

Image Segmentation II

Advanced Approaches

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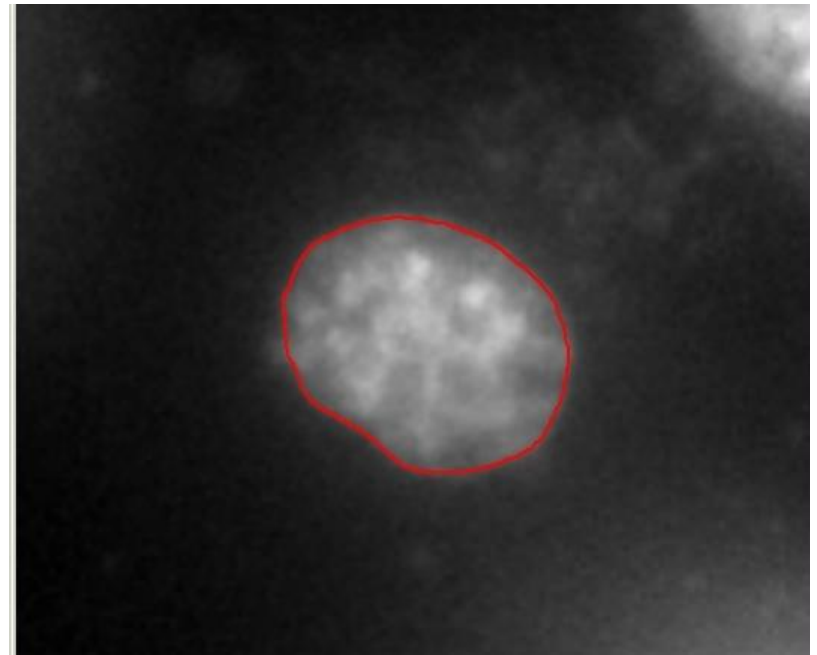
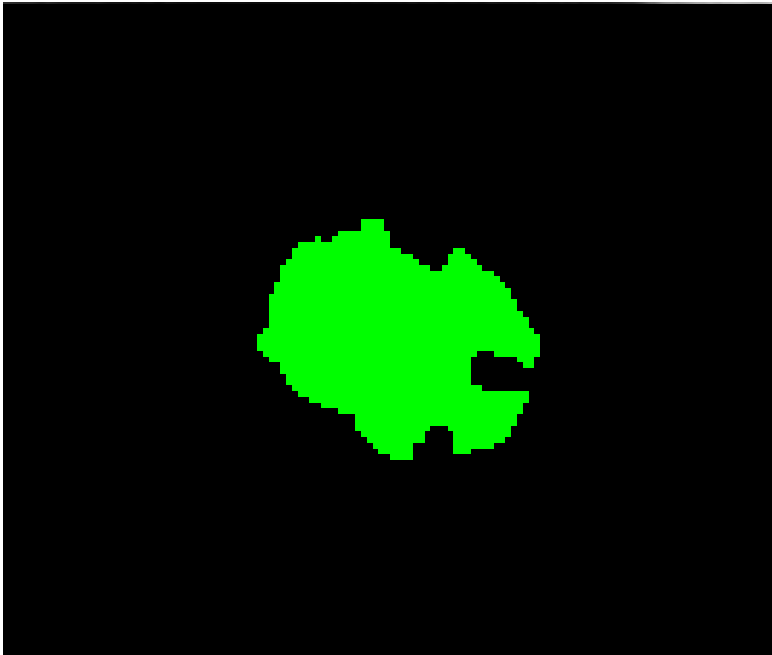
1. Segmentation I

- Digital image processing
- Segmentation basics

2. Segmentation II

- Advanced techniques

- “Some” times more information is needed in order to achieve a good segmentation



- Designed to optimize a cost or fitness function
 - Interest features detection (e.g. edges, points)
 - SIFT
 - Template matching
 - Hough transform
 - Pixel clustering
 - Statistics, probabilities
 - Graph cuts
 - Machine learning (support vector machine, neural networks...)
 - Differential equations, calculus of variations
 - Contour properties
 - Region properties

This can be a VERY long list...

