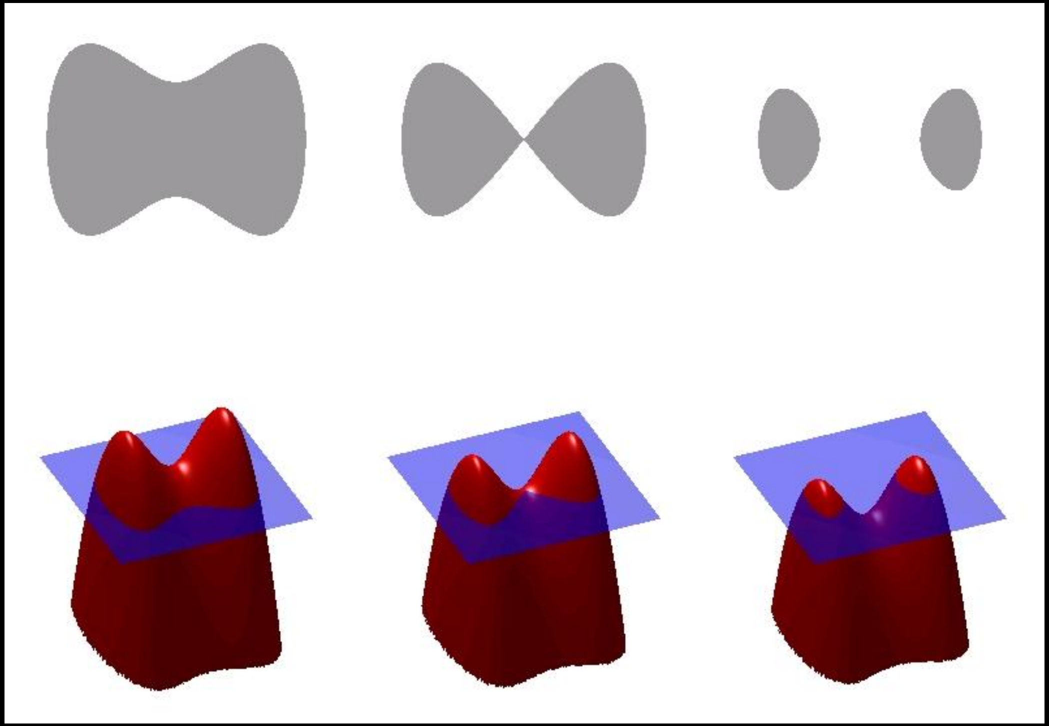
Dynamic Implicit Surfaces / Level Sets



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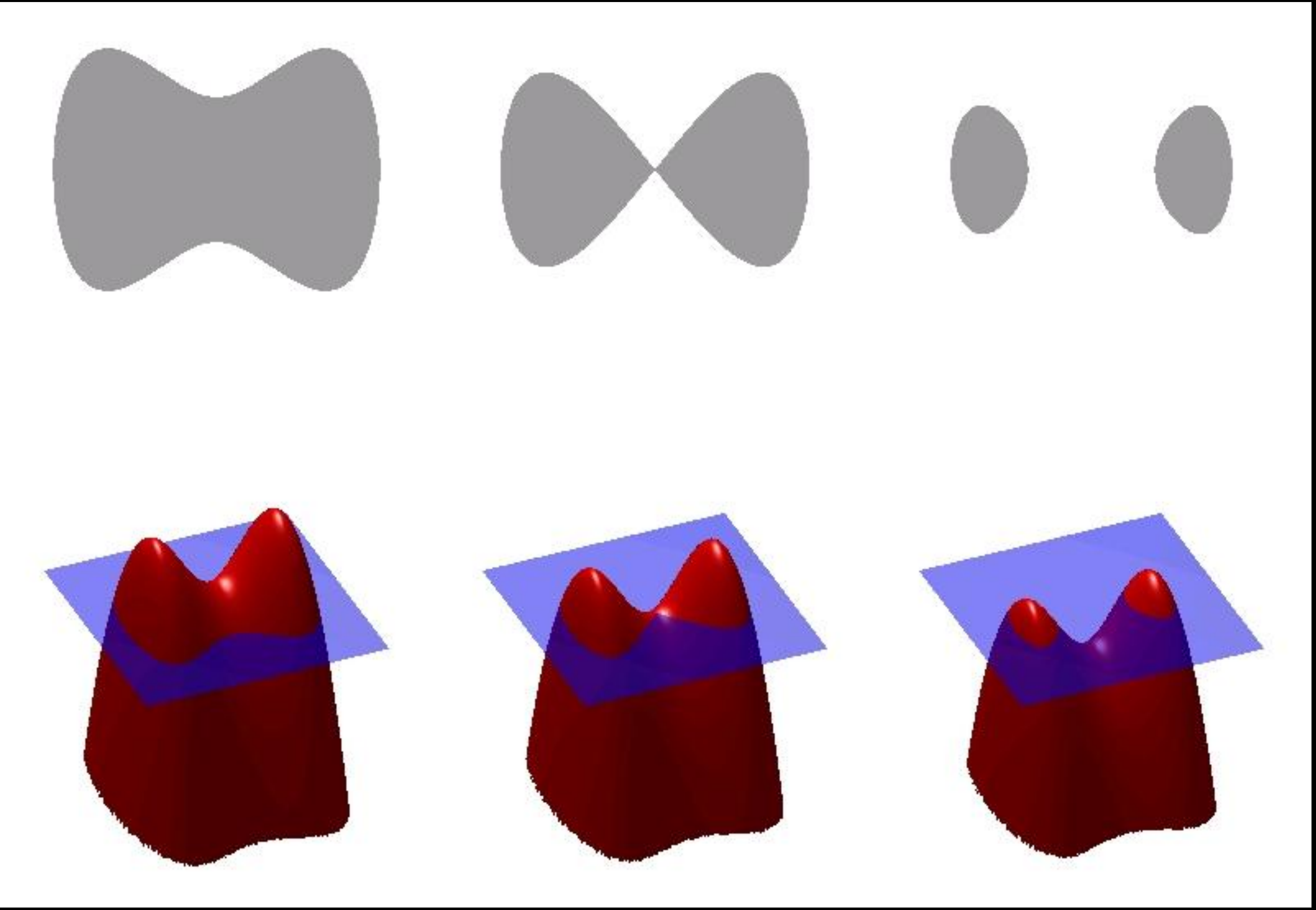
1. Introduction



