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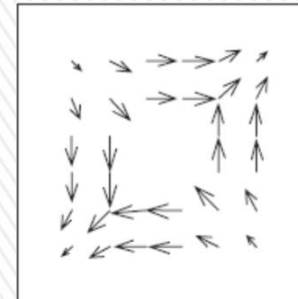
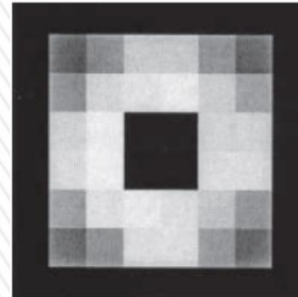
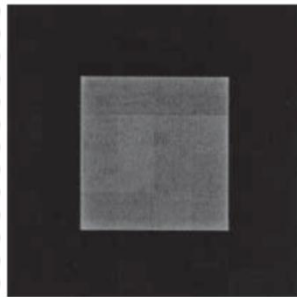
Prepaso práctico N°4

## Filtros de borde, gaussiano y morfológico

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(1) *Programa de Magister en Informática Médica, Universidad de Chile/Universidad de Heidelberg*

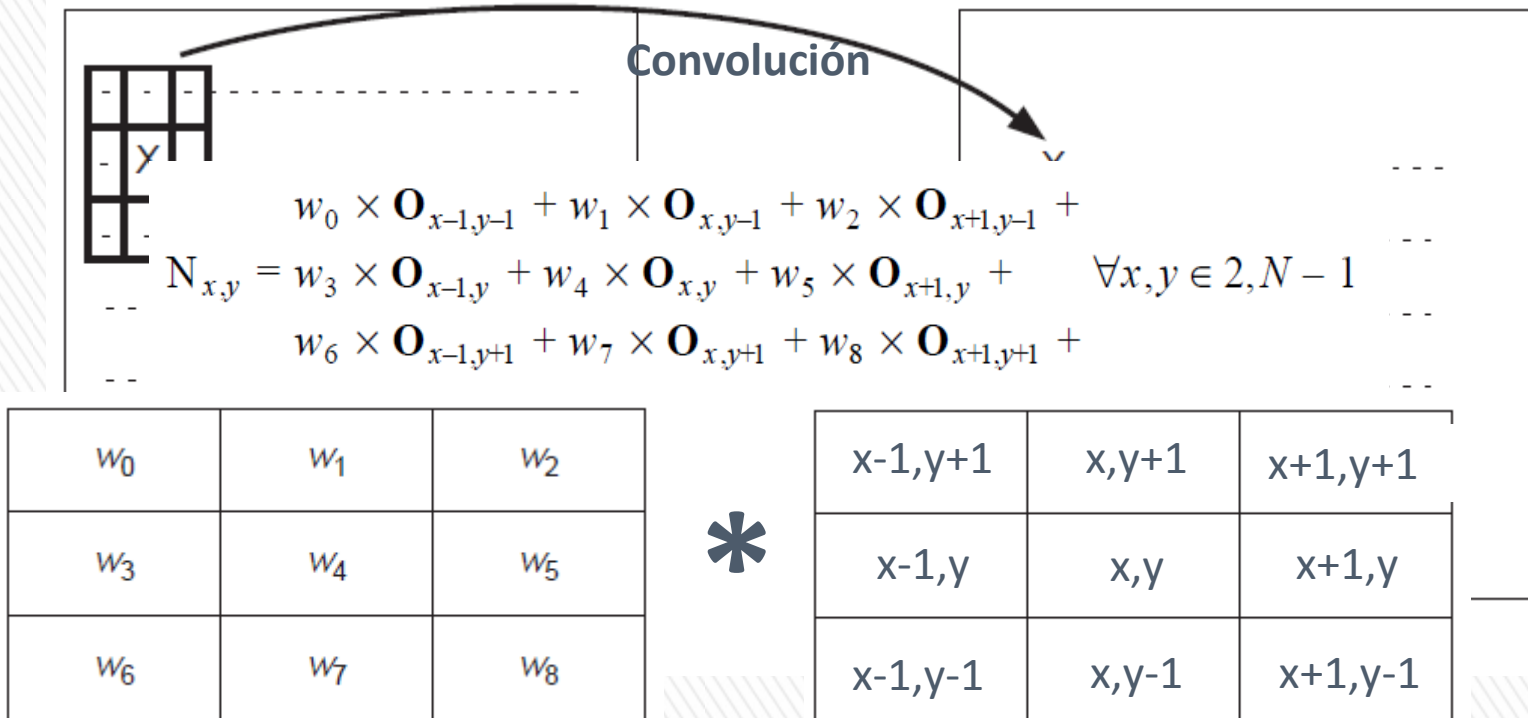
(2) *Programa de Doctorado en Ciencias con Mención en Biología Molecular, Celular y Neurociencias, Universidad de Chile.*





## Kernel y operaciones de grupo (Plantilla, matriz, template, etc)

Group operations calculate new pixel values from pixels' neighbourhoods by using a 'grouping' process. The group operation is usually expressed in terms of template convolution where the template is a set of weighting coefficients. The template is usually square, and its size is usually odd to ensure that it can be positioned appropriately. The size is normally used to describe the template; a  $3 \times 3$  template is three pixels wide by three pixels long

