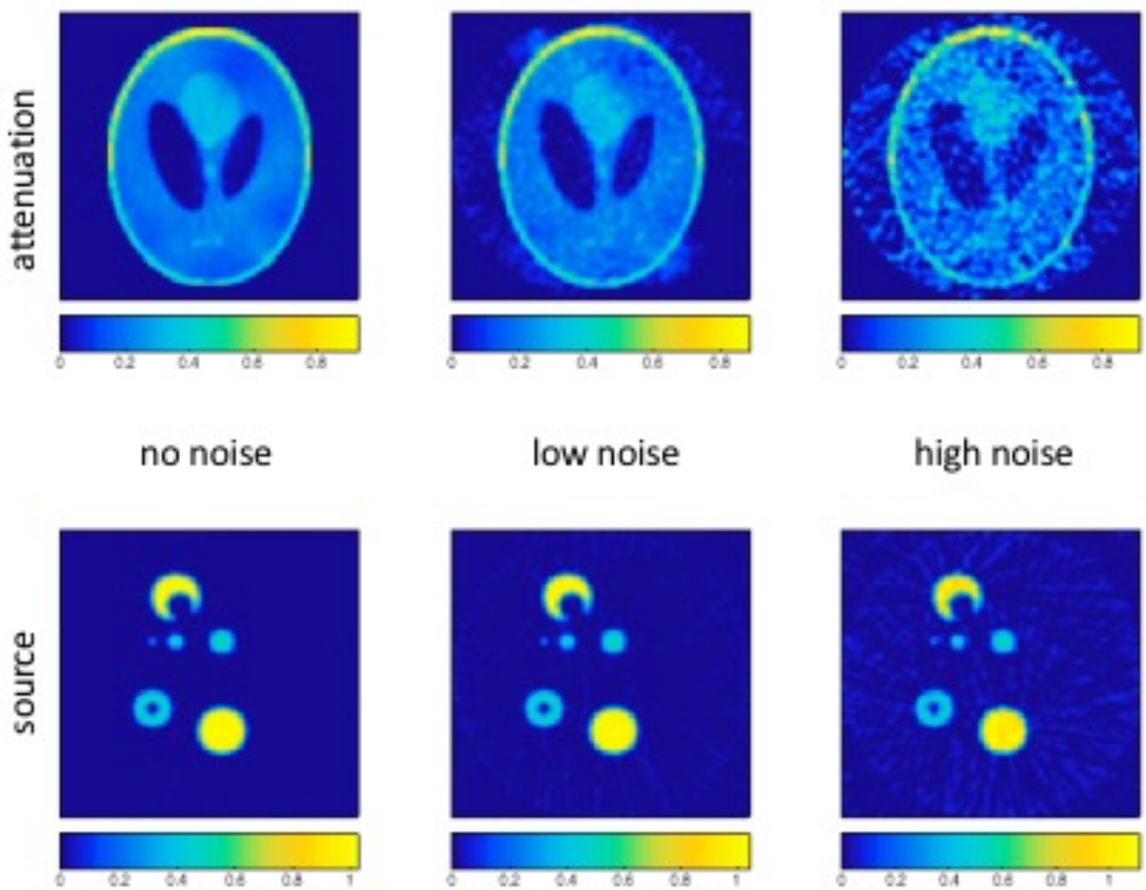


simultaneous source and attenuation reconstruction (Newmann series and inverse formula)



- *Simultaneous source and attenuation reconstruction in SPECT using ballistic and single scattering data*. M. Courdurier, F. Monard, A. Osses and F. Romero. Inverse Problems, 2015

simultaneous source and attenuation reconstruction (Algebraic reconstruction)



- *Algebraic reconstruction of source and attenuation in SPECT using first scattering measurements*. E. Cueva, A. Osses, J.-C. Quintana, C. Tejos, M. Courdurier, P. Irarrázaval, Trends in Mathematics Series, Birkhäuser, 2018.

3D reconstruction experiment with real data. Experiment

Figure: Phantom used in the experiment.

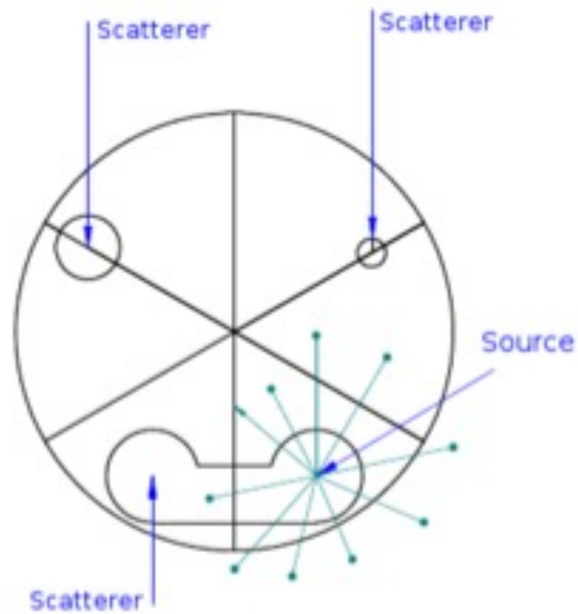


Figure: Schematic of the phantom used.

(Fixed point reconstruction)

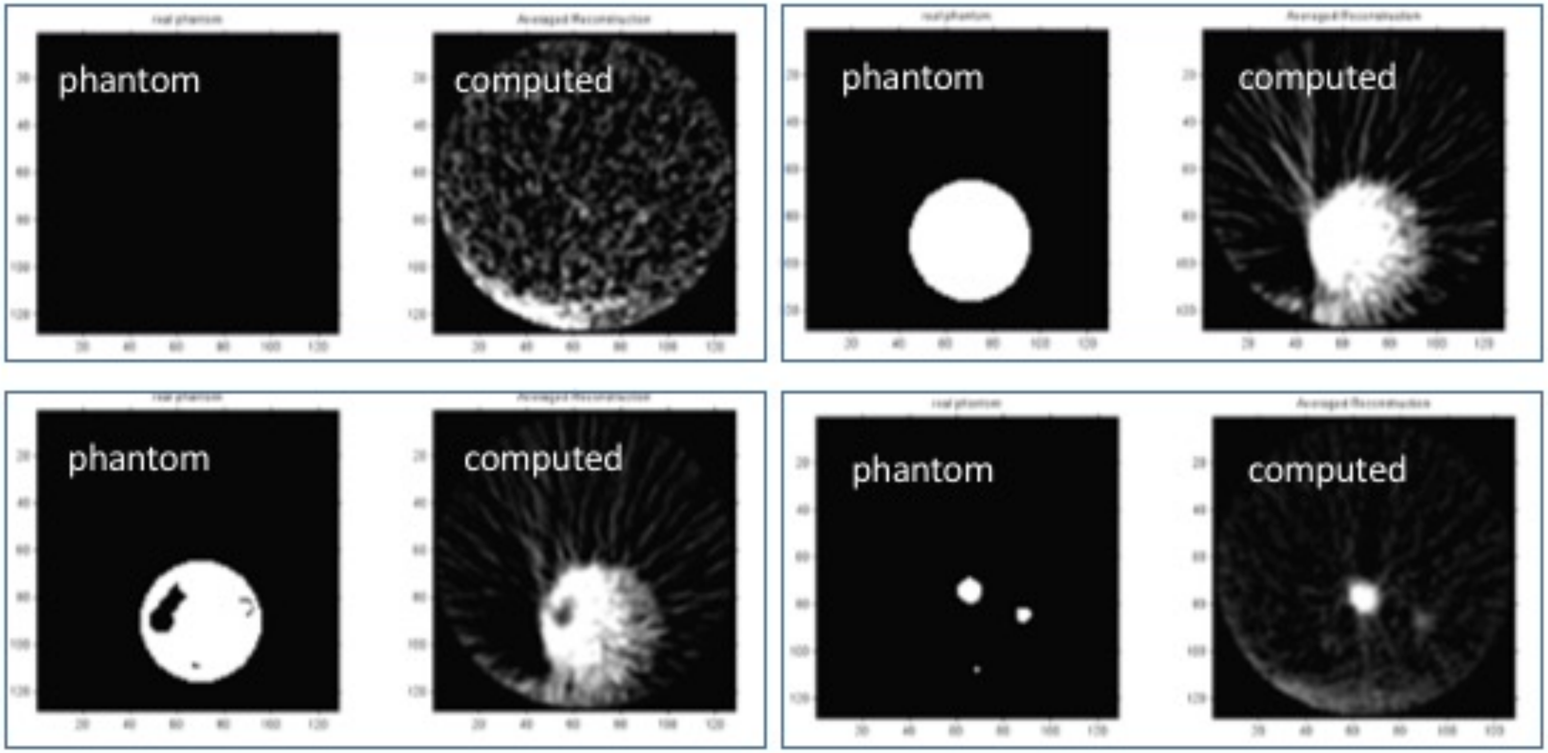


Figure: Attenuation reconstruction averaged at four different heights. Comparison of real phantom versus reconstruction.