https://en.wikipedia.org/wiki/Level_set_method

Laboratory for SCIAN Scientific Image Analysis

1. Introduction

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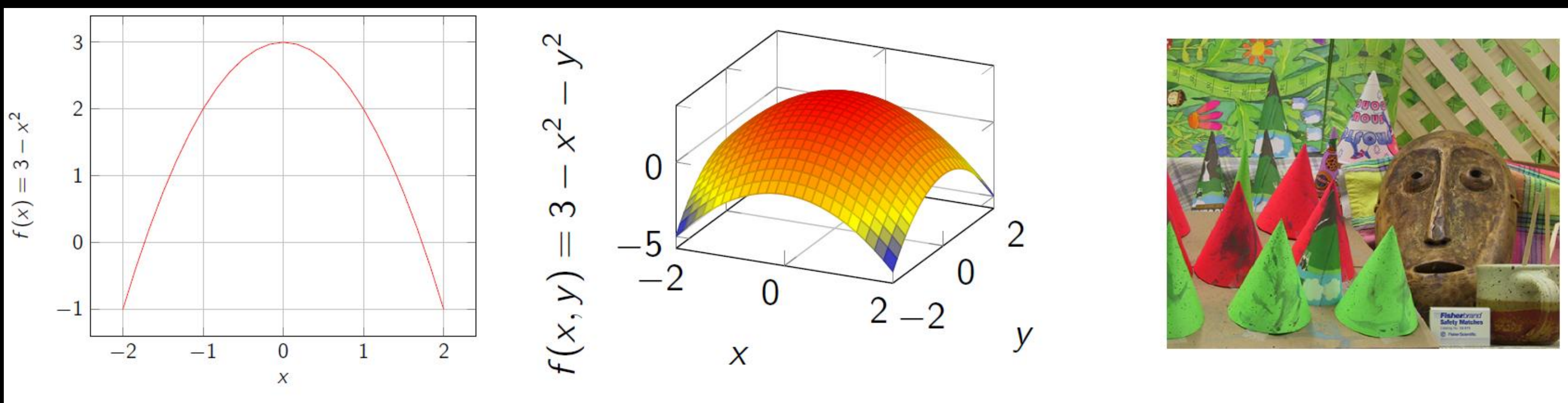
- Images as functions
  Functions…

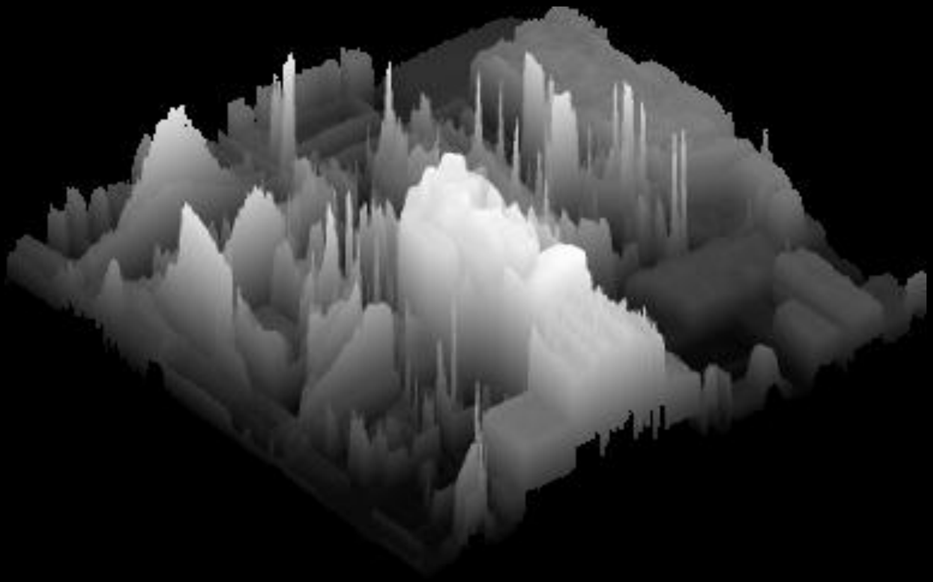


(a) $\Omega \in [-2 \dots 2]$ (b) $\Omega \in [-2 \dots 2] \times [-2 \dots 2]$ (c) $\Omega \in [1 \dots m] \times [1 \dots n]$