- Template matching

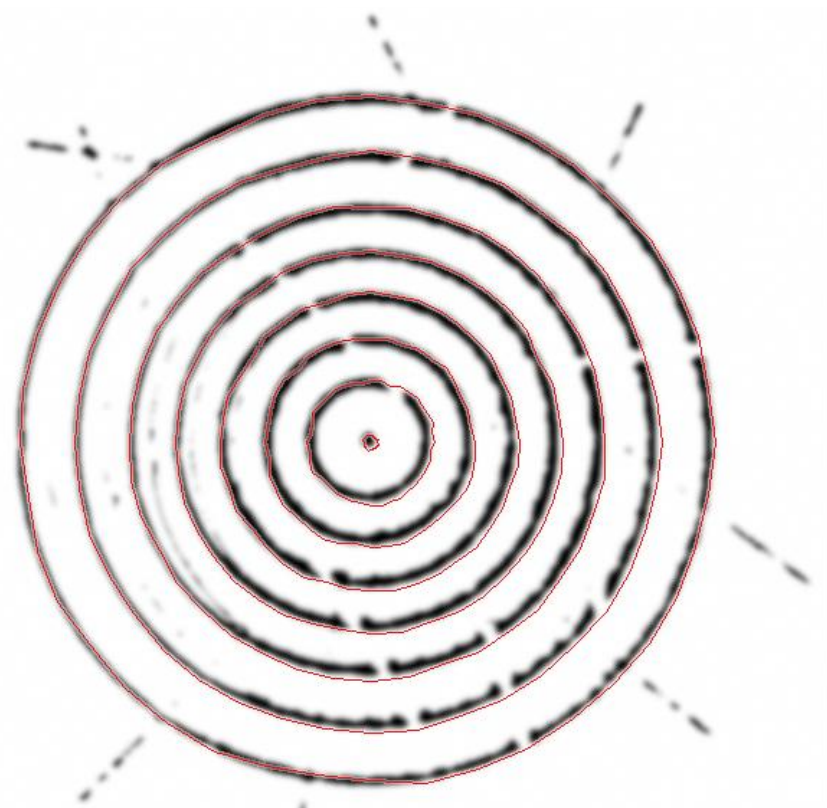
- “Classic model” Hough transform
- Applies to circles, line segments and a variety of shapes

Hough P (1959)

If we can detect edges we can approximate a circle (center, radius)

n connected edges and m ($=n$ or $<> n$) circles

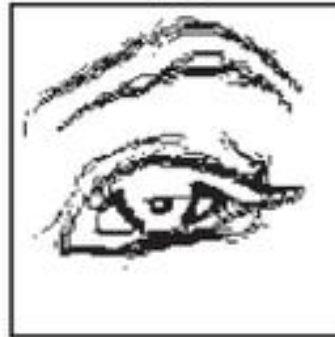
A test is performed to determine the circles with “best fit”



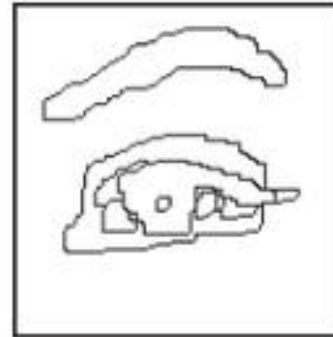
- Variational methods
 - Based on energy minimization, defining integral models
 - Idea: to include desirable features on segmented images (like homogeneous regions, short or smooth ROI boundaries)
 - Optimum solutions found by partial differential equations
 - Examples: Mumford-Shah, Ambrosio-Tortorelli, Chan-Vese (details in the book from Aubert & Kornprobst 2006)