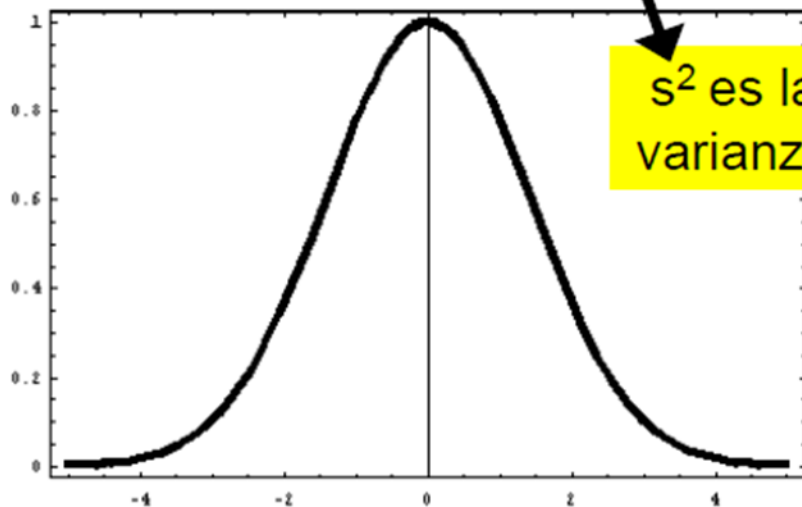




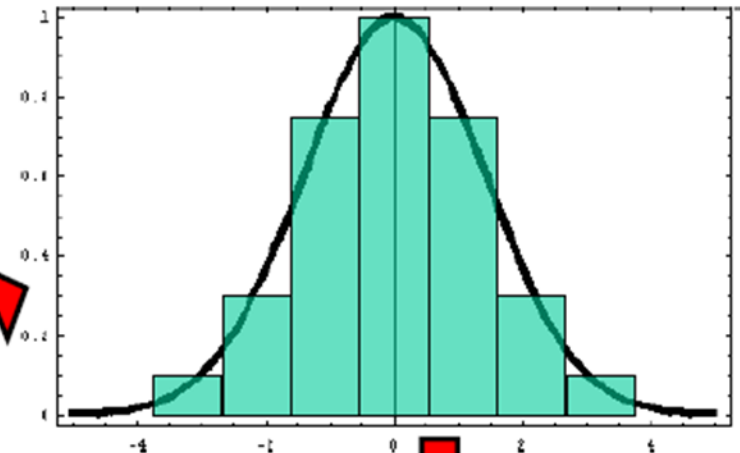
- **Suavizado gaussiano:** media ponderada, donde los pesos toman la forma de una campana de Gauss.
- **Ejemplo.** Suavizado gaussiano horizontal.

### Campana de Gauss

$$f(x) = e^{-x^2/s^2}$$



### Campana discreta



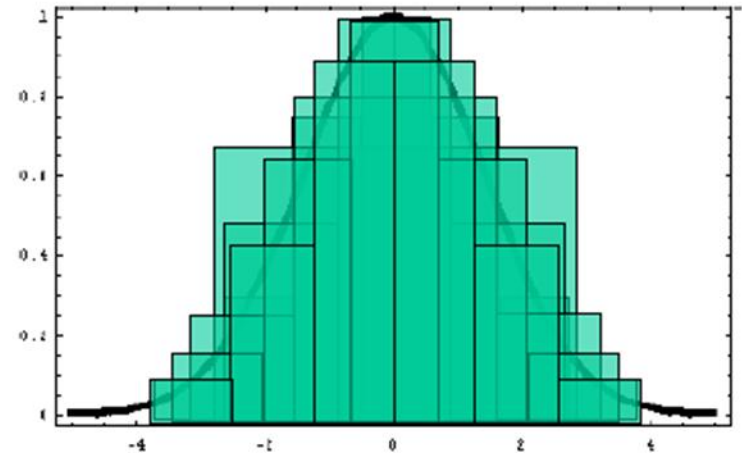
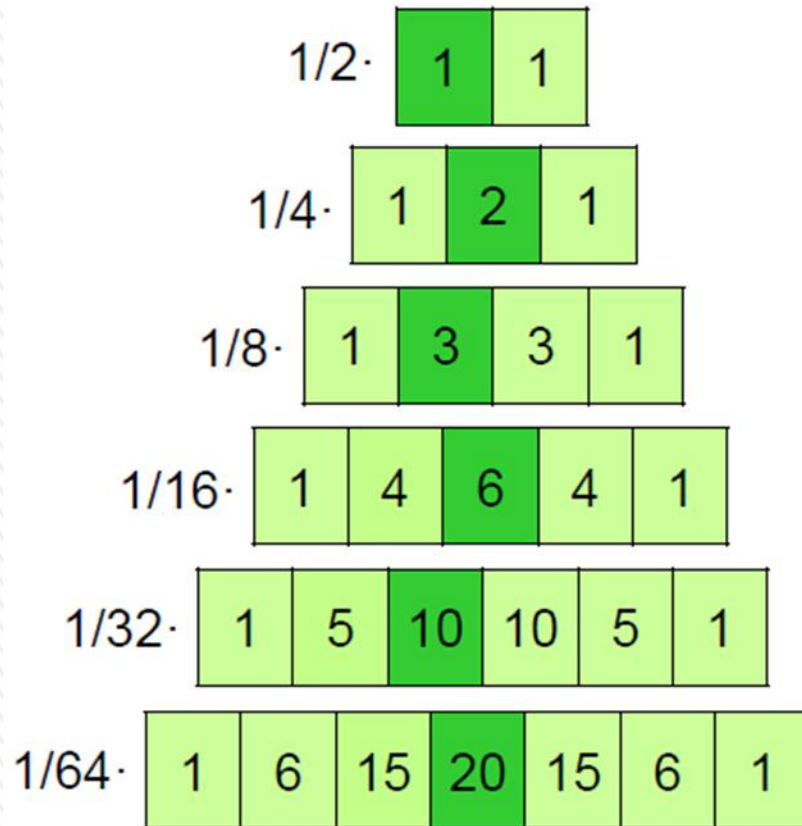
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1	6	15	20	15	6	1
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- **¡Magia!** Las filas del triángulo de Pascal forman discretizaciones de la campana de Gauss.