1. Introduction

- Images as functions
  Images…

(a) Tsukuba

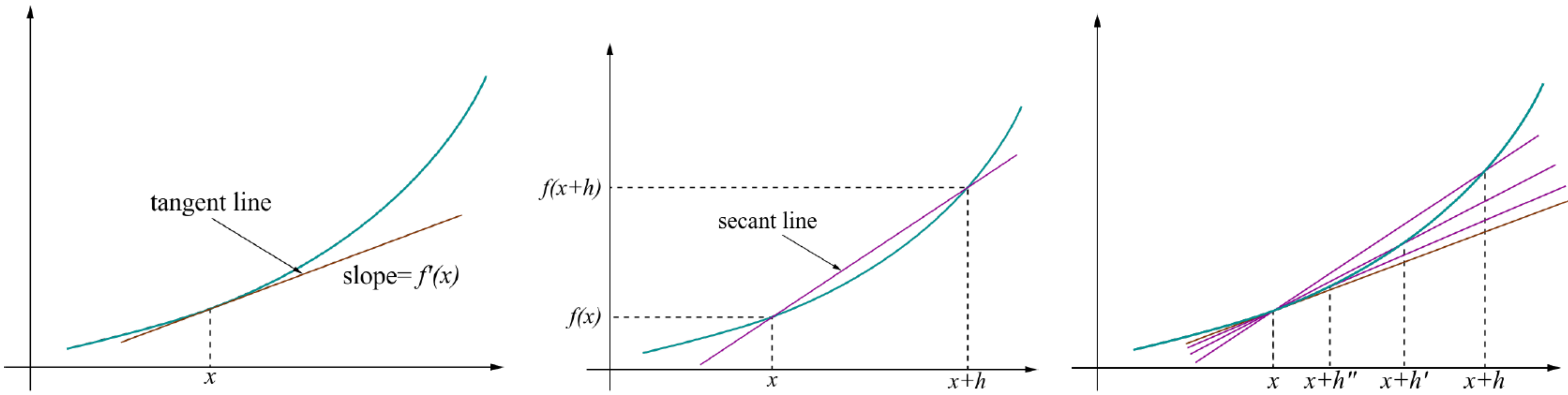


(b) Tsukuba

1. Introduction

- Derivatives

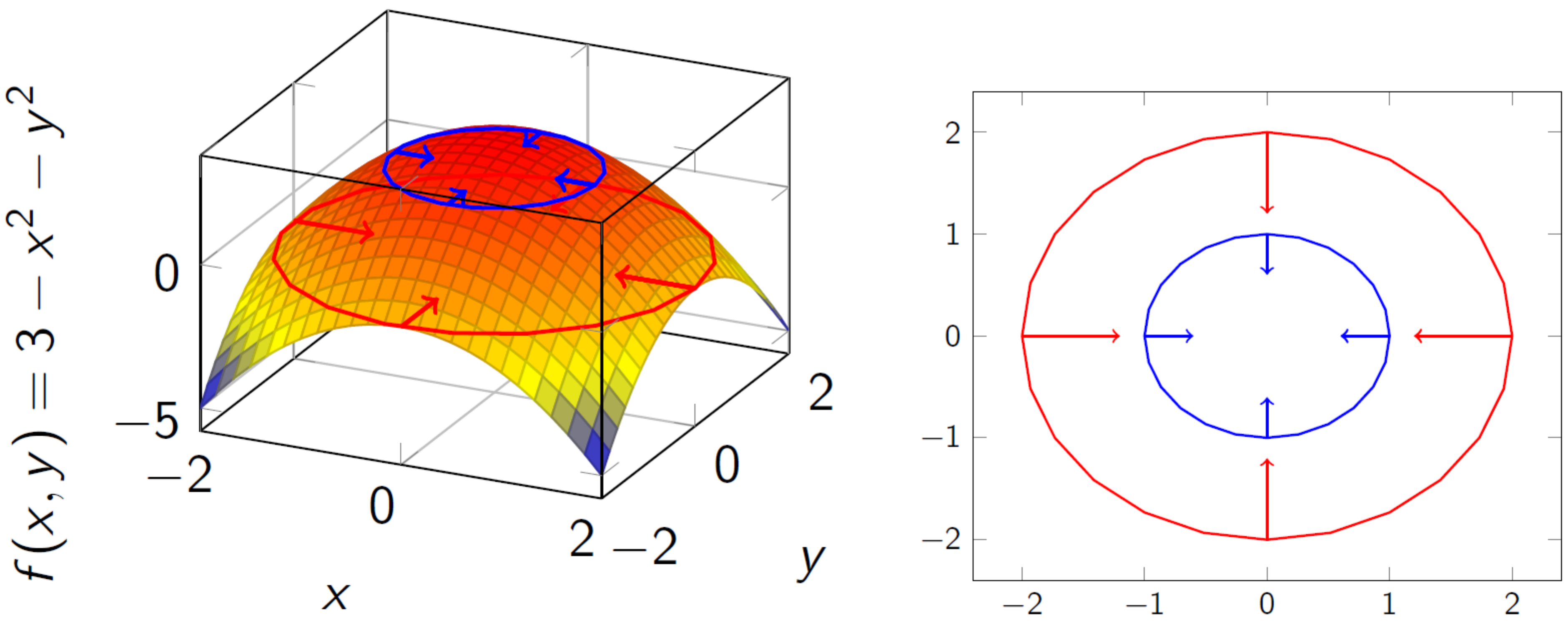
$$f(x)' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



(a) Tangent (b) Secant (c) Limit of secant

1. Introduction

- The gradient is a vector, ∇f, pointing in the direction of the greatest increase.
- In 2D cartesian coordinates, with f = f(x,y)
  ∇f = [∂f/∂x, ∂f/∂y]



(a) 3D plot of ∇f (b) Seen from above

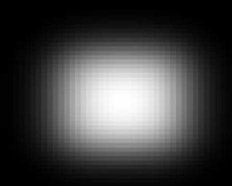
1. Introduction

- Diffusion – A model for a function that evolves in time…

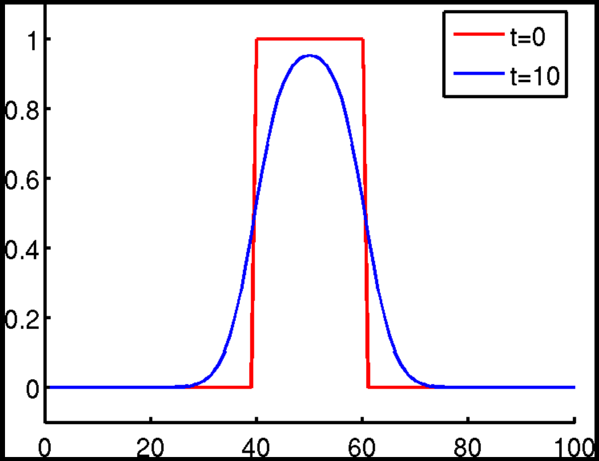
  $I_t = \mathrm{DIV}\,(\nabla I)$

  …temporal change in the image is due to “movement” of particles due to diffusion. If ∇I is a continuously differentiable vector field, then
  $\mathrm{DIV}\,(\nabla I) = \nabla\cdot(\nabla I) = (\nabla I)_x + (\nabla I)_y$