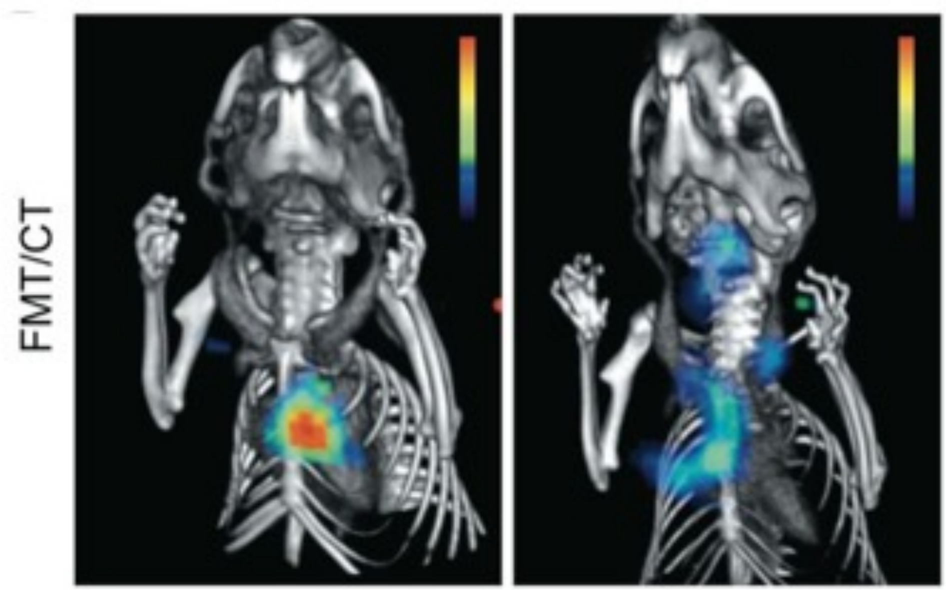
non published results

4.- Fluorescence molecular tomography FMT

currect research topic: mathematics of light-sheet microscopy

Fluorescence molecular tomography FMT

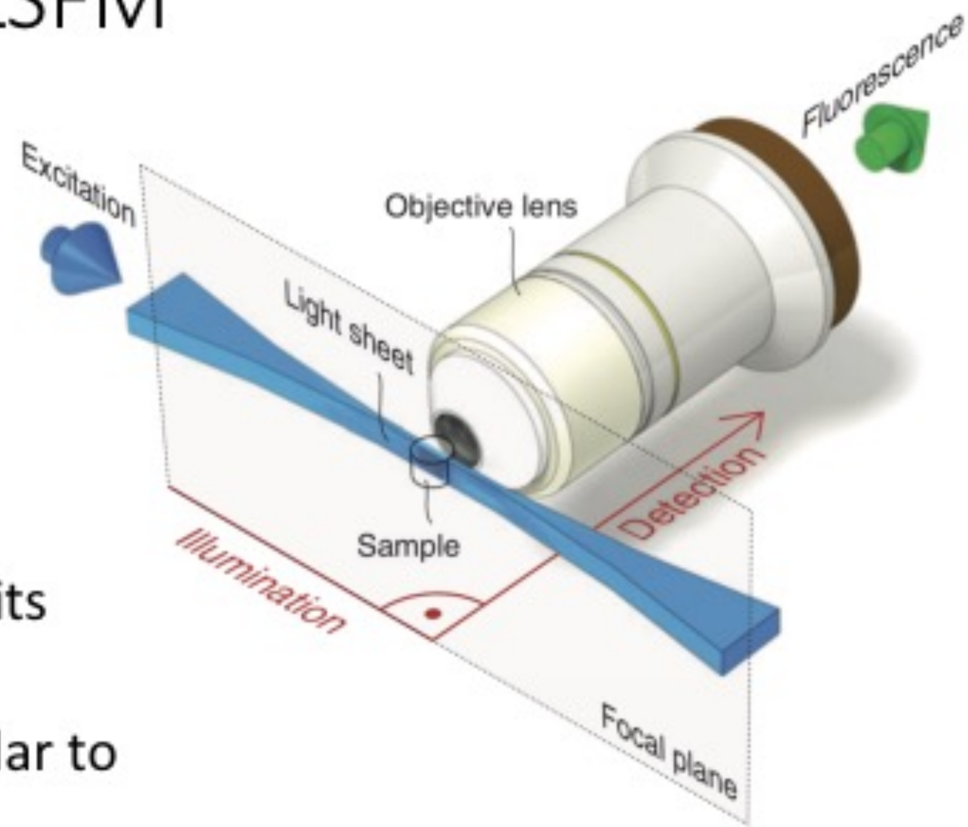
- fluorescent biochemical markers are injected in a tissue
- they are excited using an external light source
- after excitation the markers emit a near infrared light that can be measured outside the tissue
- this emmited light used to recover the distribution of the markers inside the tissue



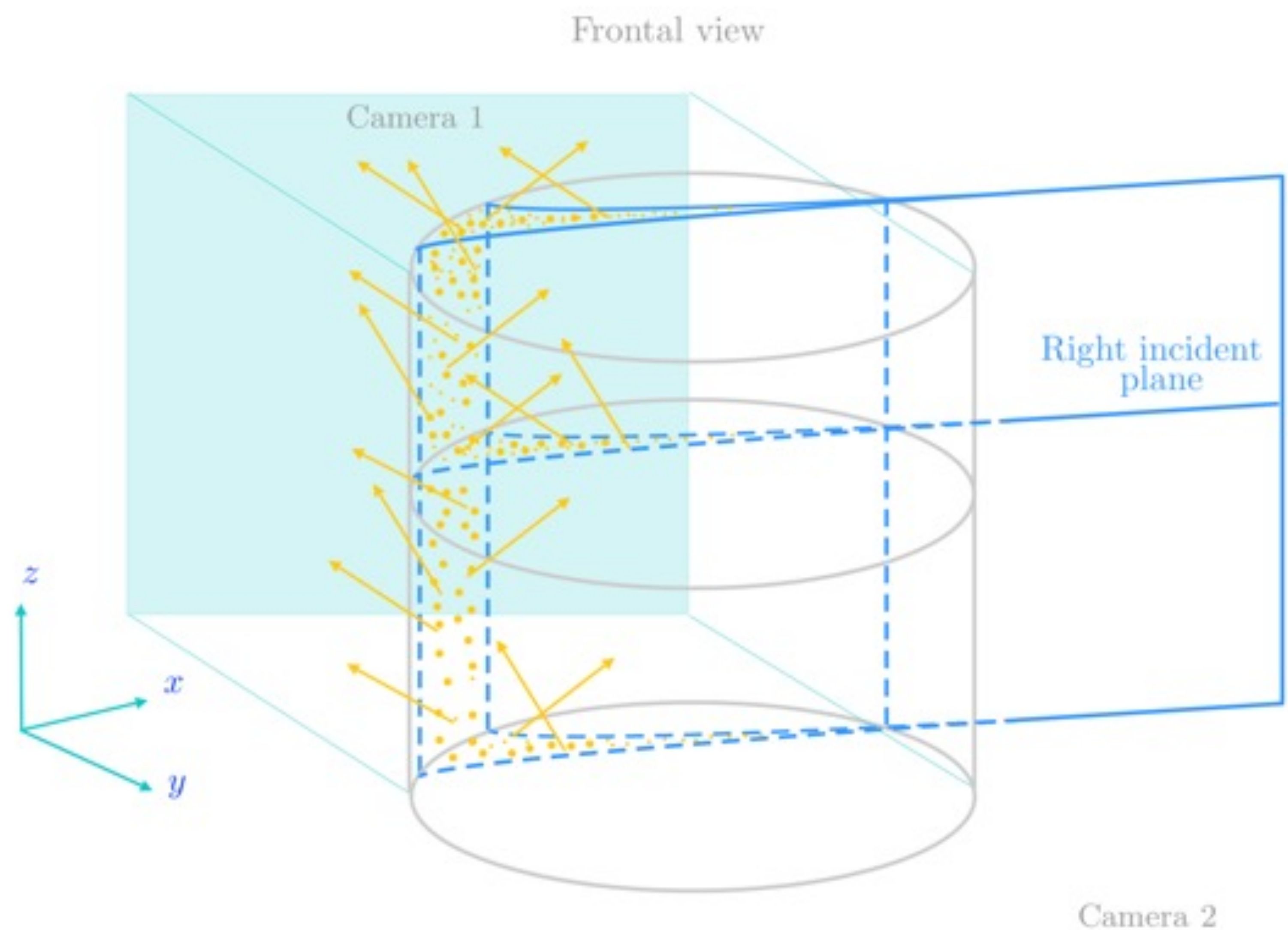
credit: H. Sager et al., Science Translational Medicine (2016)

light-sheet microscopy LSFM

- a thin laser illuminates by planes an optically near-transparent tissue
- the tissue has been labeled with a fluorophore
- these illumination sheets excite fluorescence in the sample
- this is detected with a camera as it exits the object
- the light-sheet planes are perpendicular to the axis of the fluorescence camera detection.

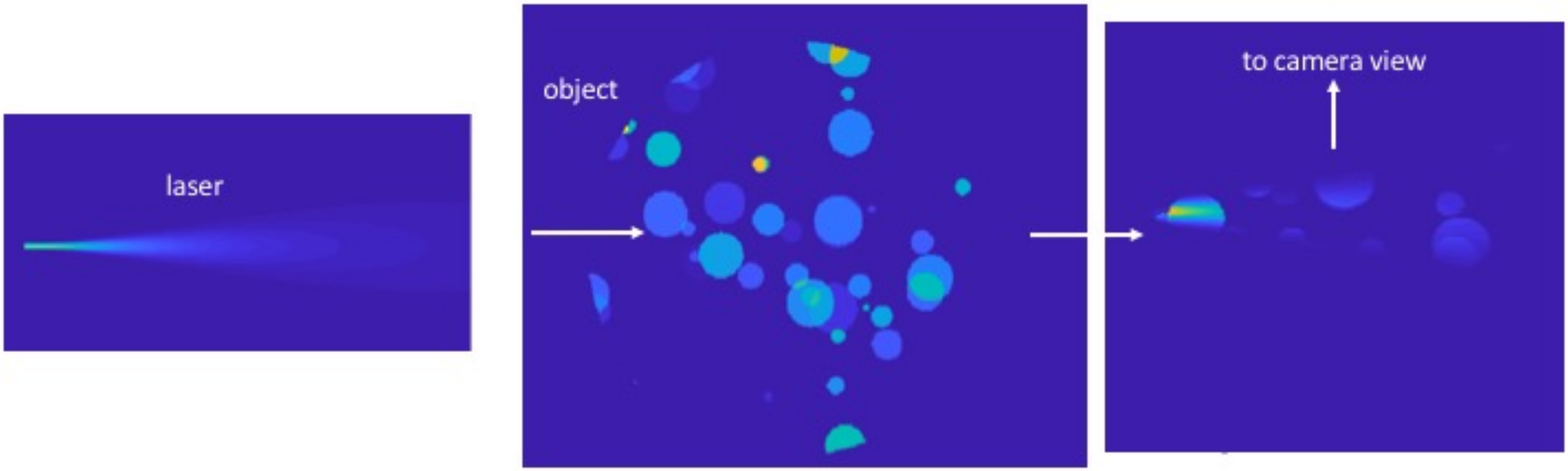


credit: J. Huisken and D. Y. Stainier, Development (2009)


Frontal view
Camera 1
Right incident plane
z
x
y
Camera 2

The model

- illumination step: Fermi-Eyges Pencil beam
- nonlinear gaussian spread

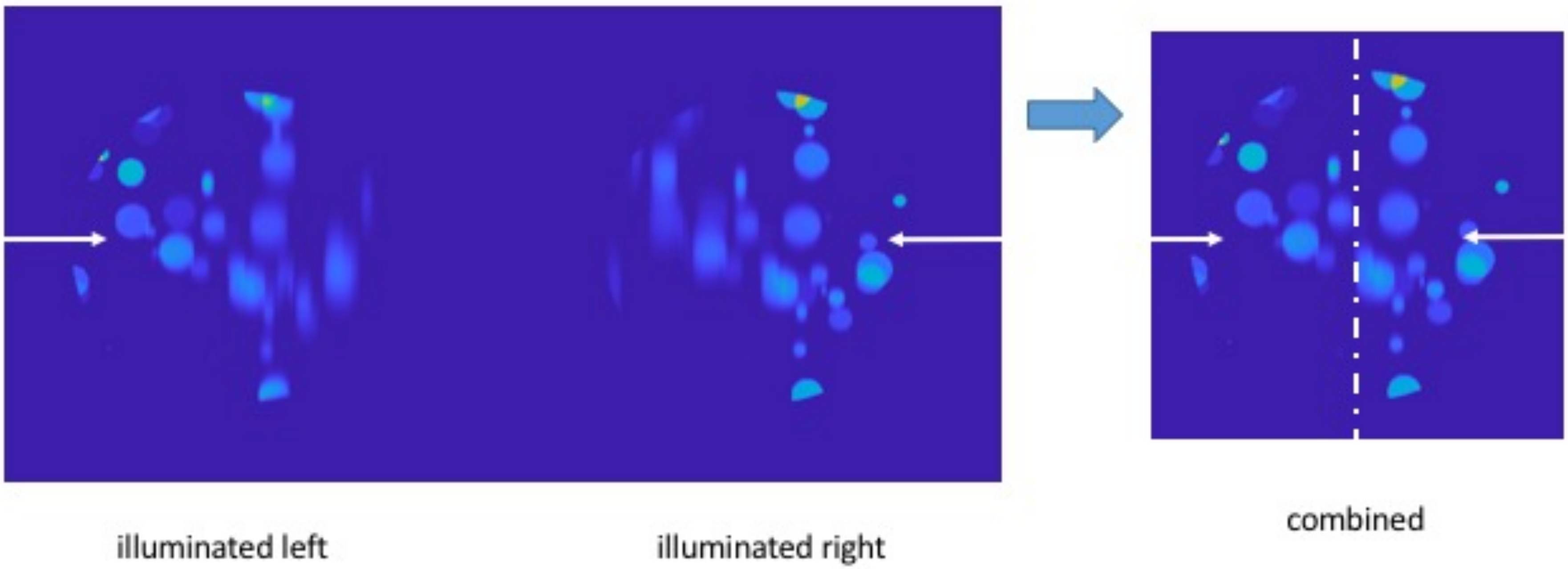


$$v_{y_0}(x,y) = \exp\left(-\int_{-R}^{x} a(s,y_0)ds\right)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(y-y_0)^2}{2\sigma^2}\right)$$

Ref: E. Cueva, A. Osses, M. Courdurier, V. Castañeda. Mathematical modelling for the Light Sheet Fluorescence Microscopy image reconstruction, in preparation.