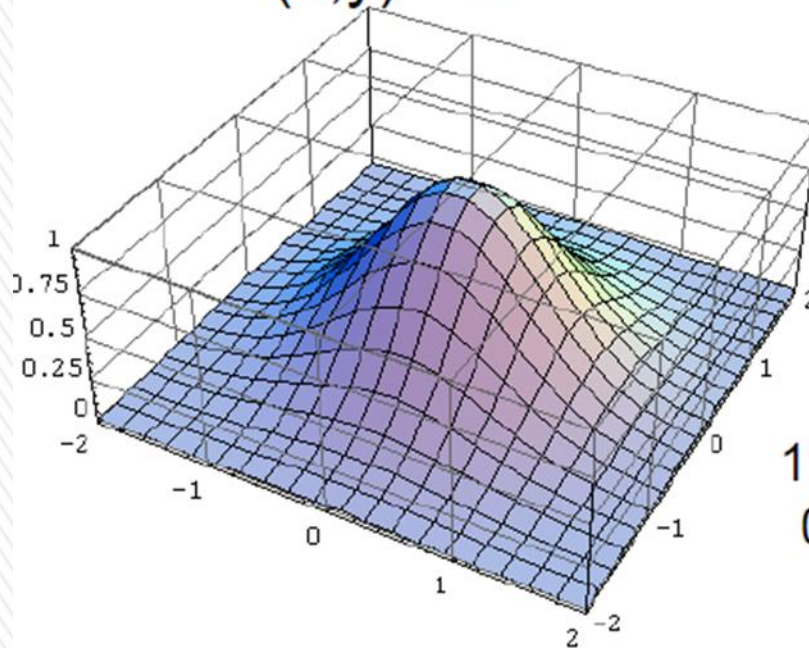




- Normalmente, el suavizado gaussiano se aplica en dos dimensiones. Los pesos de la máscara dependen de la **distancia al píxel central**.

## Campana de Gauss 2D

$$f(x,y) = e^{-(x^2+y^2)/s^2}$$



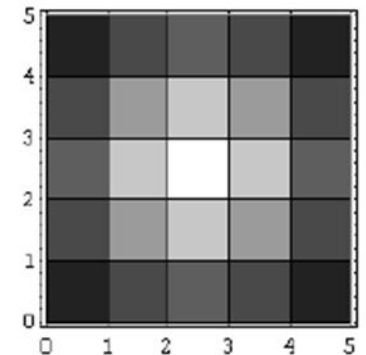
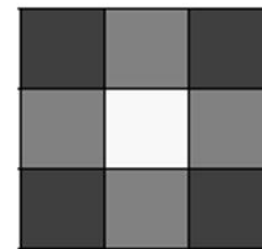
## Máscara gaussiana de 3x3

1	2	1
2	4	2
1	2	1

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1: blanco  
0: negro



EJEMPLO

## Operadores de suavizado.



- **Efecto de niebla.** Dada una imagen bien definida, queremos simular una niebla (objetivo *empañado*).
- **Idea:** calcular una media ponderada entre la imagen original y un suavizado gaussiano de la imagen.

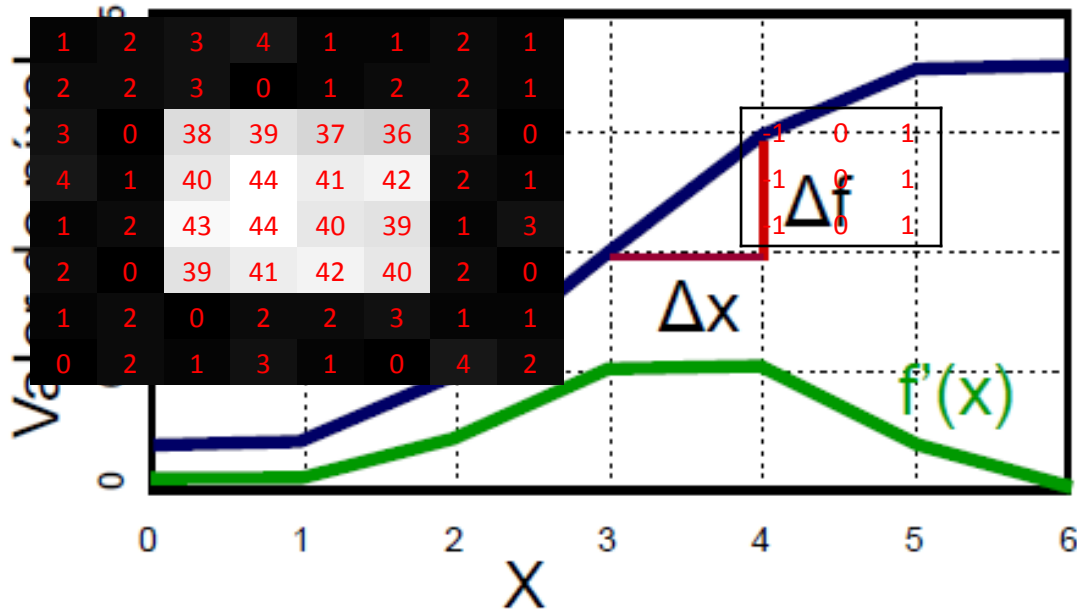
A Imagen original



B. Suaviz. gauss. 40x40

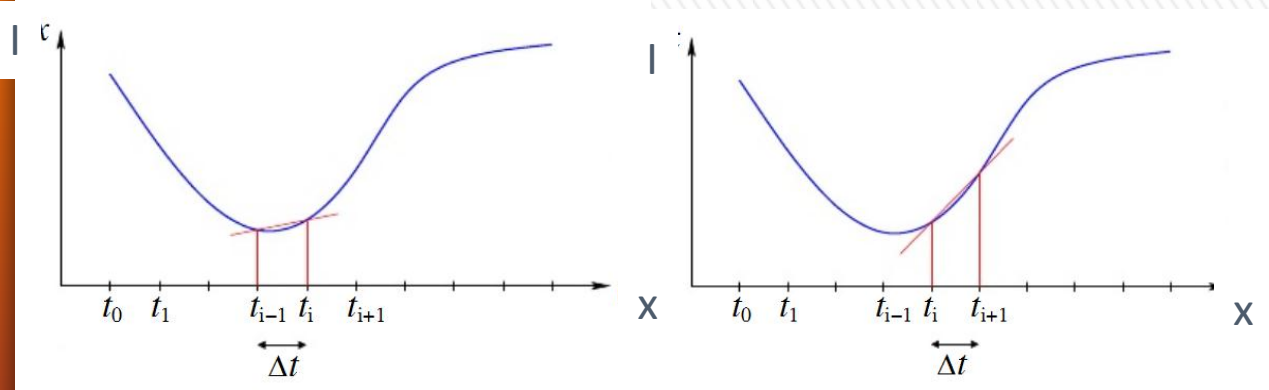


# Detectores de Borde



4	3	0	-4	-1	2	-1	-4
4	38	39	-5	-4	-32	-37	-7
3	72	80	-2	-3	-72	-78	-7
3	113	124	-3	-10	-112	-113	-6
3	115	126	1	-8	-118	-117	-5
4	78	83	2	-5	-80	-78	-4
4	37	42	5	-3	-38	-40	-7
4	0	1	2	-2	2	0	-5

← Derivada



$$\approx \frac{I_i - I_{i-1}}{\Delta x}$$

$$\approx \frac{I_{i+1} - I_i}{\Delta x}$$

$\Delta x = 1$

-1	0	+1
-1	0	+1
-1	0	+1





Filtro capaz de determinar los bordes por la determinación de un gradiente de intensidad.