(a) Time t = 0



(b) Time t = 1



(c) Graphs

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2. Level Sets

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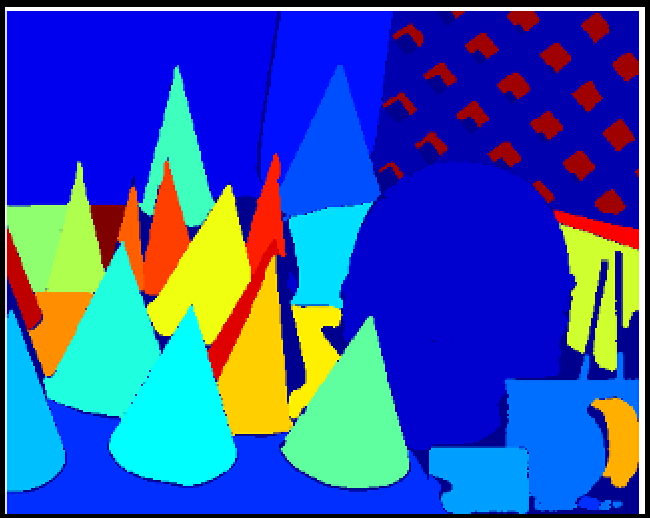
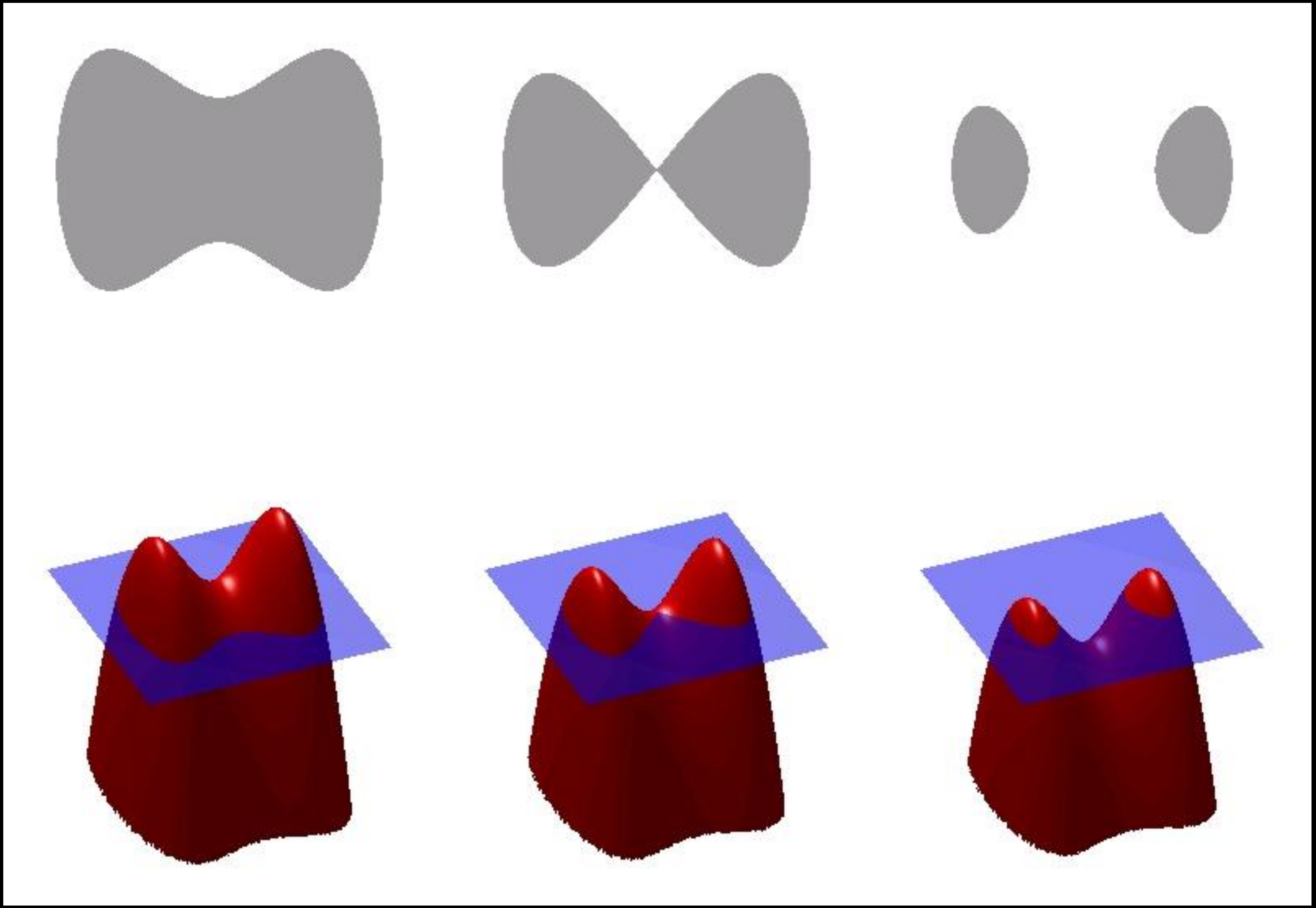
- Segmentation: joining individual pixels into meaningful groups.



- Here each group is assigned a different number, and each pixel belonging to a particular group is displayed using the group's number (mapped to a lookup color table).

2. Level Sets



https://en.wikipedia.org/wiki/Level_set_method

2. Level Sets

- A set can be defined by enclosing its members in curly brackets, e.g. C = {2, 4, 51}. Alternatively, we can identify the set members by a logical statement such as

  $\{ x \mid P(x) \}$,

  which means the set of all x for which P(x) is true. We can use the sets for the image segments (regions of interest), where $\{ x \mid P(x) \}$ describes the set of points belonging to a given region.