The model

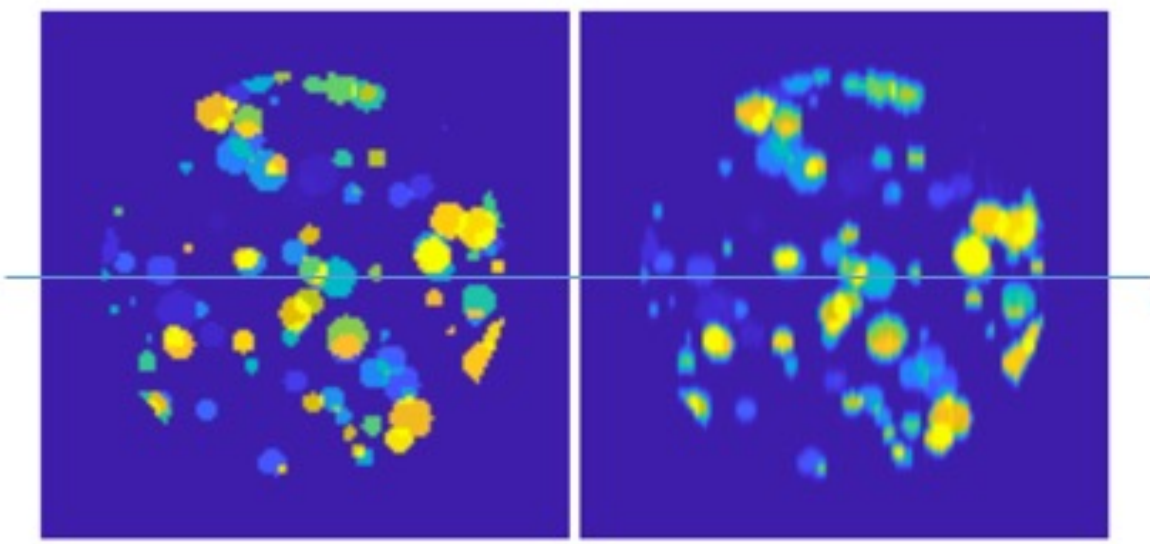
- detection step: attenuated rays transform
- weighted X-ray transform or convolution



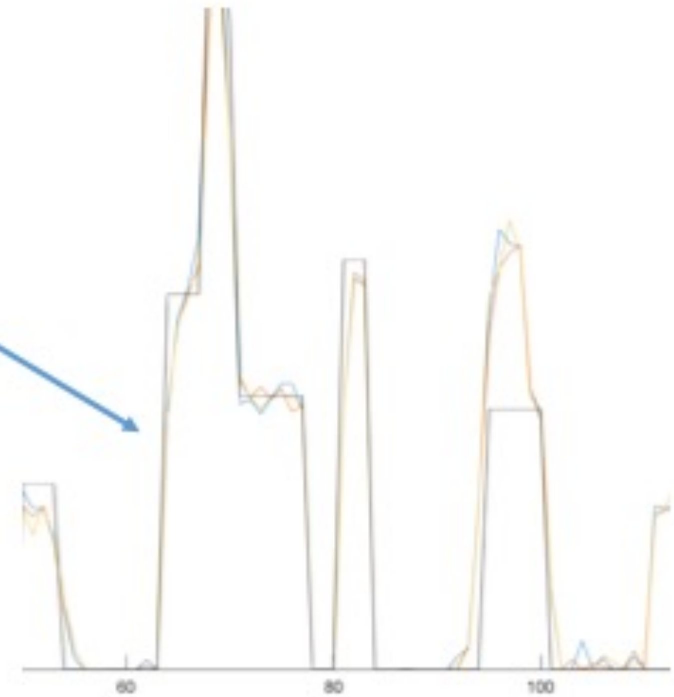
$$p(x, y_0) = \int_{\mathbb{R}} \mu(x,y) v_{y_0}(x,y) \exp\left(- \int_y^{\infty} a(x,t)dt \right) dy$$

Algebraic reconstruction

$$M\boldsymbol{\mu} = \boldsymbol{b}$$



real (syntetic) reconstructed



Second part (wave phenomena)

2.- magnetic resonance

current research topic: enhanced phase contrast technique for MRI-4DFlow adquisition and applications to non-invasive pressure and blood flow estimation

imaging the body... magnetism... MRI

- the so-called **Magnetic Resonance Imaging** (MRI).
- Strong magnetic fields are induced in the body with suitable forms that force the **spin of water hydrogen nuclei** into the body, to obtain:
  - images of incredible resolution inside the body
  - the speed of blood in an artery or
  - the displacement of the heart when barking
- The **first MRI machine** for human patients was built and patented in 1972 by the American physician Raymond Damadian.

Magnetic Resonance

Bloch's mathematical model
of magnetization

$$\frac{\partial \vec{M}}{\partial t} + (\vec{v} \cdot \nabla)\vec{M} = \gamma \vec{M} \times \vec{B} - \frac{M_x \hat{x} + M_y \hat{y}}{T_2} - \frac{M_z - M_0}{T_1}\hat{z}$$

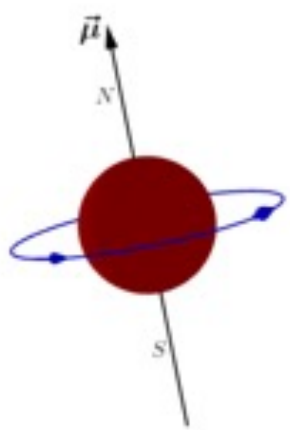
transport

spins
precesion

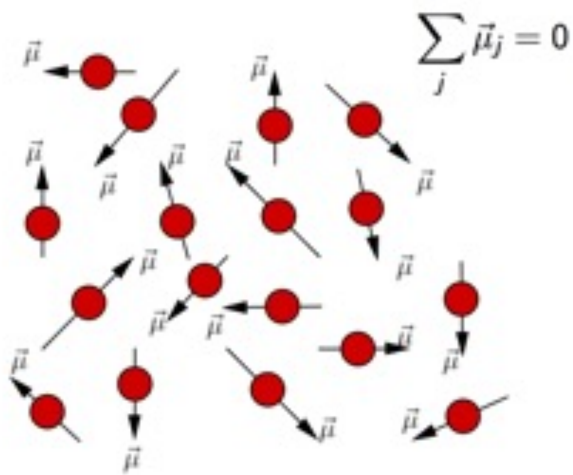
transverse
relaxation

longitudinal
relaxation

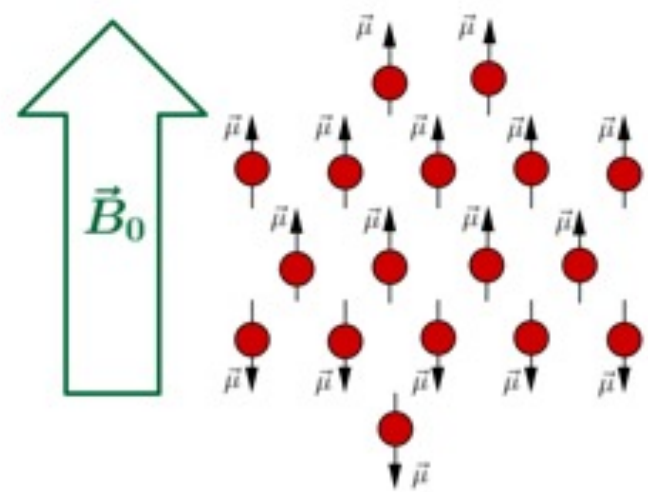
bloodstream
velocity



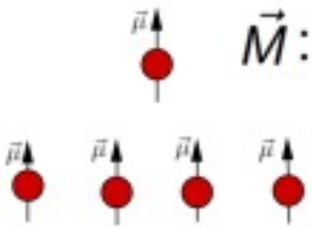
1



2



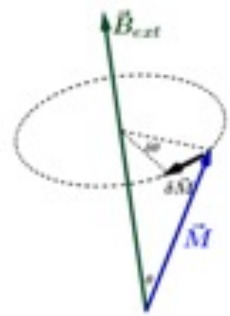
3

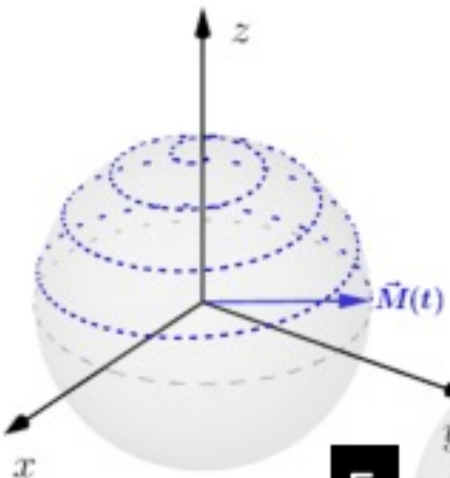


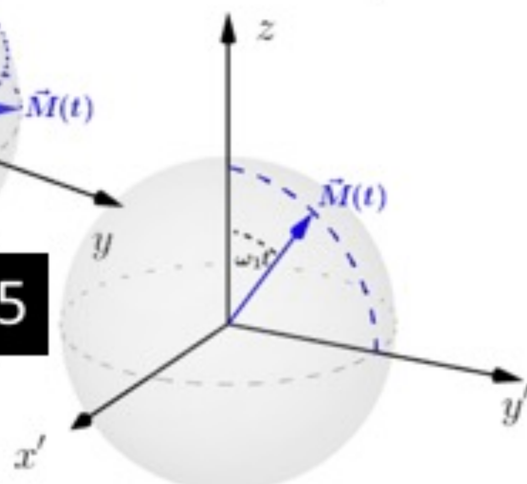
$$\vec{M} := \sum_j \vec{\mu}_j \neq 0$$

magnetization

4







5

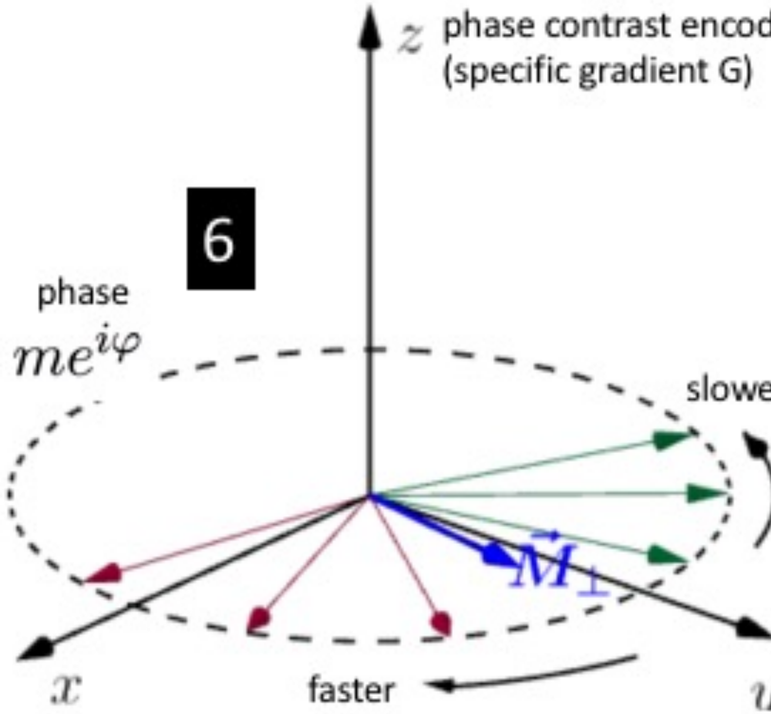
RF pulse (move M to transversal plane)

phase

$$M_x + iM_y = me^{i\varphi}$$

6

phase contrast encoding (specific gradient G)