Este gradiente se calcula utilizando una aproximación de una diferencial discreta. Utilización de dos Kernels.

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$$

Horizontal

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A}$$

Vertical

$$\mathbf{G} = \sqrt{\mathbf{G}_x^2 + \mathbf{G}_y^2}$$

Intensidad

$$\Theta = \text{atan2}(\mathbf{G}_y, \mathbf{G}_x)$$

Dirección  
del  
cambio

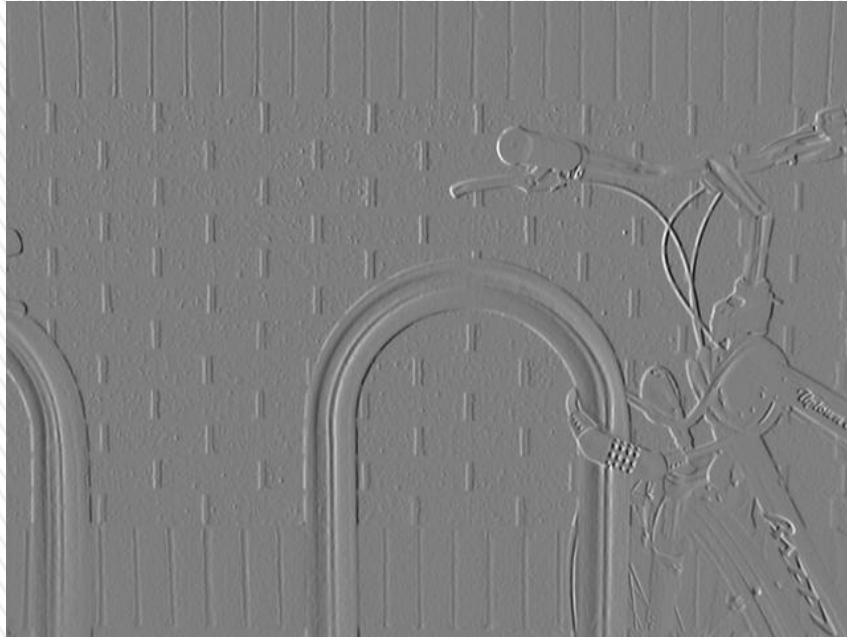
## Operador Sobel



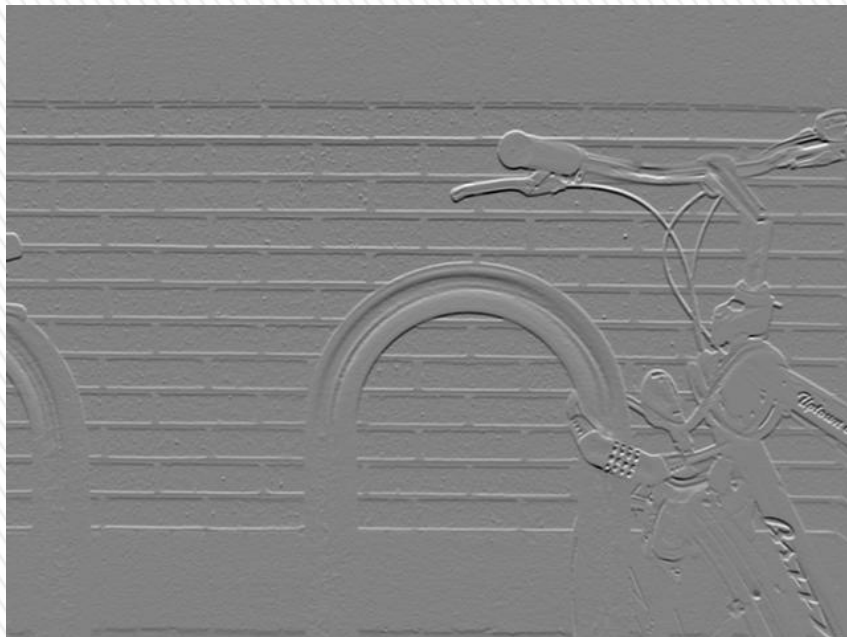
$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$$

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A}$$

# Operador Sobel



$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$$



$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A}$$

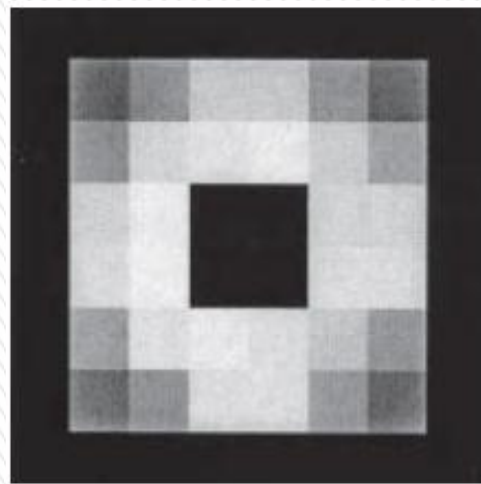
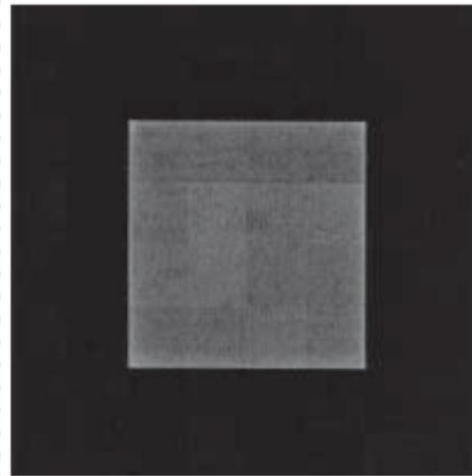
# Operador Sobel