2. Level Sets

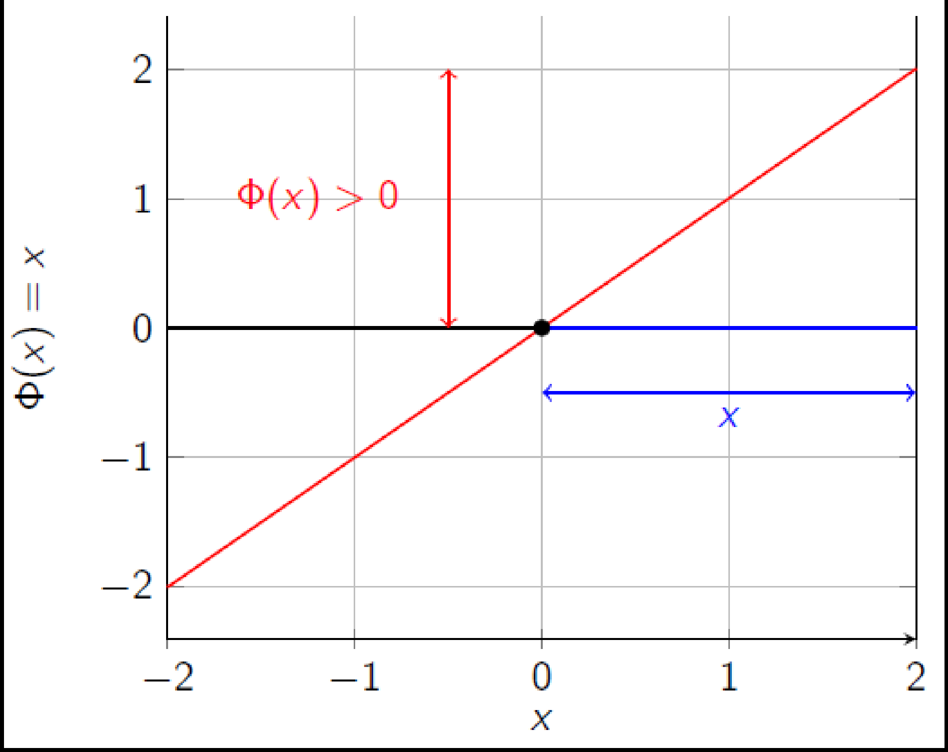
- Using the set notation and a function $\Phi(x)$, we can define the set as $\{x \mid \Phi(x) > 0\}$.

- Now we can define the following sets:

$\text{outside}(\Phi) = \{x \mid \Phi(x) < 0\}$

$\text{inside}(\Phi) = \{x \mid \Phi(x) > 0\}$

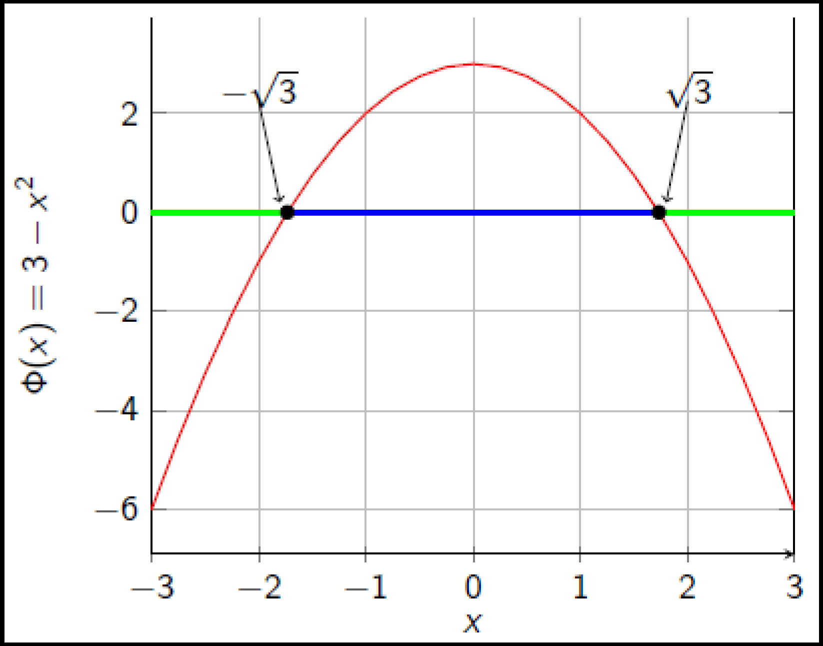
$\text{contour}(\Phi) = \{x \mid \Phi(x) = 0\}$
(also called interface)



2. Level Sets

- $\Phi(x) = 0$ is the zero level-set (contour) of the function $\Phi(x)$. This representation is said to be **implicit**.

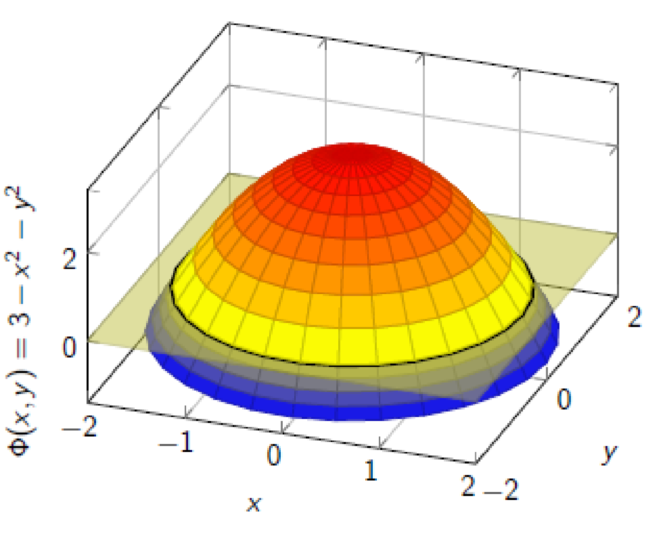
- An example (1D):
  - $\Phi(x) = 3 - x^2$
  - Outside(Φ): $\{ x \mid 3 - x^2 < 0 \}$
  - inside(Φ): $\{ x \mid 3 - x^2 > 0 \}$
  - Interface(Φ): $\{ x \mid 3 - x^2 = 0 \}$