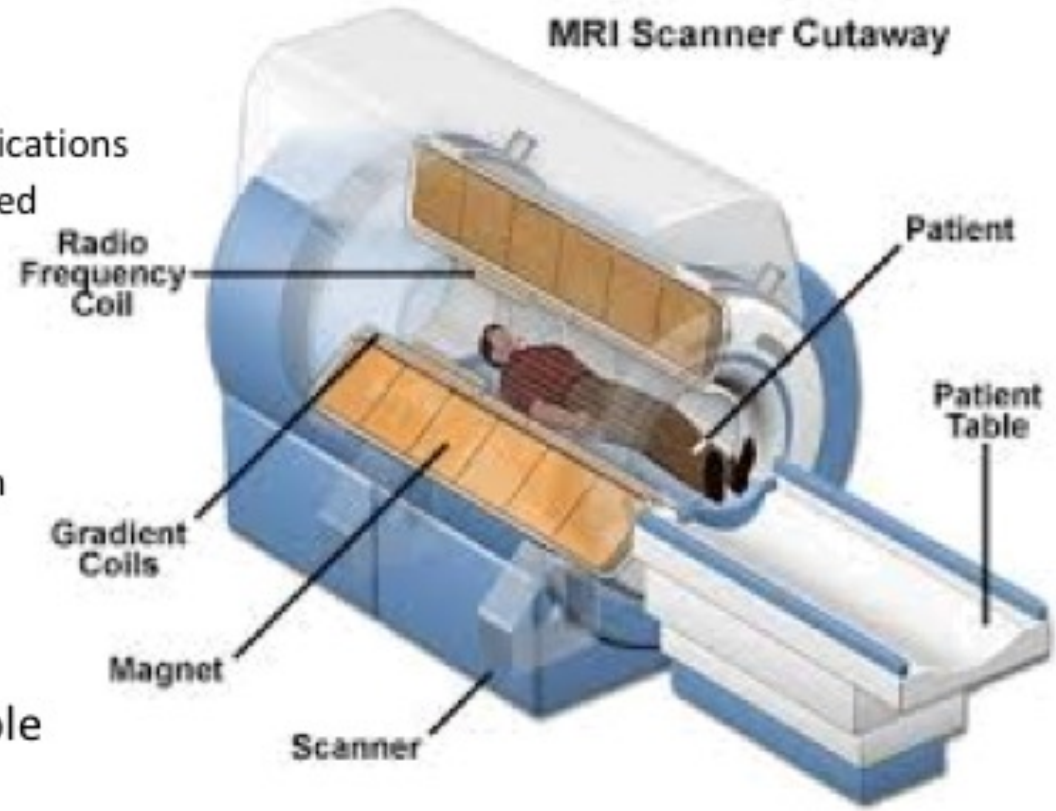


$$\frac{\partial \varphi}{\partial t} + \vec{v} \cdot \nabla \varphi = -\gamma((B_0 + \delta B) + \vec{x} \cdot \vec{G}(t))$$

bloodstream velocity

Magnetic Resonance Imaging

- Why MRI?
  - No ionizing radiation
    - Hardly any side effects, few contra indications
    - Works even in children and anesthetized patients
    - May be applied in serial examinations
  - Excellent image quality
    - High biological contrast
    - High resolution in arbitrary orientation
    - Mature technology
  - Relatively easy to set up
    - Low maintenance
  - Still advances happening and possible
  - Good reimbursement



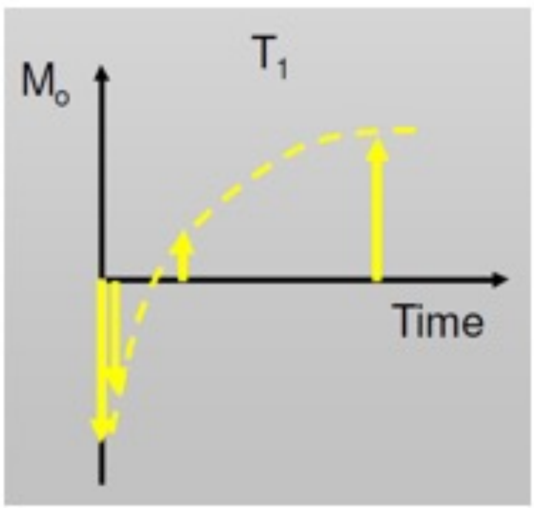
principles of MRI (video 8 min)

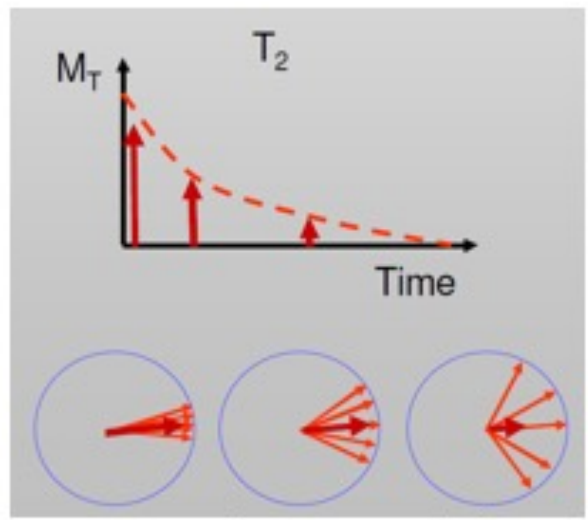
LIGHTBOX

RADIOLOGY EDUCATION

Magnetic Resonance Imaging Principles

- Relaxation
  - This relaxation process can be separated in two factors: longitudinal (or spin-lattice) and transversal (or spin-spin). The relaxation time are usually named T1 and T2.

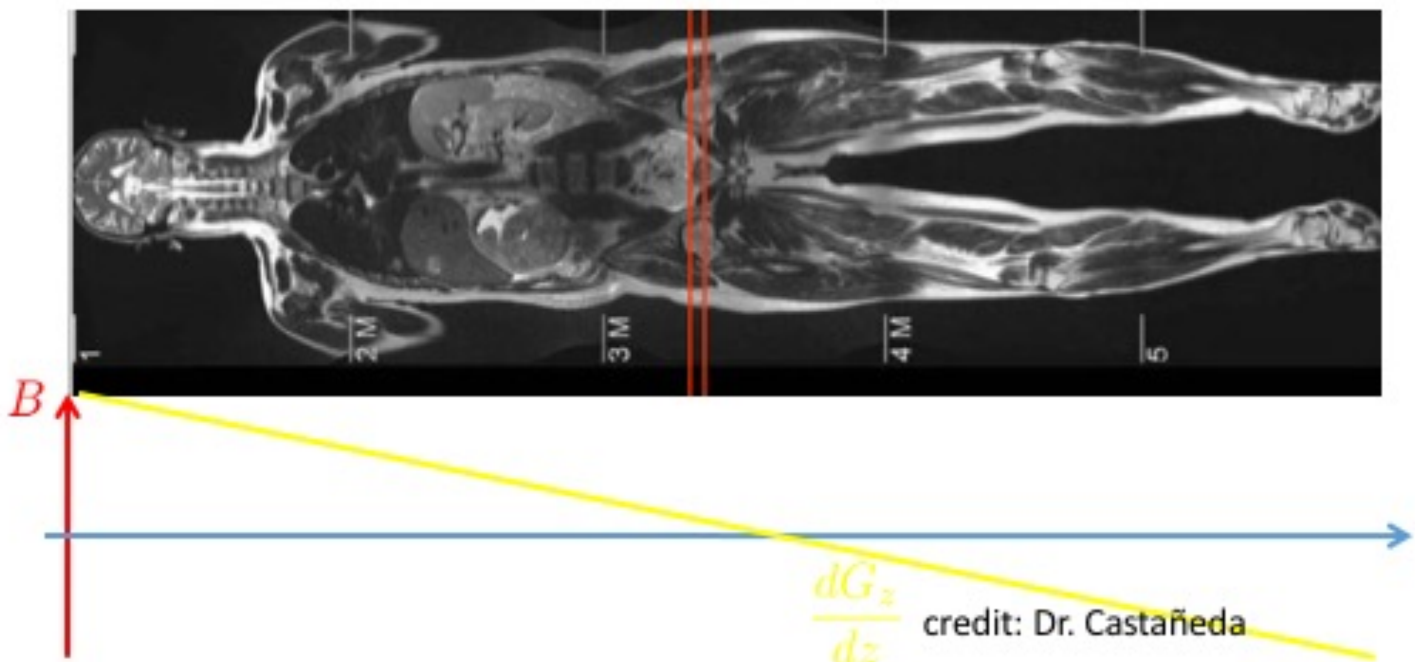




credit: Dr. Castañeda

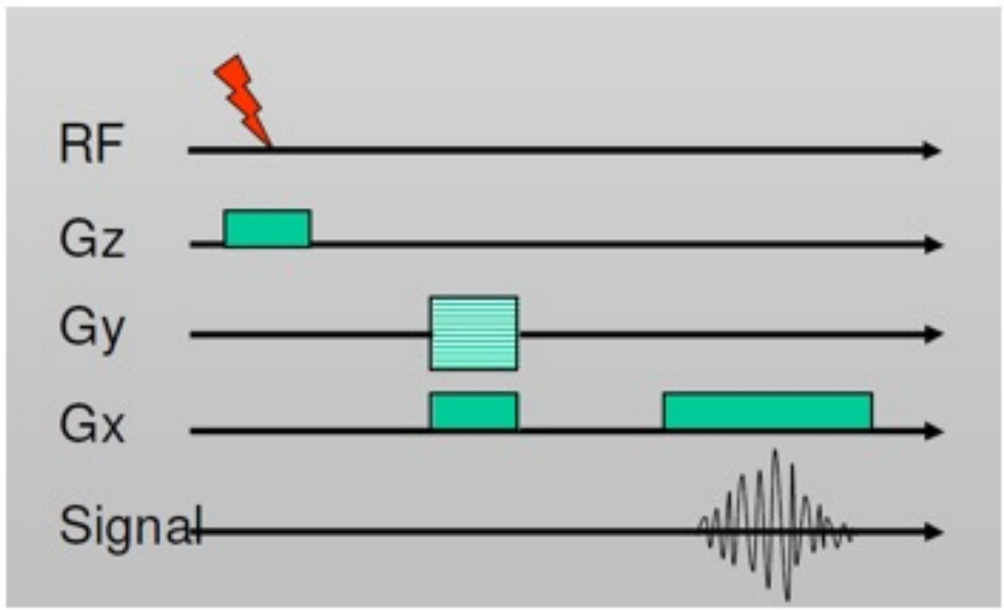
Magnetic Resonance Imaging Principles

- **Z – Enconding** (Gradient in Magnetic Field)



Magnetic Resonance Imaging Principles

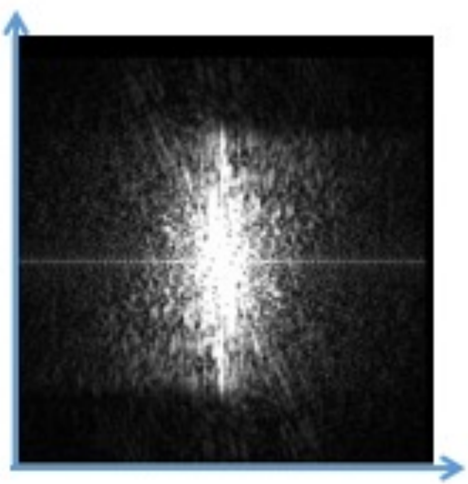
- How are all these individual steps combined ?