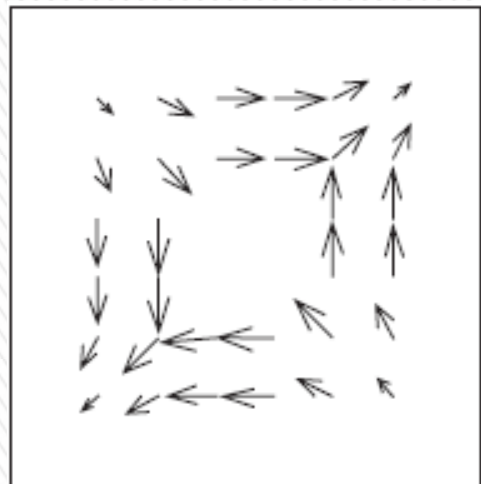
$$G = \sqrt{G_x^2 + G_y^2}$$



# Operador Sobel



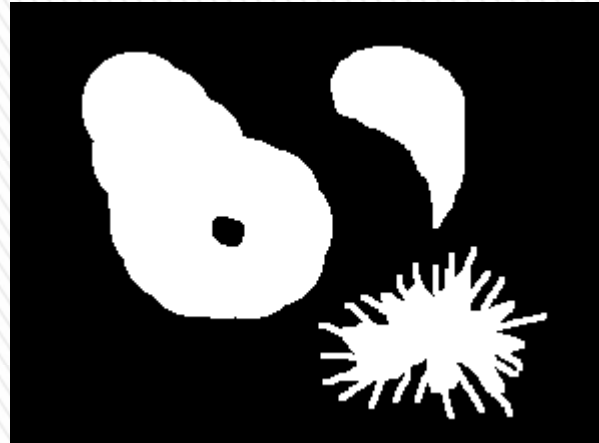
$$G = \sqrt{G_x^2 + G_y^2}$$



$$\Theta = \text{atan2}(G_y, G_x)$$

# Operadores morfológicos

Dilatación



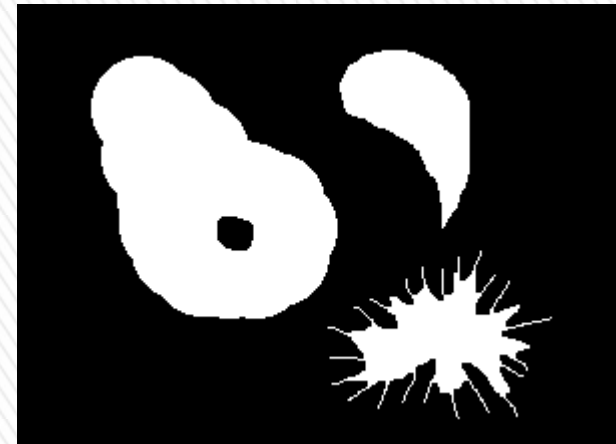
Apertura



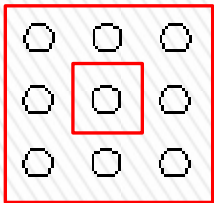
Erosión



Cierre



# Apertura

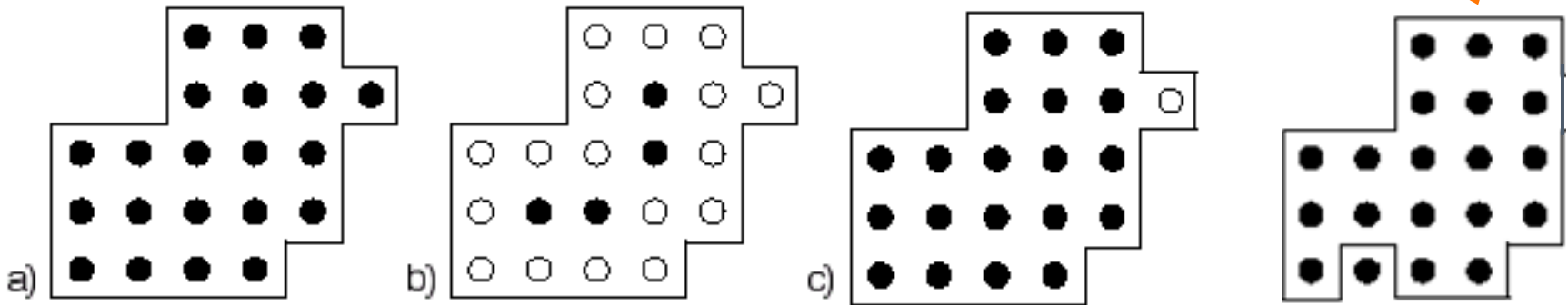


Erosión

Apertura

Dilatación

Cierre



Notación

Dilatación  $A \oplus B$

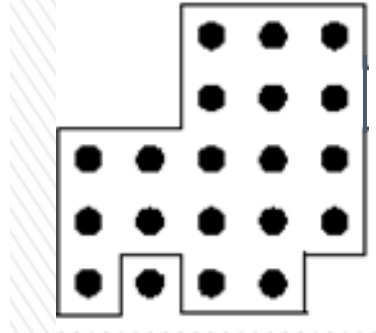
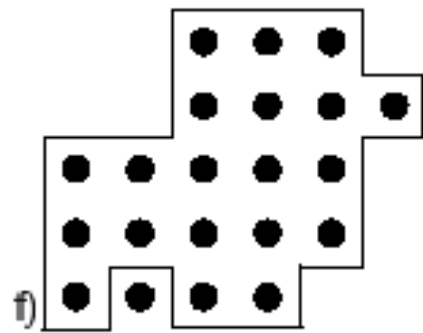
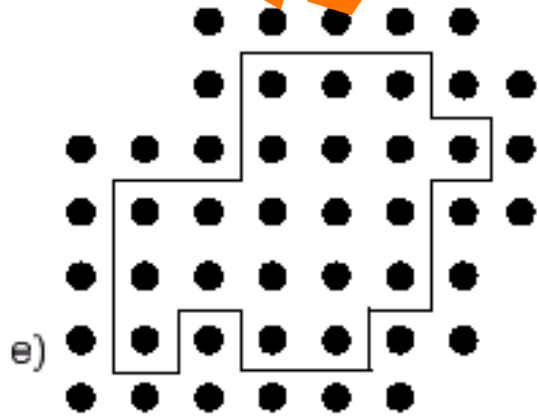
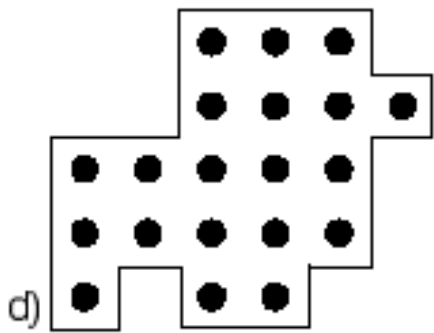
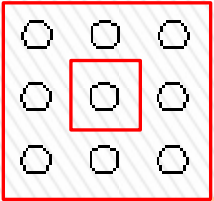
Erosión  $A \ominus B$