image I



main discontinuities in I



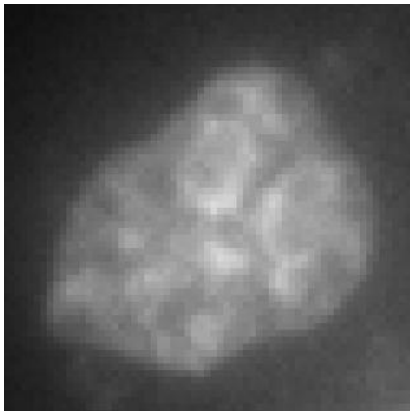
ROI boundaries B



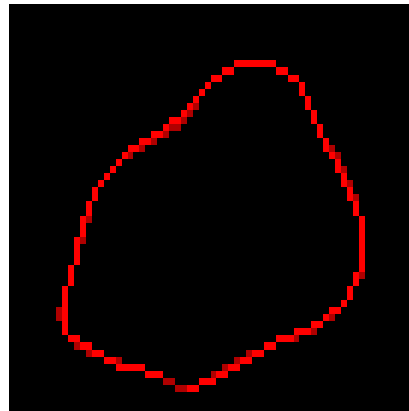
piecewise smooth
image J

$$E[J, B] = C \int d\vec{x} (I(\vec{x}) - J(\vec{x}))^2 + A \int_{D/B} \vec{\nabla} J(\vec{x}) \cdot \vec{\nabla} J(\vec{x}) d\vec{x} + B \int_B ds$$

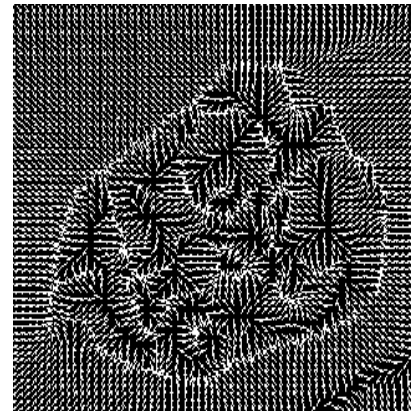
- Active contour models
 - Optimization of different properties



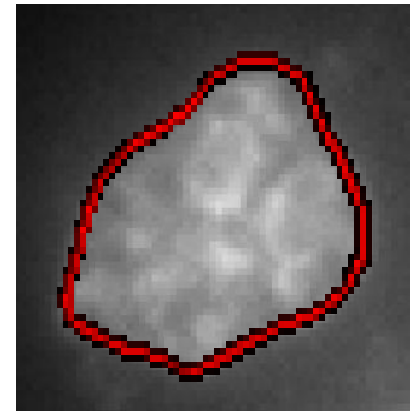
input
 image
 + initial guess



contour $C(s)$
 - elasticity
 (contraction)
 - rigidity
 (bending, cornering)