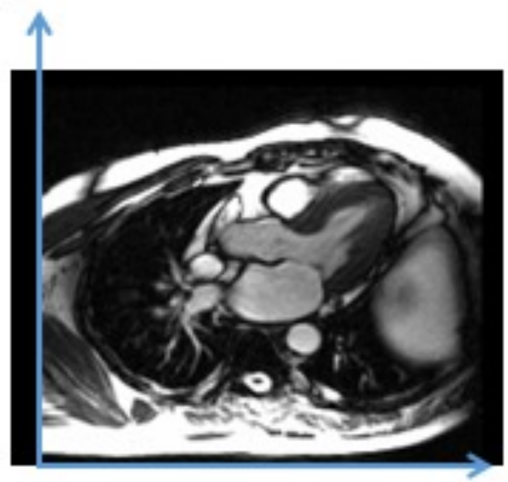


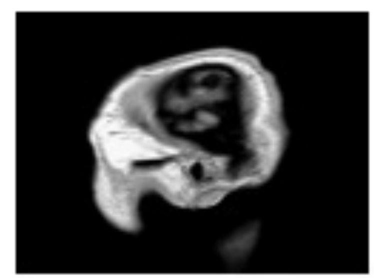
credit: Dr. Castañeda

Magnetic Resonance Imaging

- Image Reconstruction
  - Using data only from one slice and coding this information in phase and frequency, we are able to sample to so called "k-space". Its 2D Fourier transform yields the desired image. Essentially, the 2D FFT separates out the phase shift and the different frequencies







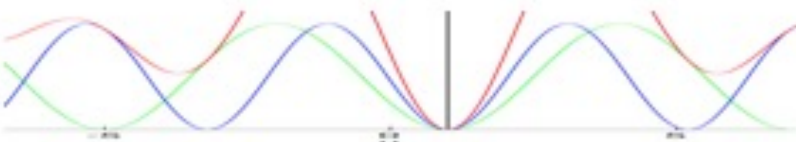
credit: Dr. Castañeda

Dual-venc method: better MRI bloodstream estimation

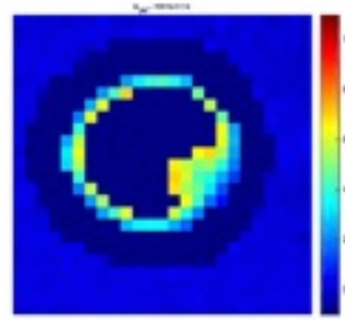
$$u_{pc} = \frac{\varphi_1 - \varphi_2}{\gamma \Delta M_1}\ ,\ \Delta M_1 = (M_1(G_1) - M_1(G_2))$$

$$venc(G_1, G_2) = \left|\frac{\pi}{\gamma \Delta M_1}\right|$$

$$u_{true} \in \{u_{pc} + 2k venc, \quad k \in \mathbb{Z}\}$$

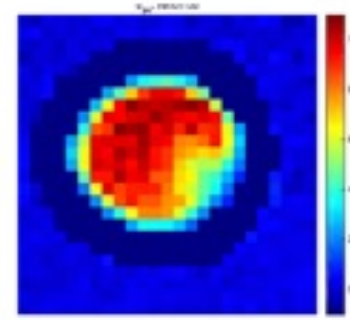


- *A least-squares dual-venc (LSDV) method for effective velocity de-aliasing in PCMRI.* H. Carrillo,1, A. Osses, S. Uribe, C. Bertoglio. To be submitted to Magnetic Resonance in Medicine Journal.



clean but incorrect

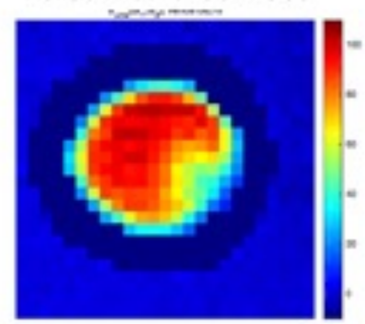
(a) venc 70



good but noisy

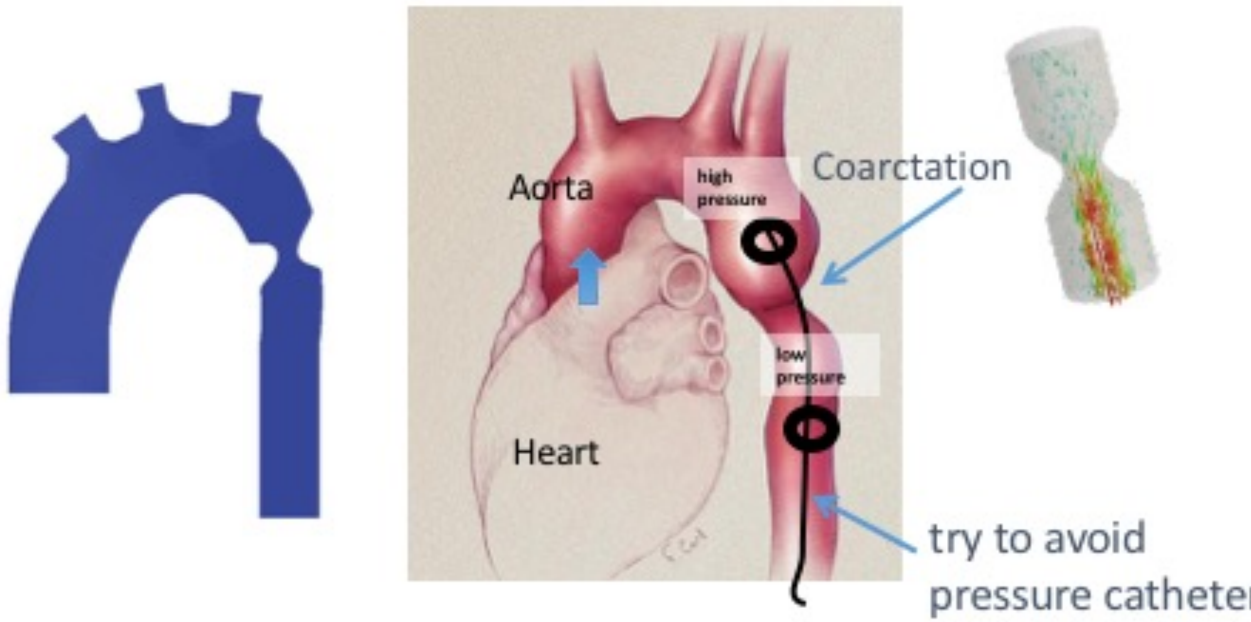
(b) venc 150

clean and correct



Cardiovascular medicine: pressure gradients

- Non invasive estimation of **pressure gradients** in arteries



Bertoglio et al. 2017

Navier-Stokes Equations:

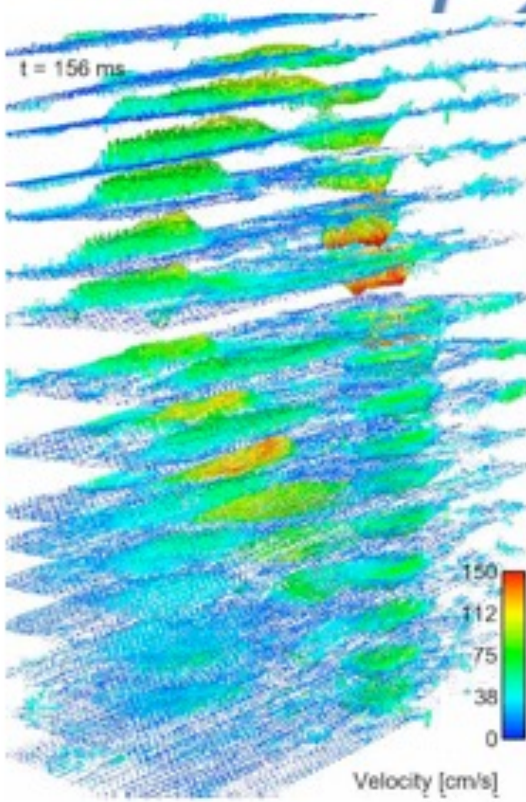
$$\rho_f \frac{\partial \boldsymbol{u}_f}{\partial t} + \rho_f (\boldsymbol{u}_f \cdot \boldsymbol{\nabla}) \boldsymbol{u}_f - \mu \Delta \boldsymbol{u}_f + \boldsymbol{\nabla} p = \boldsymbol{0}, \quad \text{in} \quad \Omega^f,$$

?

$$\boldsymbol{\nabla} \cdot \boldsymbol{u}_f = 0, \quad \text{in} \quad \Omega^f$$

Phase contrast MRI

- MRI 4-dimensional flow measures of the speed of the **bloodstream**



Courtesy of KCL

Measurements of velocity field

Spatial resolution: 1.5-3.0 mm3

Temporal resolution: 30-40 ms

Noise around 10% of maximal velocity (set for each acquisition)

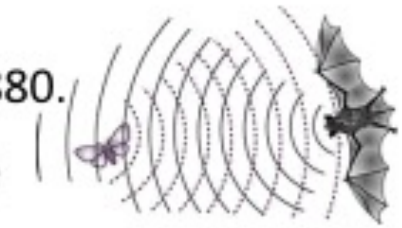
- Relative pressure estimation from velocity measurements in blood flows: state-of-the-art and new approaches. C. Bertoglio, R. Núñez, F. Galarce, D. Nordsletten, A. Osses. *Int. J. Numer. Meth. Biomed. Engng. 2017*

3.- ultrasound, elasticity imaging

current research topic: bone porosity estimation by ultrasound

obstacles... ultrasound

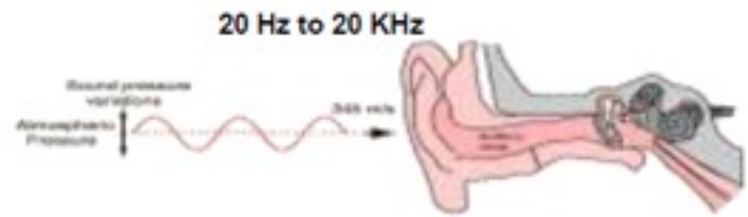
- Ultrasound date back to 1930 and basically detect **obstacles** and allow you to see them on a screen. Used today to follow the fetal development.
- Ultrasound was first used with **hydrophones** to detect icebergs, after the tragic sinking of the Titanic, or submarines during the first war world.
- Pierre Curie discovered the basis of **ultrasound transducers** in 1880.
- The key **mathematical rule** behind ultrasound is that used by the bat when hunting in the dark: when emitting echoes before an obstacle, the distance to it is proportional to the time it takes to **go and return**.
- This assume that most of the body is water and that the sound propagates in it at a **constant speed** and in straight lines, this is not completely true and the images are somewhat confusing.



d=t/v

Ultrasound

- What is Ultrasound?

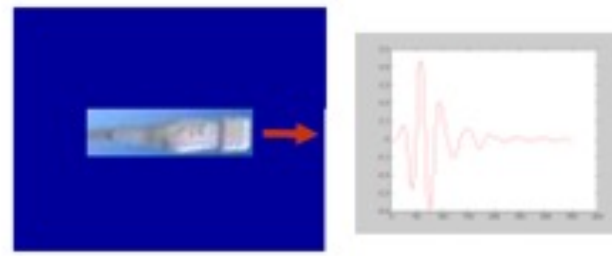


- Medical ultrasound uses frequencies in the range of 500 KHz to 30 MHz.
- For imaging 1MHz to 10 MHz.
- Intravascular imaging up to 30 MHz. (Higher the frequency better the resolution!!)

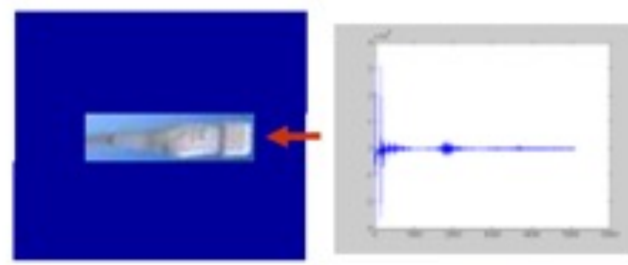
credit: Dr. Castañeda

Ultrasound

- Workflow
  1. An ultrasound probe is used to generate a short burst of the ultrasound signal, which is sent into the tissue.