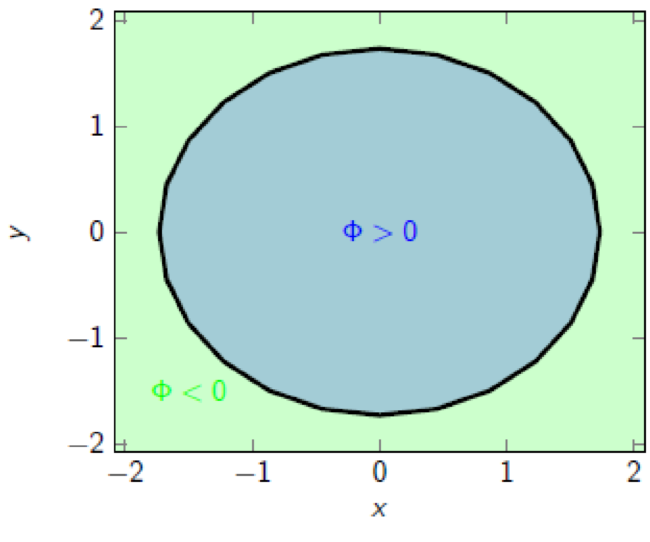
2. Level Sets

- Example (2D):
  - $\Phi(x) = 3 - x^2 - y^2$
  - Outside(Φ): $\{ x \mid 3 - x^2 - y^2 < 0 \}$
  - inside(Φ): $\{ x \mid 3 - x^2 - y^2 > 0 \}$
  - Interface(Φ): $\{ x \mid 3 - x^2 - y^2 = 0 \}$

    the contour is a line... while Φ can be seen as a surface



(a) 2D plot

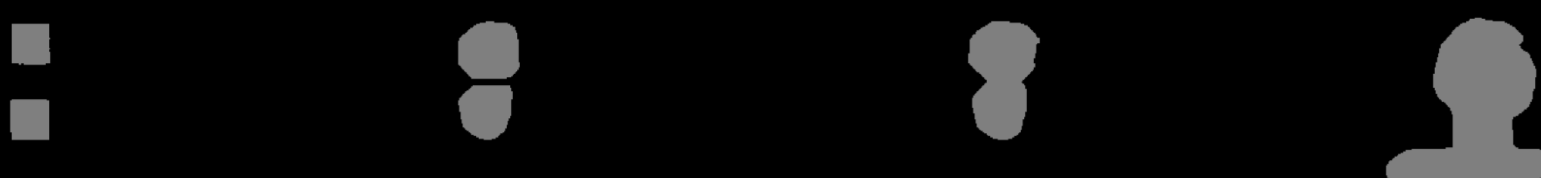


(b) Seen from above

2. Level Sets

Segmentation Example:

- A level-set segmentation for the image at the right is started with two individual “seeds” (t = 0) with no connectivity.
- Around t = 13 the seeds “fused” together and, therefore, the topology changed.



t = 0 t = 12 t = 14 t = 199

2. Level Sets

EXPLICIT models (e.g. "snakes")

- Contours directly available
- Inside of segment:
  - ...searching complicated
- One segment per contour
- Handling of topological changes
  - ...via ad-hoc methods
- Numerical stability:
  - ...depends on the curve
- Implementation:
  - ...depends on dimensionality
- Numerically (computation) efficient

IMPLICIT models (e.g. "level sets")

- Contours have to be "reconstructed"
- Inside of segment:
  - ...searching trivial
- Several segments per contour
- Handling of topological changes
  - ...implicit
- Numerical stability:
  - ...depends on derivatives
- Implementation:
  - ...extensible up to n-dimensions
- Numerically (computation) more complex

3. Dynamic Implicit Interfaces

- Similar to the diffusion model…

$$\Phi_t + \underline{V} \cdot \nabla\Phi = 0$$
$$\Phi_t + u\,\Phi_x + v\,\Phi_y = 0$$

- Motion along the normal direction

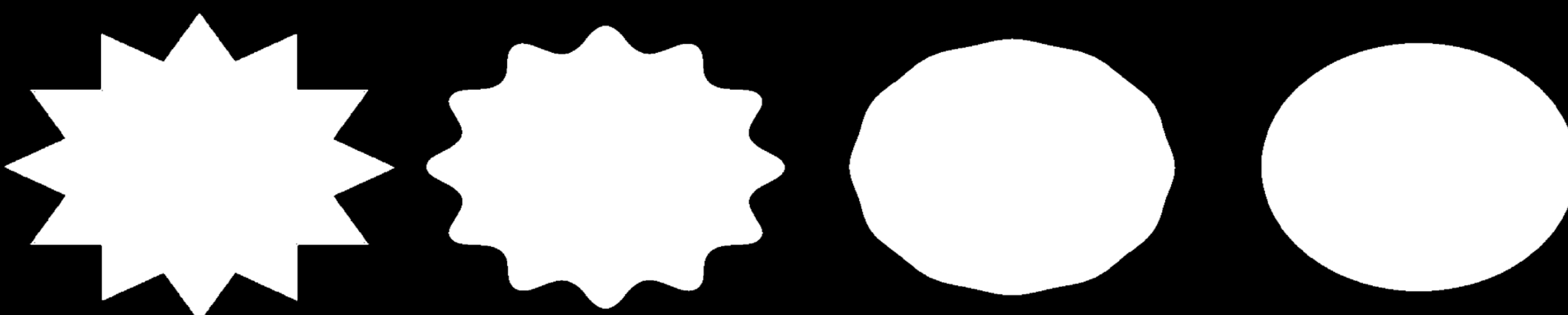
$$\Phi_t + v_n |\nabla\Phi| = 0$$

- Mean curvature motion (MCM)

$$v_n = -\alpha\,\mathrm{DIV}\,\underbrace{(\nabla\Phi / |\nabla\Phi|)}_{\textbf{Normal direction}}$$