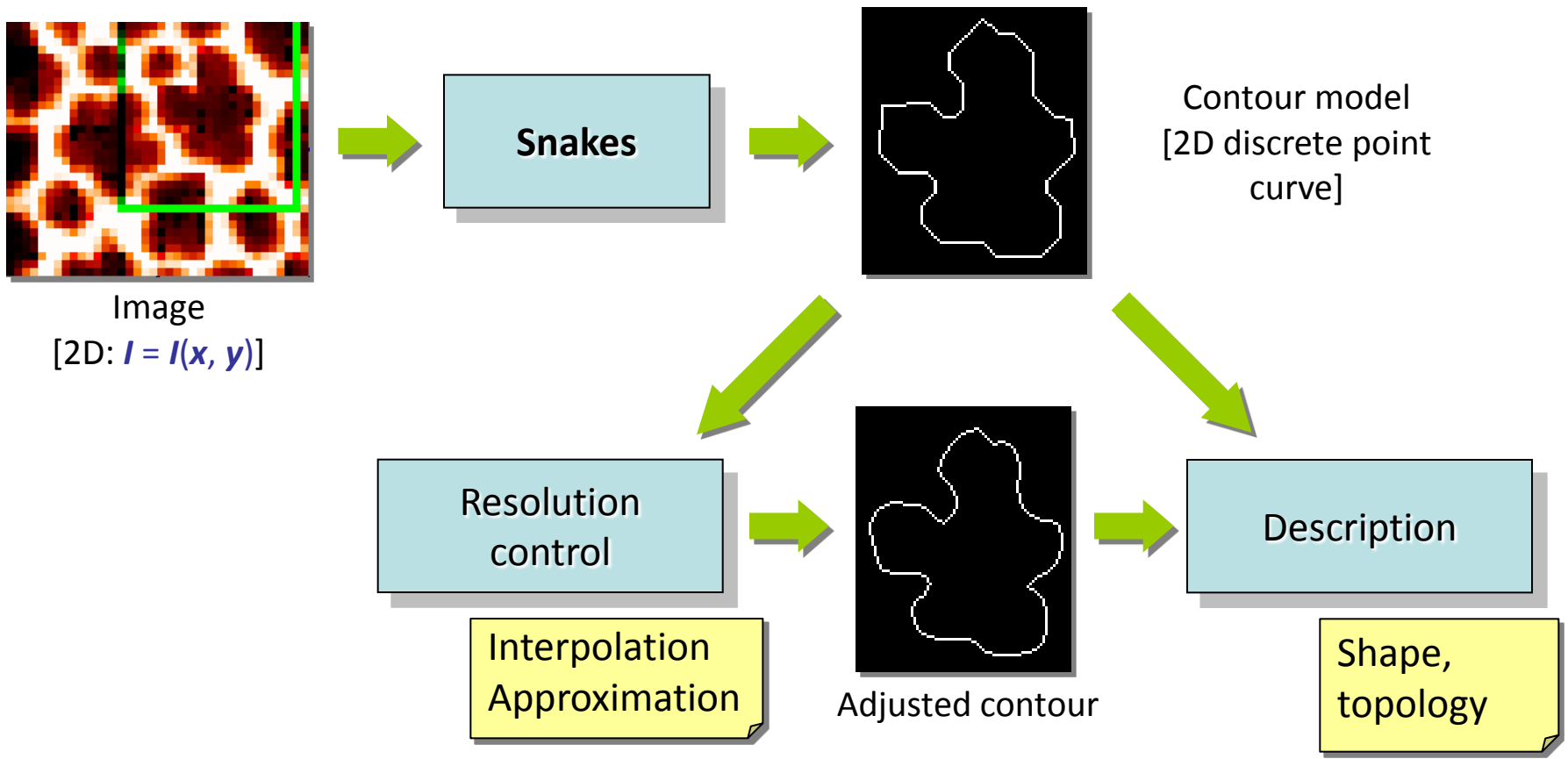
force field
 - repulsion
 - attraction



output: force balance
 minimal energy

First active contours approach:
 Kass, Witkin & Terzopoulos (1988)
 "Snakes"

- 2D active contours or *snakes*



1 contour function \rightarrow 1 ROI (this is called *parametric approach*)

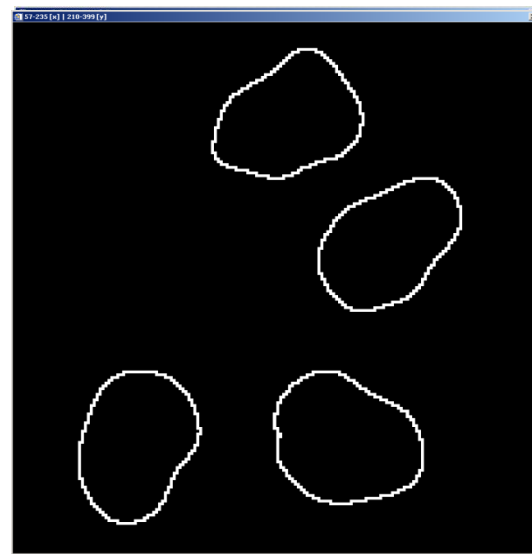
- 2D parametric curve

$$C = C(s) = [x(s), y(s)]$$

$s \in [0, 1]$ (arbitrary length)

- 2D discretization

$$C = \{[x_i, y_i]; i = 0..n\}$$

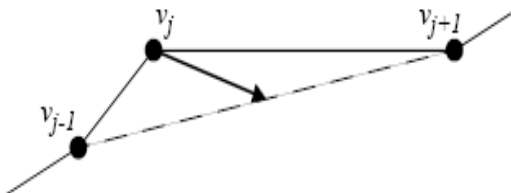


- Snakes: optimization derived from a **variational** approach

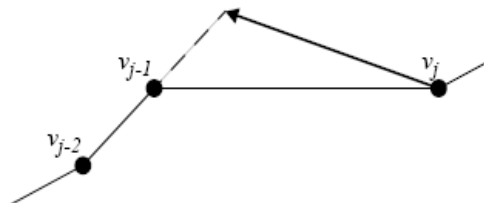
- Minimization of an **integral functional**
 “a snake minimizes its energy”

$$E = \int_0^1 \frac{1}{2} \left[\alpha \left| \frac{\partial C(s)}{\partial s} \right|^2 + \beta \left| \frac{\partial^2 C(s)}{\partial s^2} \right|^2 \right] + E_{ext} [C(s)] ds$$