  2. The same ultrasound probe is usually used to Record the echoes (RF signals) returned back from the tissue.

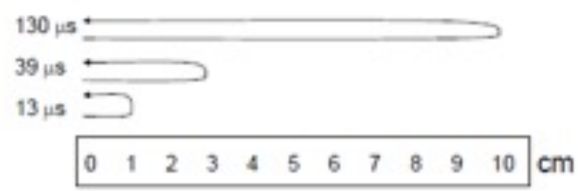


  Different tissues show different reflection properties.

credit: Dr. Castañeda

Ultrasound

- The sound speed in soft tissue is about 1540 m/s
- As round-trip increases, reflector's distance increases
- For c = 1540 m/s = 1.54 mm/micro s:

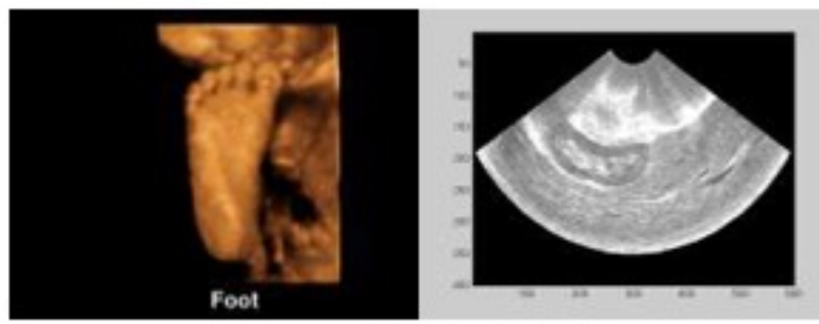


3. Processing the RF signals
   - Envelope detection by Hilbert Transform
   - Dynamic range reduction by *log function*
   - Decimation for data reduction (5100 samples into 300 samples)

credit: Dr. Castañeda

Ultrasound

- Examples:

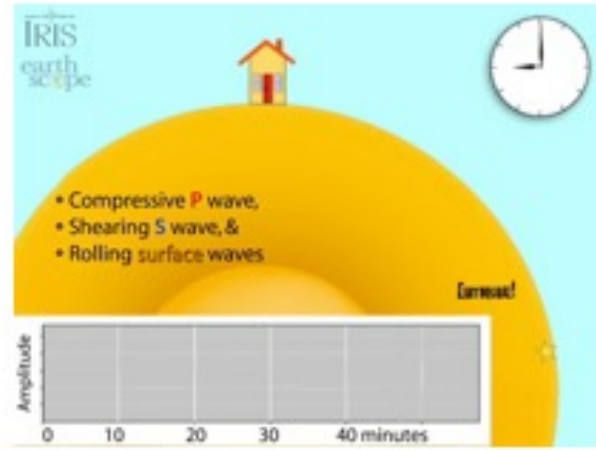


credit: Dr. Castañeda

Ultrasound elastography

- Elastography is used in the detection of breast or liver cancer, and use **elastic P or S waves** (the same of **earthquakes**!) to detect hardness tissues.
- It is a sophisticated form of the ancient technique of **palpation** to detect hardness parts inside the body.

www.iris.edu



p-wave

s-wave

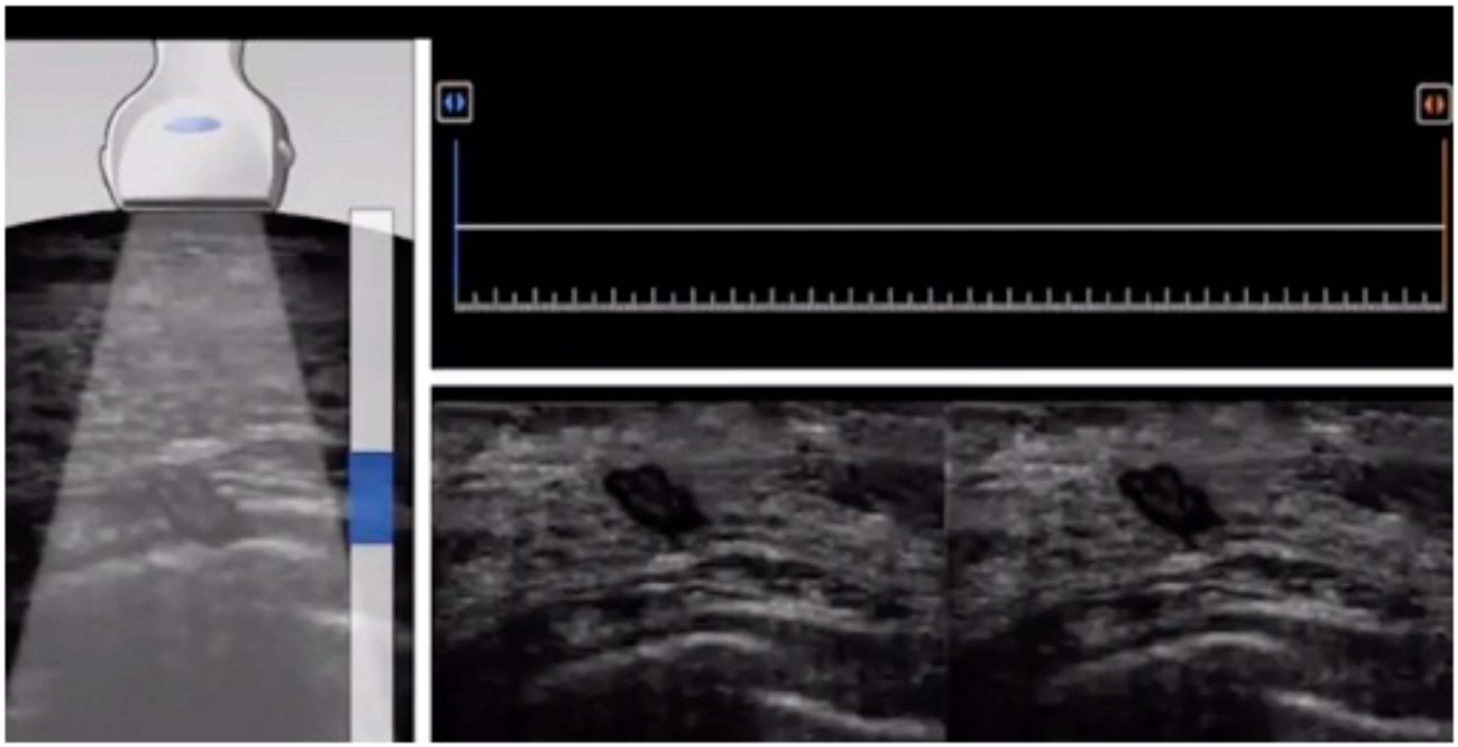
principles of elastography (soft tissue)

- non invasive observation of the internal displacements by an standard imaging modality such as ultrasound or MRI
- the medium is deformed by a given external force (load, vibration, acoustic radiation) or by an internal displacement (cardiac/respiration cycles, blood flow)
- dynamic elastography: time dependent or periodic forces
- quasi-static elastography: quasi-static external force
- cancer is usually stiffer than healthy tissue
- shear modulus: one of the most wide ranging physical parameters for differentiating tissue (several orders of magnitude)

Ultrasound elastography

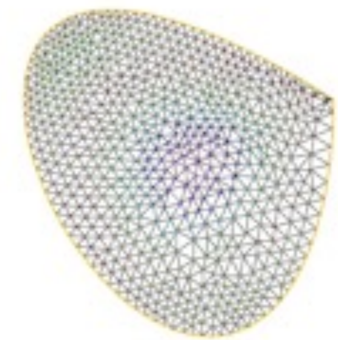
Toshiba medical systems

ultrasound transducer



wave

Elasticity v/s viscoelasticity (soft tissue: brain, liver, skin, breast, prostate, etc.)

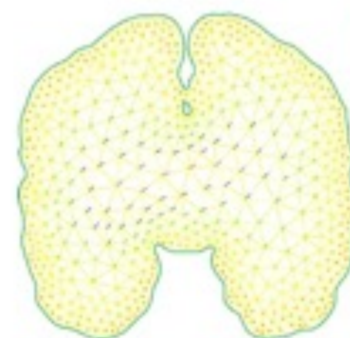


- Most of living tissues have a **mix of solid/fluid behaviour** : skin, blood, bones, muscles, liver, brain, lungs, etc.
- This is called **visco-elasticity**. Viscosity is a property of fluids, elasticity is a property of solids.
- The idea is to obtain images of the **viscoelastic parameters** and the **fading memory**.

$$\mathcal{P}u := \partial_t^2 u - \underbrace{\nabla \cdot \left(2\overline{\mu}_0 \epsilon(u) + \overline{\lambda}_0 (\nabla \cdot u) I\right)}_{\text{elasticity}}$$

$$+ \underbrace{\nabla \cdot \int_0^t (2\tilde{\mu}(t-s)\epsilon(u(s)) + \tilde{\lambda}(t-s)(\nabla \cdot u)(s) I) ds}_{\text{viscoelasticity}} = 0$$

Maxwell viscoelastic mathematical model

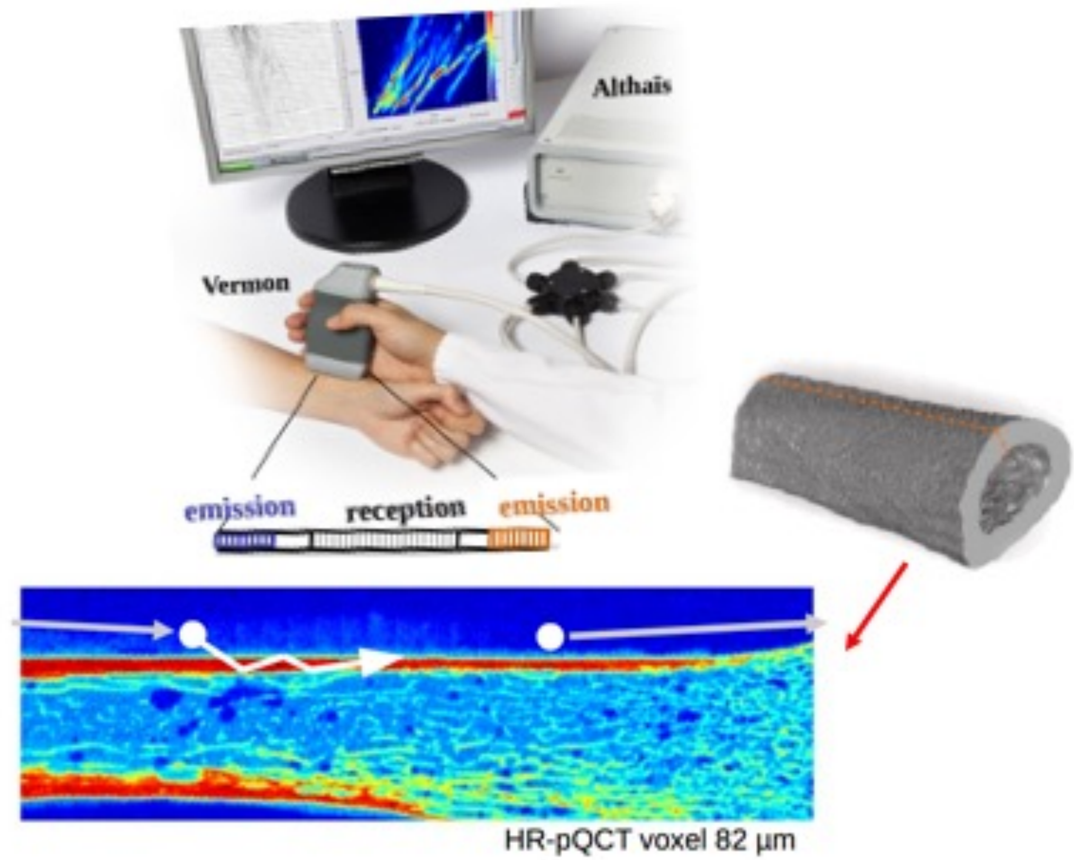


- *Logarithmic stability in determination of a 3D viscoelastic coefficient and a numerical example*