- $\Phi_t - \alpha\,\mathrm{DIV}\,(\nabla\Phi / |\nabla\Phi|) = 0$

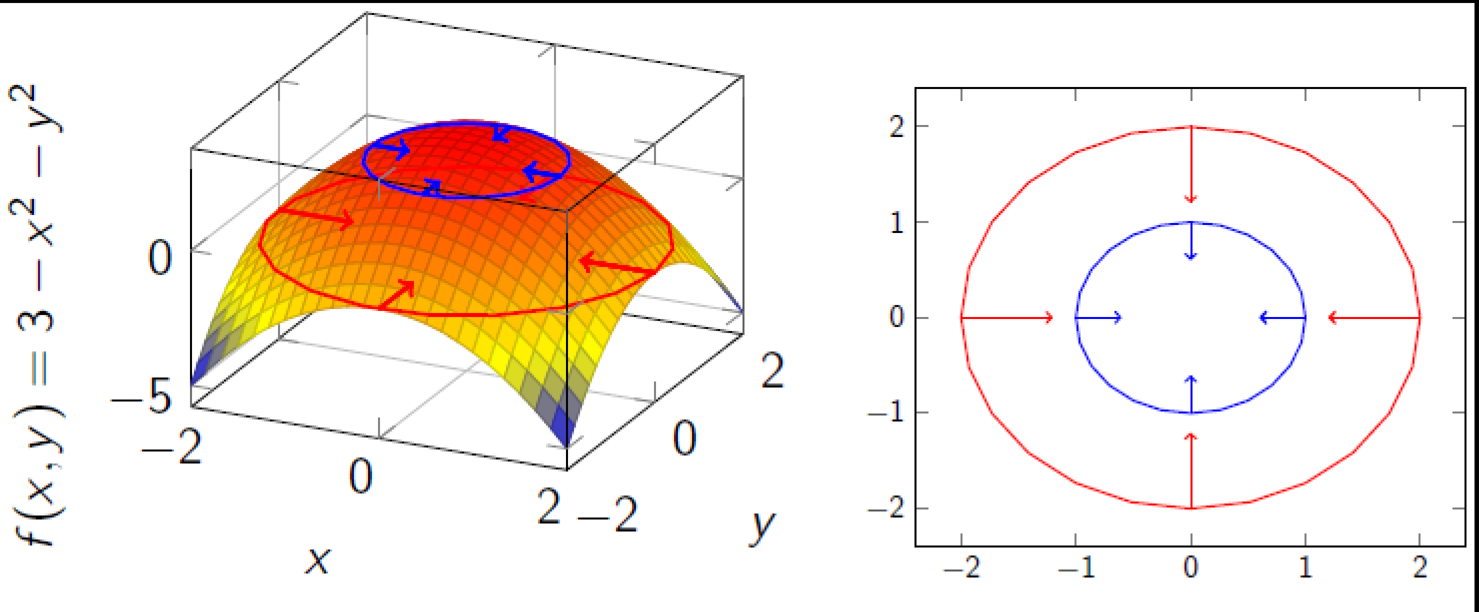
  $\Phi_t = \alpha\,\mathrm{DIV}\,(\nabla\Phi / |\nabla\Phi|)$



$t \rightarrow$

3. Dynamic Implicit Interfaces

- The normal vectors are perpendicular to the surface tangent plane (for a given point).



(a) 3D plot of ∇f (b) Seen from above

4. Algorithms

1. V. Caselles et al., 1993 "A Geometric Model for Active Contours in Image Processing"

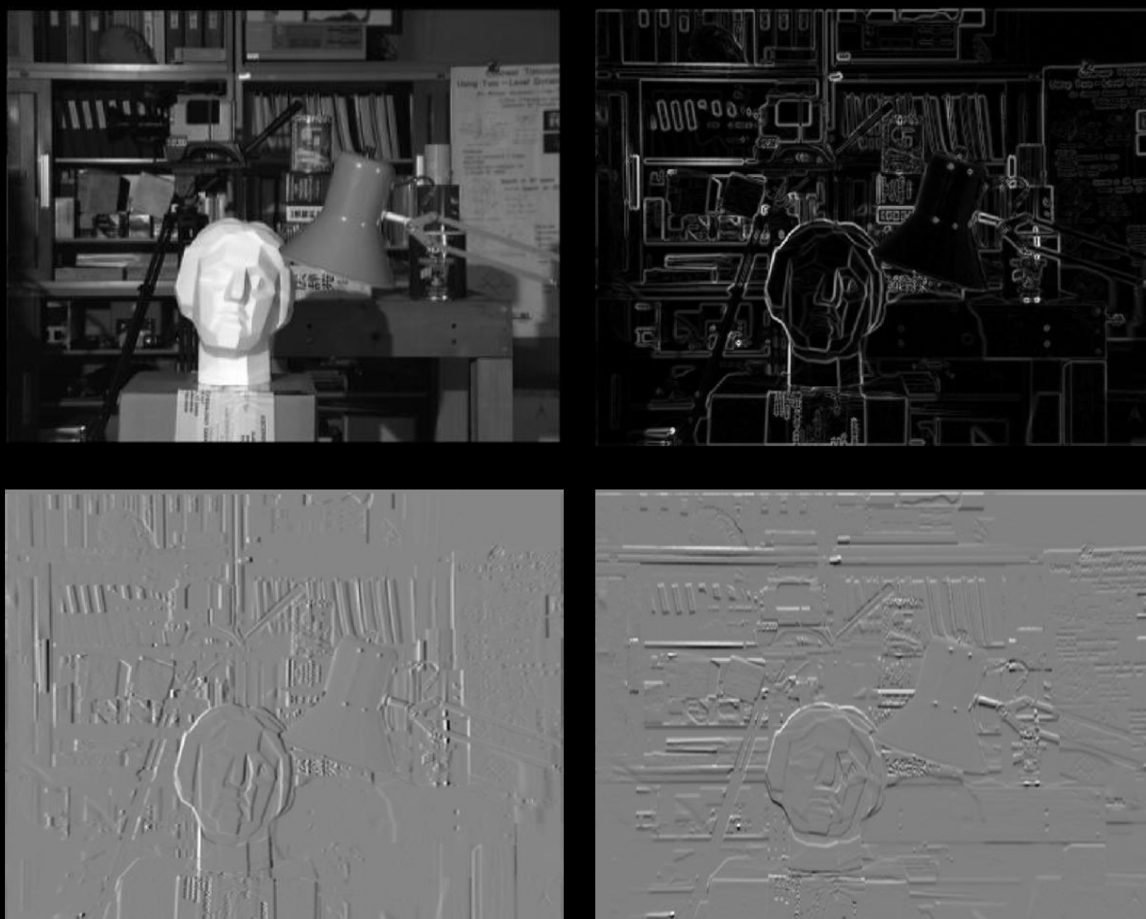
$$\Phi_t = \underbrace{g(|\nabla I|) DIV\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right)|\nabla\Phi|}_{\text{minimises local curvature}} + \underbrace{g(|\nabla I|)c|\nabla\Phi|}_{\text{balloon force}}$$