Apertura  $A \circ B = (A \ominus B) \oplus B,$

Cierre  $A \bullet B = (A \oplus B) \ominus B,$

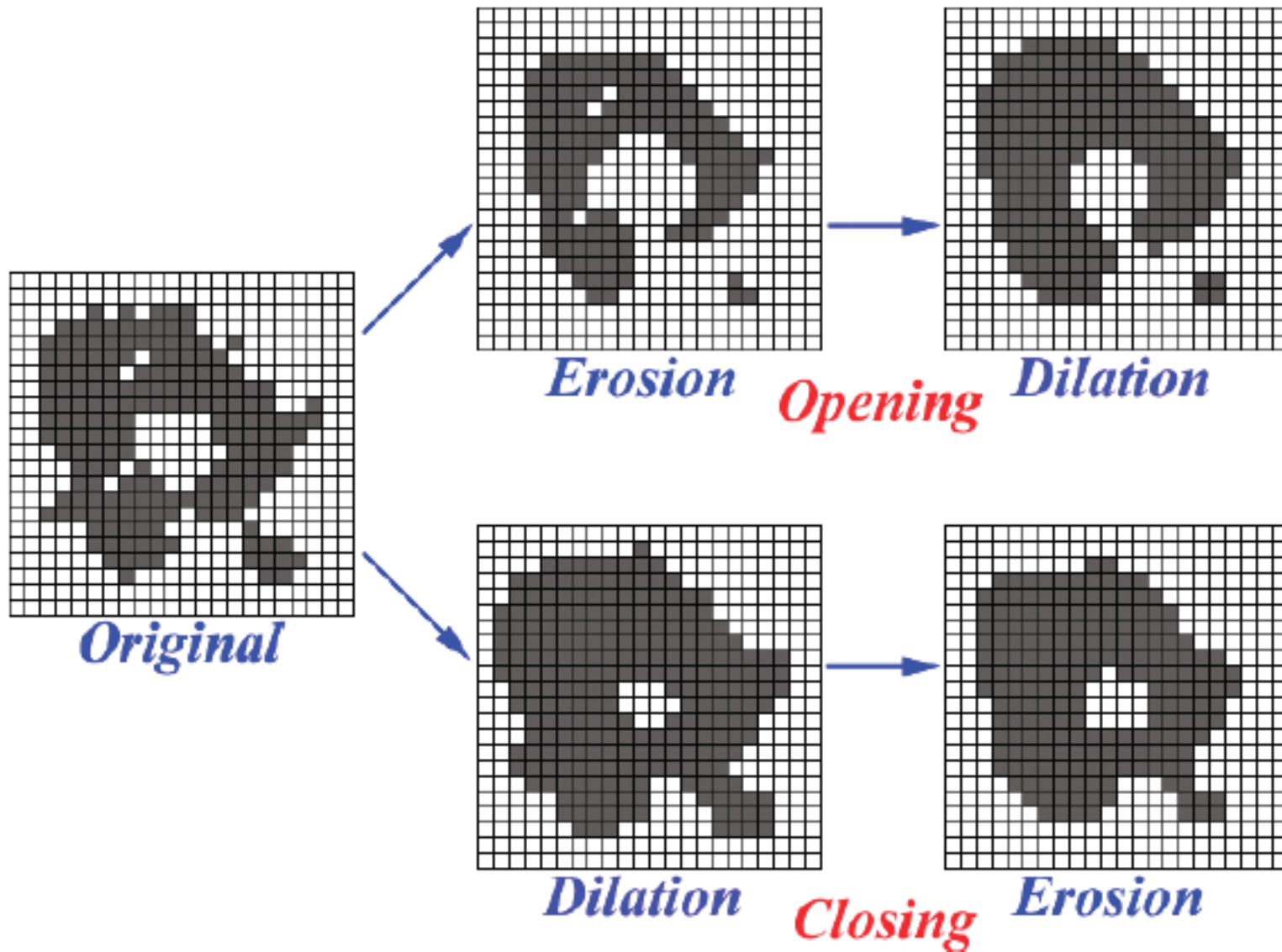


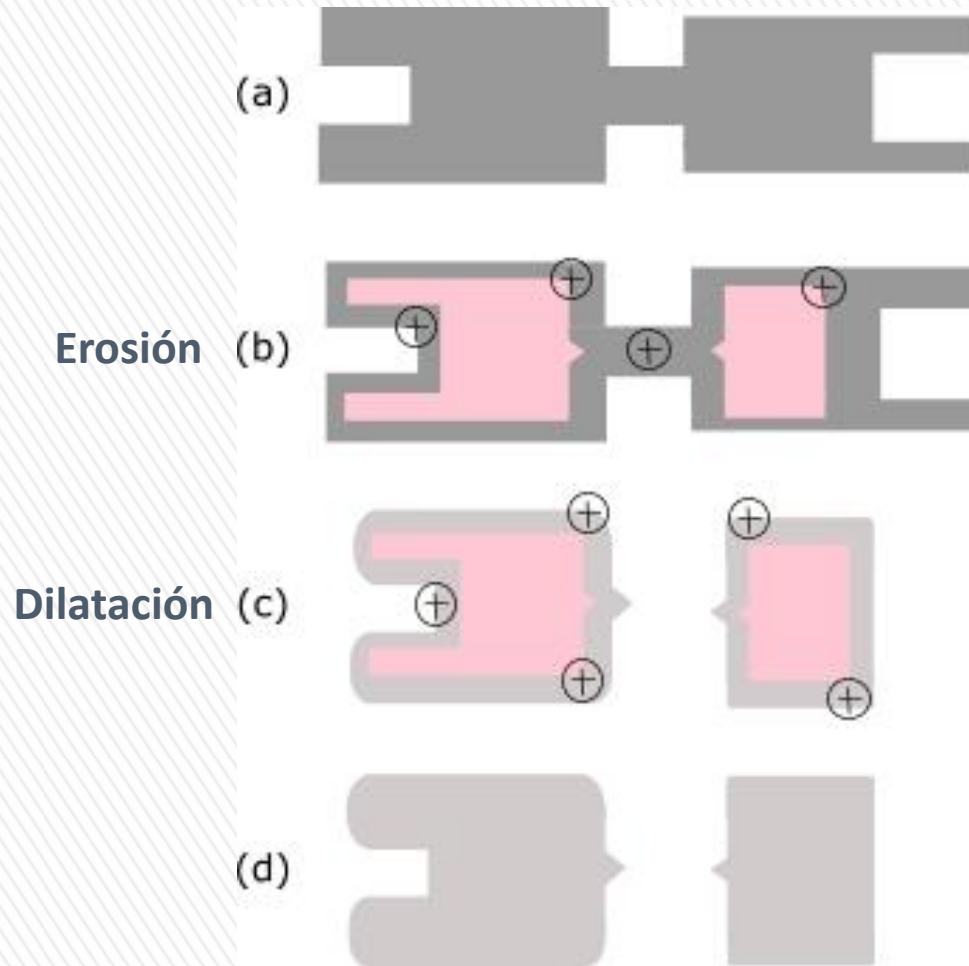
# Cierre



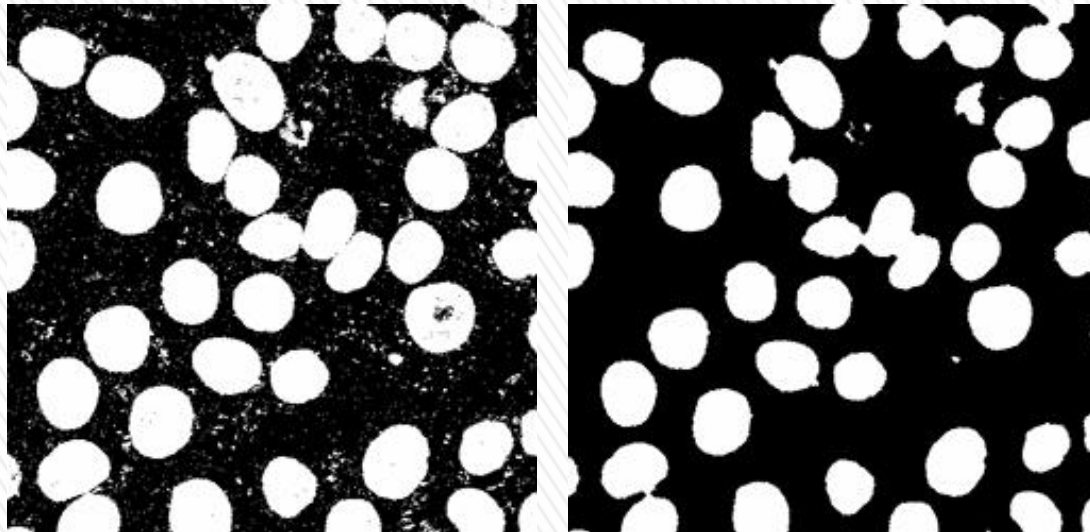


# Open-Close

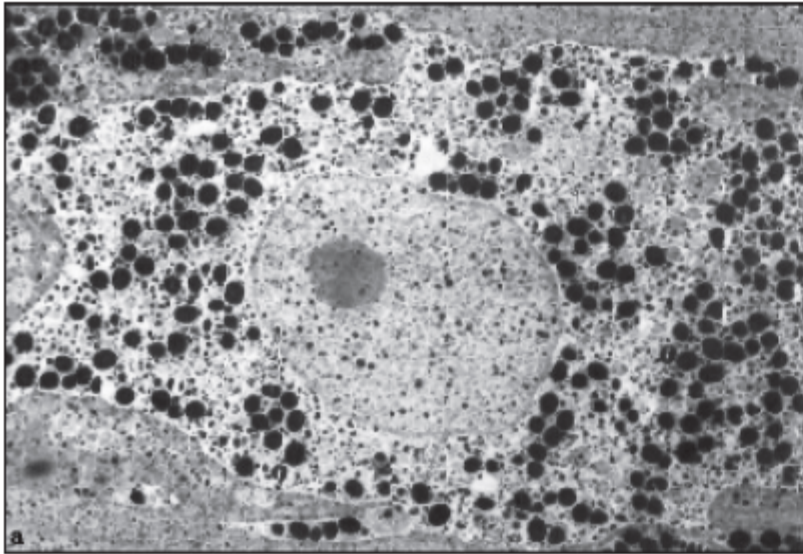




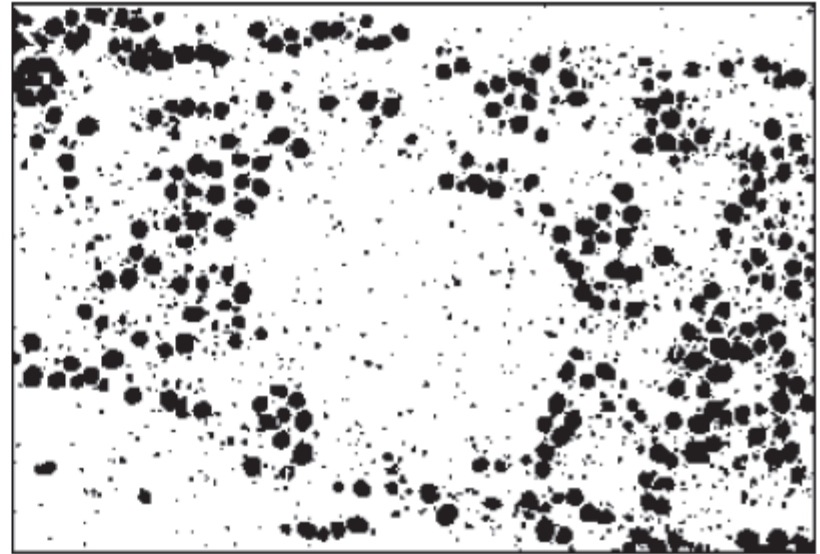