M. de Buhan, A. Osses, Inverse Problems, 2010.

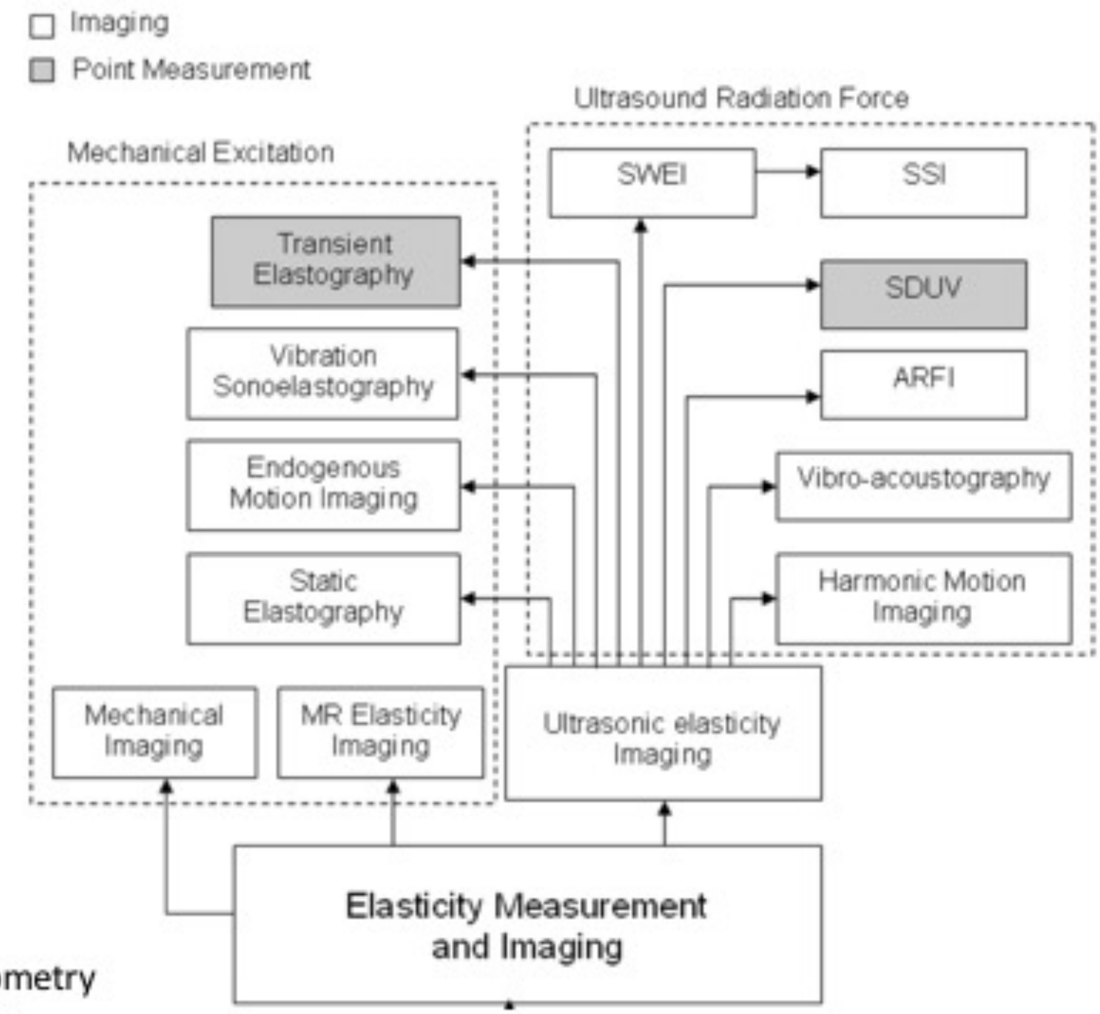
Ultrasonic guided waves and bone tomography

- a new development for **osteoporosis** detection instead of the classic X-rays
- allow bone **porosity** and bone **thickness** estimation
- mathematical model: homogenized elastic material, **guided waves** (as optical fibers)



Images coutesy of Jean-Gabriel Minonzio,
LIB, France and CMM, Chile

elastography:
there are
many modalities!

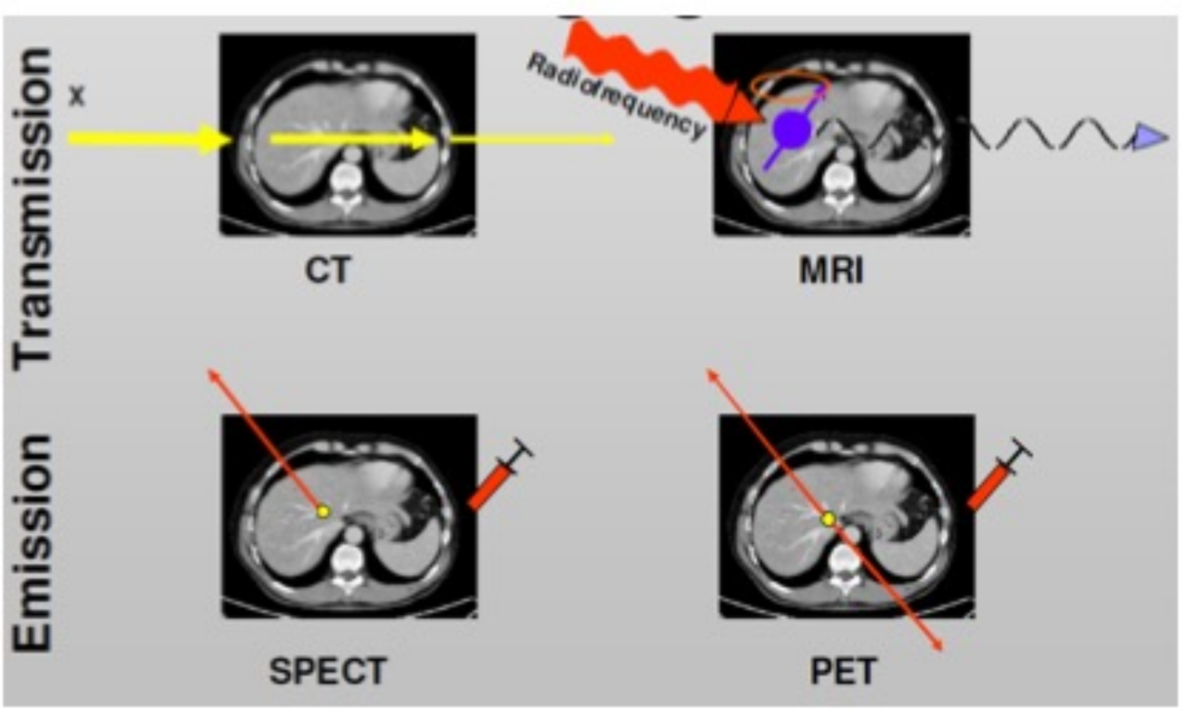


SWEI: Shear wave elasticity imaging
SSI: Supersonic shear imaging
SDUV: Shearwave dispersion ultrasound vibrometry
ARFI: Acoustic radiation force impulse imaging

credit: Sarvazyan 2011

Summary: one possible classification, not complete...

- Tomographic Imaging

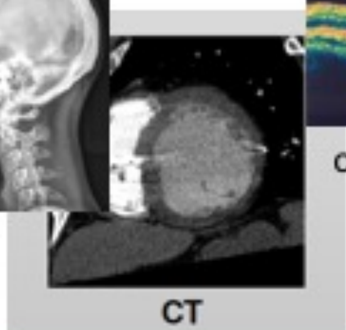


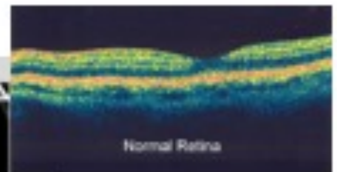
credit: Dr. Castañeda

+ ultrasound
+ EI

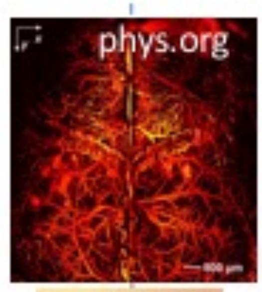
Another possible classification... (I like that)

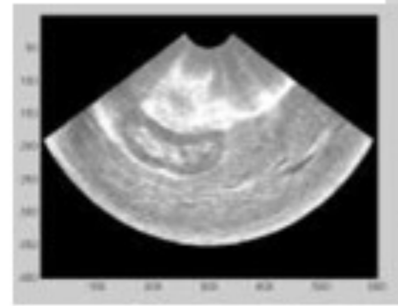
X-rays



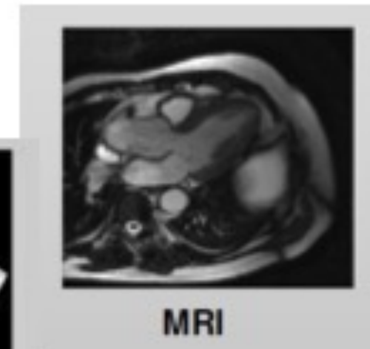


optical coherence tomography OCT





ultrasound



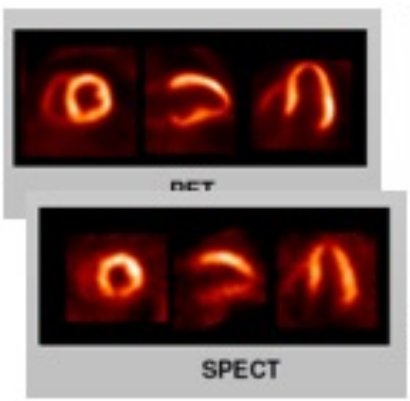
Magnetic resonance modalities: dMRI, MRE, 4DFlow, etc.