$$\Phi_t = \underbrace{\left(g(\nabla I) DIV\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right) + g(|\nabla I|)c\right)}_{\text{normal velocity}}|\nabla\Phi|$$

- g: monotonically descending function
- I: input image
- c: "balloon" force weight… $c > 0 \rightarrow$ inflation, $c < 0 \rightarrow$ contraction

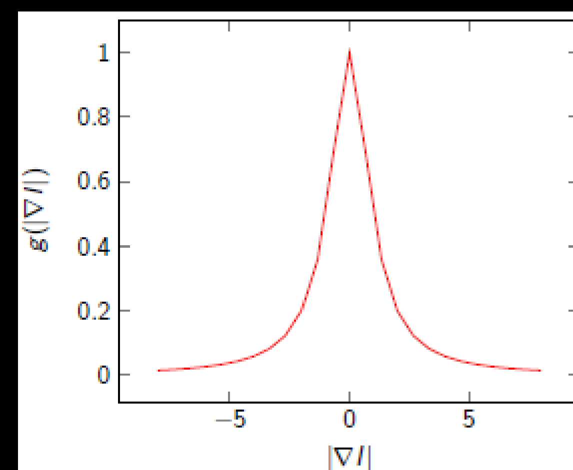
4. Algorithms

- g(∇I) acts as a stopping function for the moving contour when it reaches ROI edges (i.e. when |∇I| is "big")



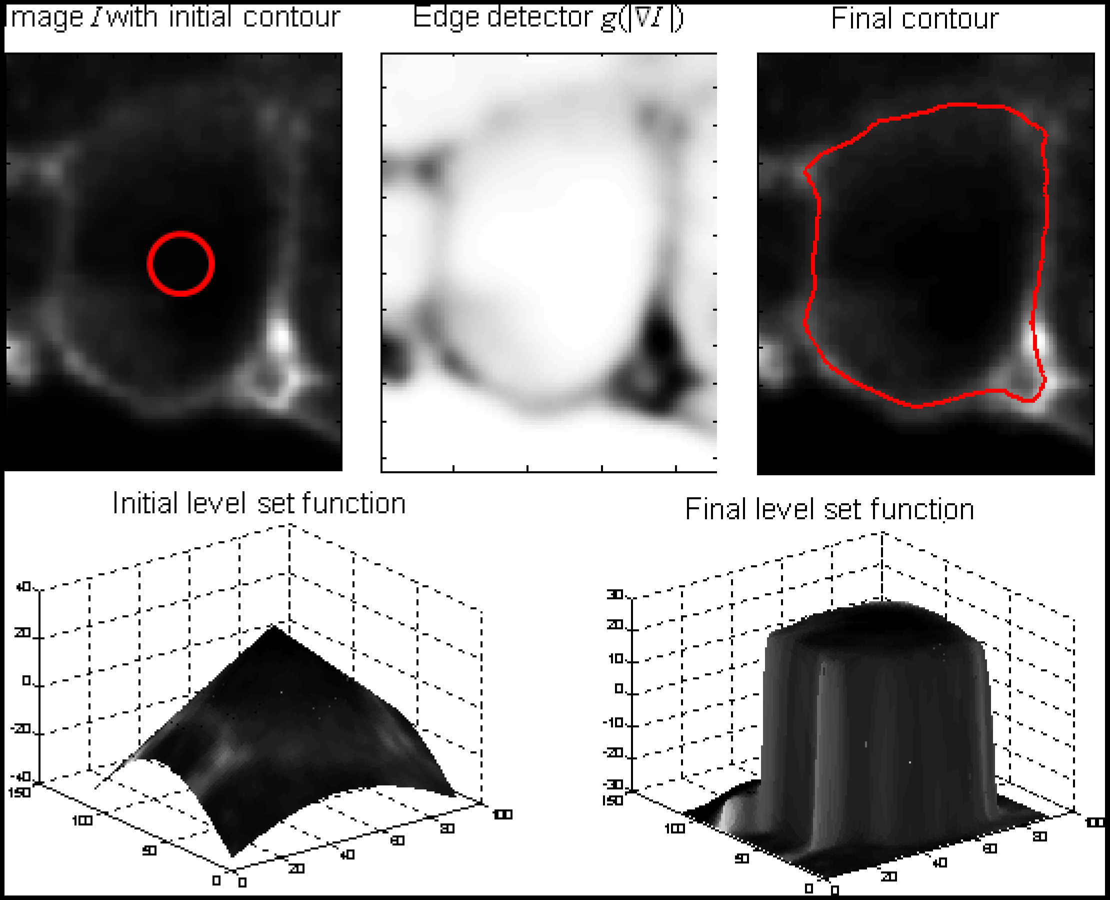
$$g(|\nabla I|) = \frac{1}{1 + \left(\frac{|\nabla I|}{\lambda}\right)^2}$$

λ: weight factor



4. Algorithms

2. Subjective Contours
   (Sarti & Sethian, PNAS 2002; Zanella et al. 2010 TIP)



http://ieeexplore.ieee.org/document/5280277/?arnumber=5280277&tag=1

References

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  S. Osher, R. Fedkiw

- A. Sarti, J.A. Sethian

- www.math.ucla.edu/~sjo/