## Ejemplo Open-Close



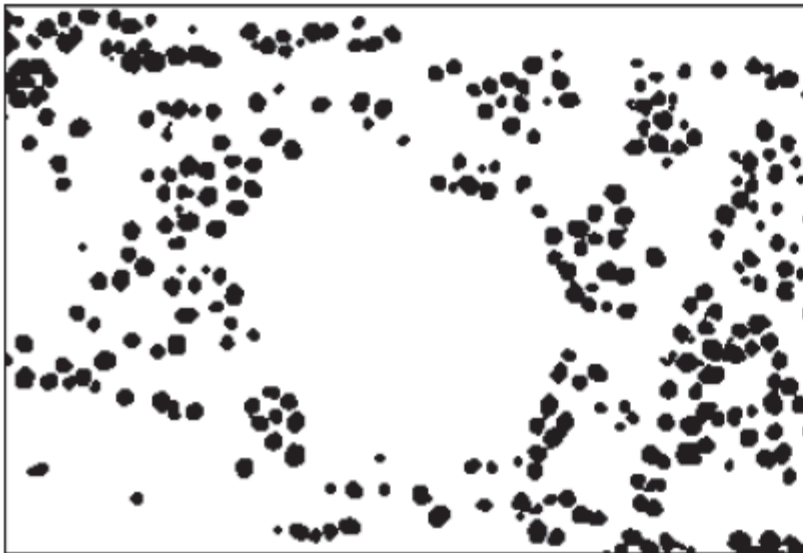
# Aplicaciones de operadores morfológicos binarios



(a)



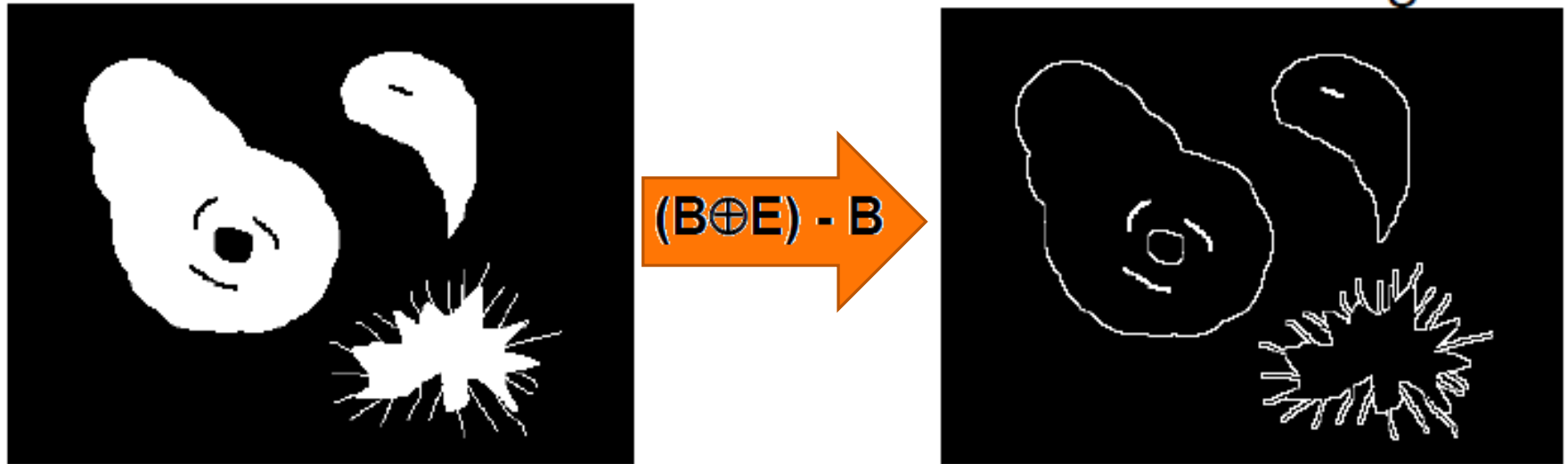
(b)



(c)

# Otros usos de operadores morfológicos elementales

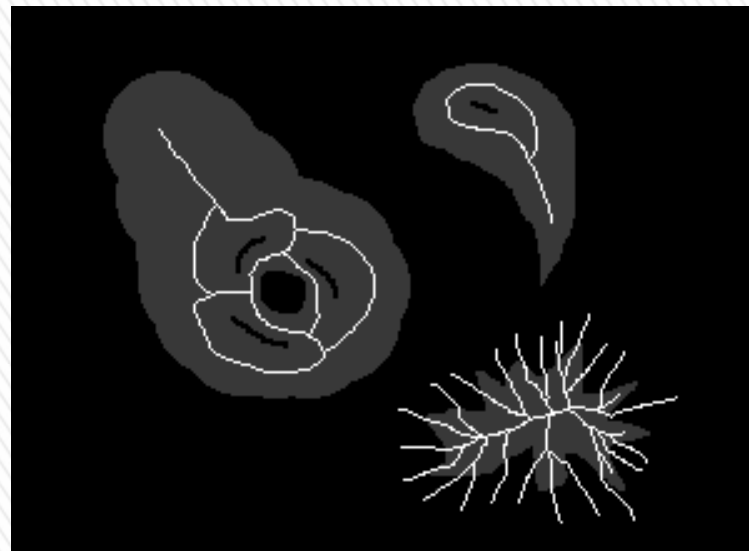
## Borde morfológico



## Thinning (Skeleton)

Erosiones sucesivas,  
excepto:

- si se pierde continuidad o
- hay un único punto



## Operadores de bordes.

- Otra forma más sencilla (aproximada) es usar máscaras de convolución adecuadas, por ejemplo de **Laplace**.
- La **función de Laplace** es la segunda derivada de la gaussiana.