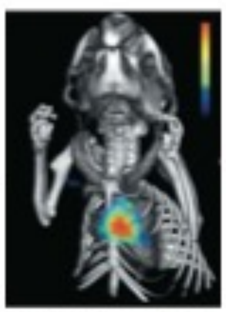
PHOTON TRANSPORT

HYBRID

photo-acoustic tomography

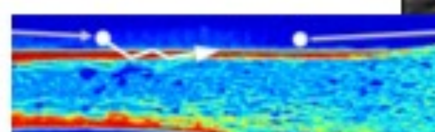
WAVE PROPAGATION



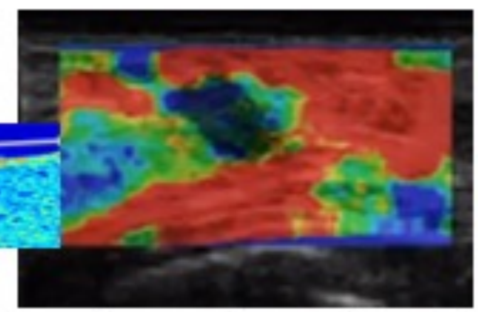


Fluorescence FMT

Nuclear medicine



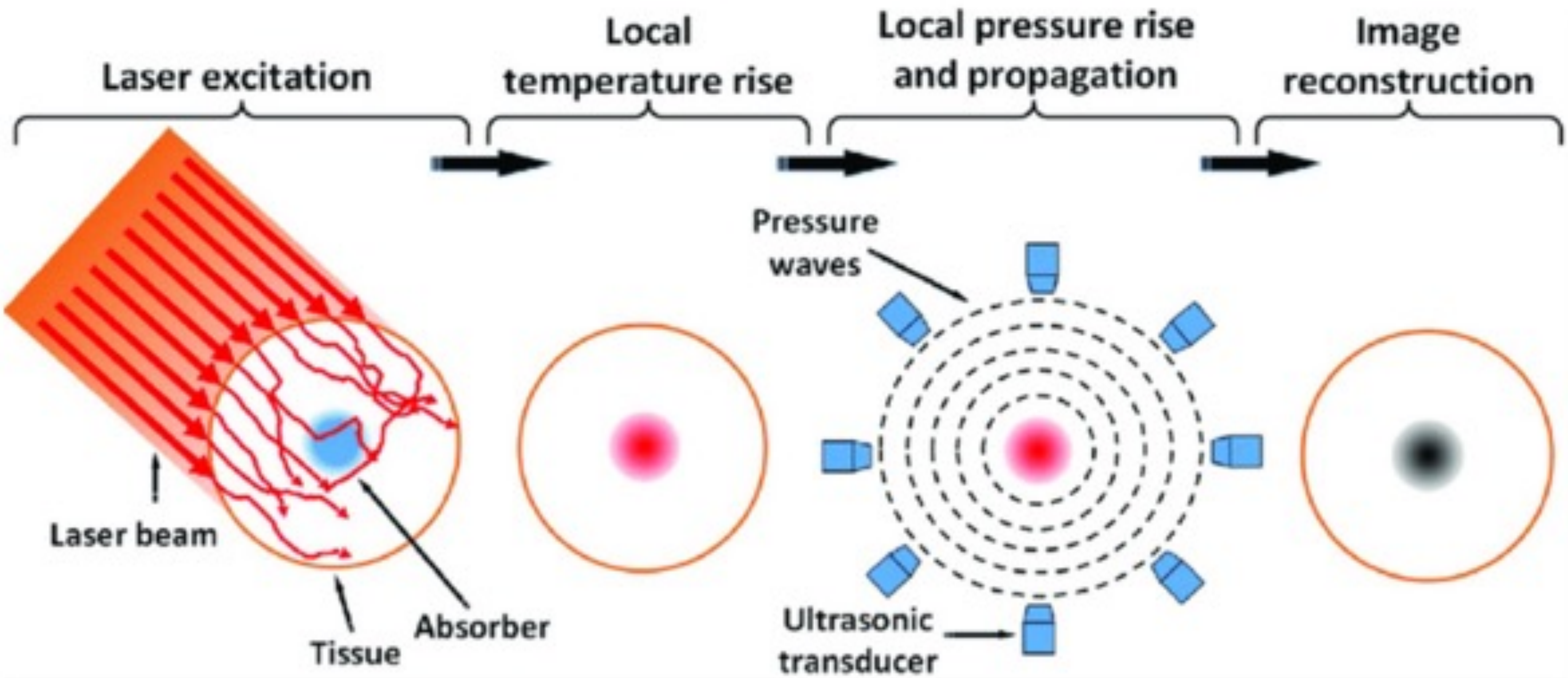
ultrasound bone porosity estimation



elastography modalities: ARFI, SWEI, MREI, etc

Hybrid methods: photo-acoustic tomography (PAT)

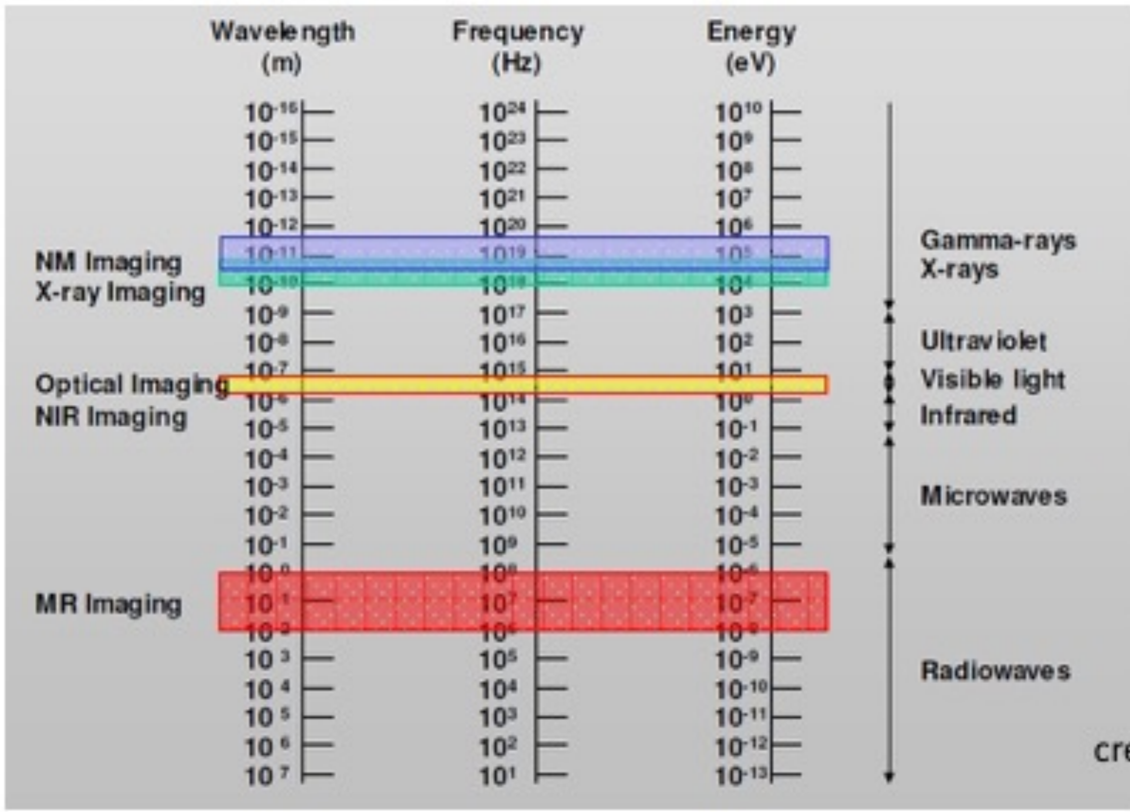
based on the photoelectric effect : Hertz 1997, Einstein 1905 nobel prize



credit: Ali-Reza et al. Neonatal brain resting-state functional connectivity imaging modalities, Photoacoustics 10, 2018

Summary

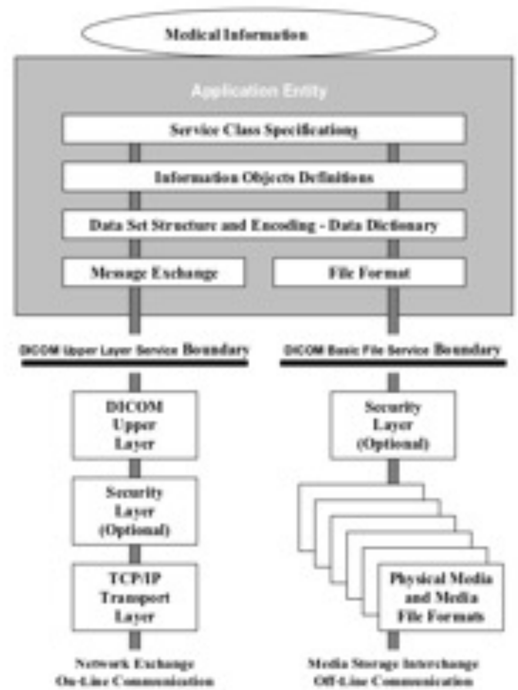
- Tomographic Imaging



credit: Dr. Castañeda

3D imaging format?

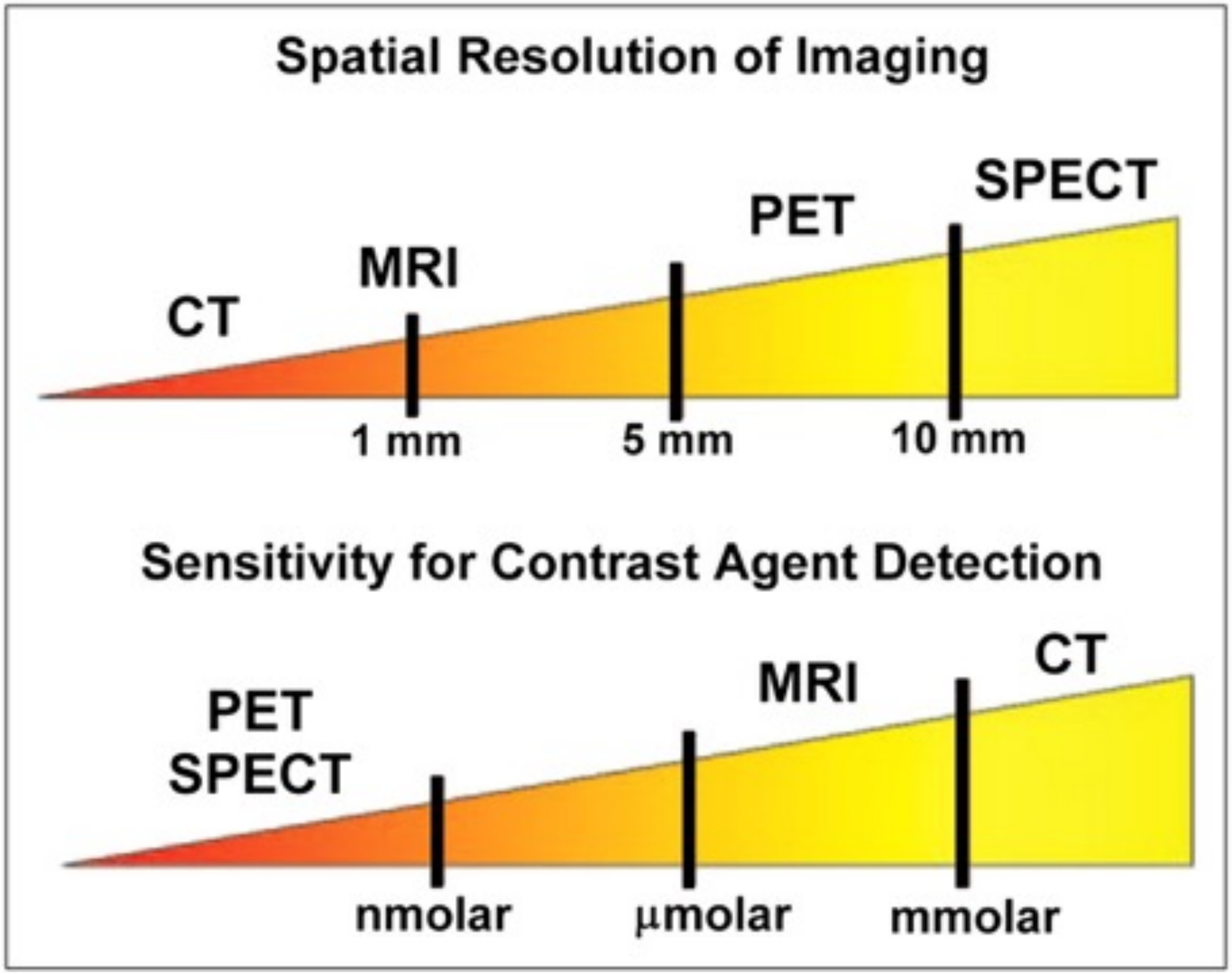
- Digital Imaging and Communications in Medicine (DICOM)
  - standard for handling, storing, printing, and transmitting information in medical imaging
  - File format
  - Network communications protocol
  - Owned by National Electrical Manufacturers Association (NEMA)



http://medical.nema.org/Dicom/2011/

credit: Dr. Castañeda

Summary



credit: web

A basic bibliography

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- J.T. Bushberg et al. *The Essential Physics of Medical Imaging*, Third Edition, Lippincott Williams and Wilkins, Philadelphia, 2012.
- Miles N. Wernick and John N. Aarsvold (eds.) *Emission Tomography, the fundamentals of PET and SPECT*, Elsevier, London, 2004.
- Videos MRI:
  - https://www.youtube.com/watch?v=zf5oX01bRgk (1hora, School of Medicine, Washington University)
  - https://www.youtube.com/watch?v=Ok9ILIYzmaY (9 minutos, Lighbox Radiology, Australia)
- A. Sarvazyan et al. *An overview of elastography - an emerging branch of medical imaging*. In: Curr. Med. Imaging Rev. 7(4) (2011), pp. 255–282

- Thanks to Dr. Víctor Castañeda for sharing some of his slides.

The end