Elasticity term
(coefficient α)

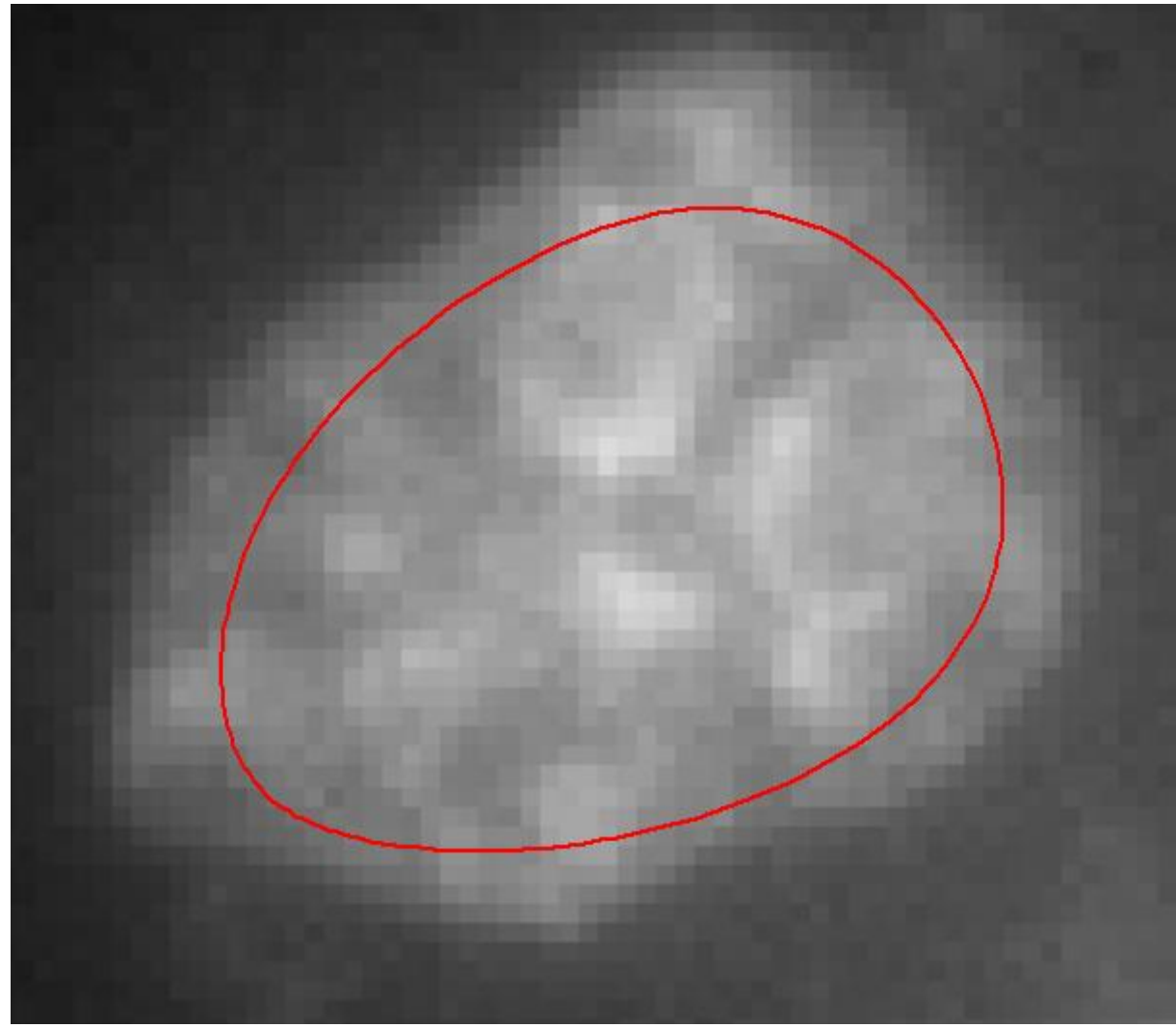


Internal energy,
contour dependant
Rigidity term
(coefficient β)

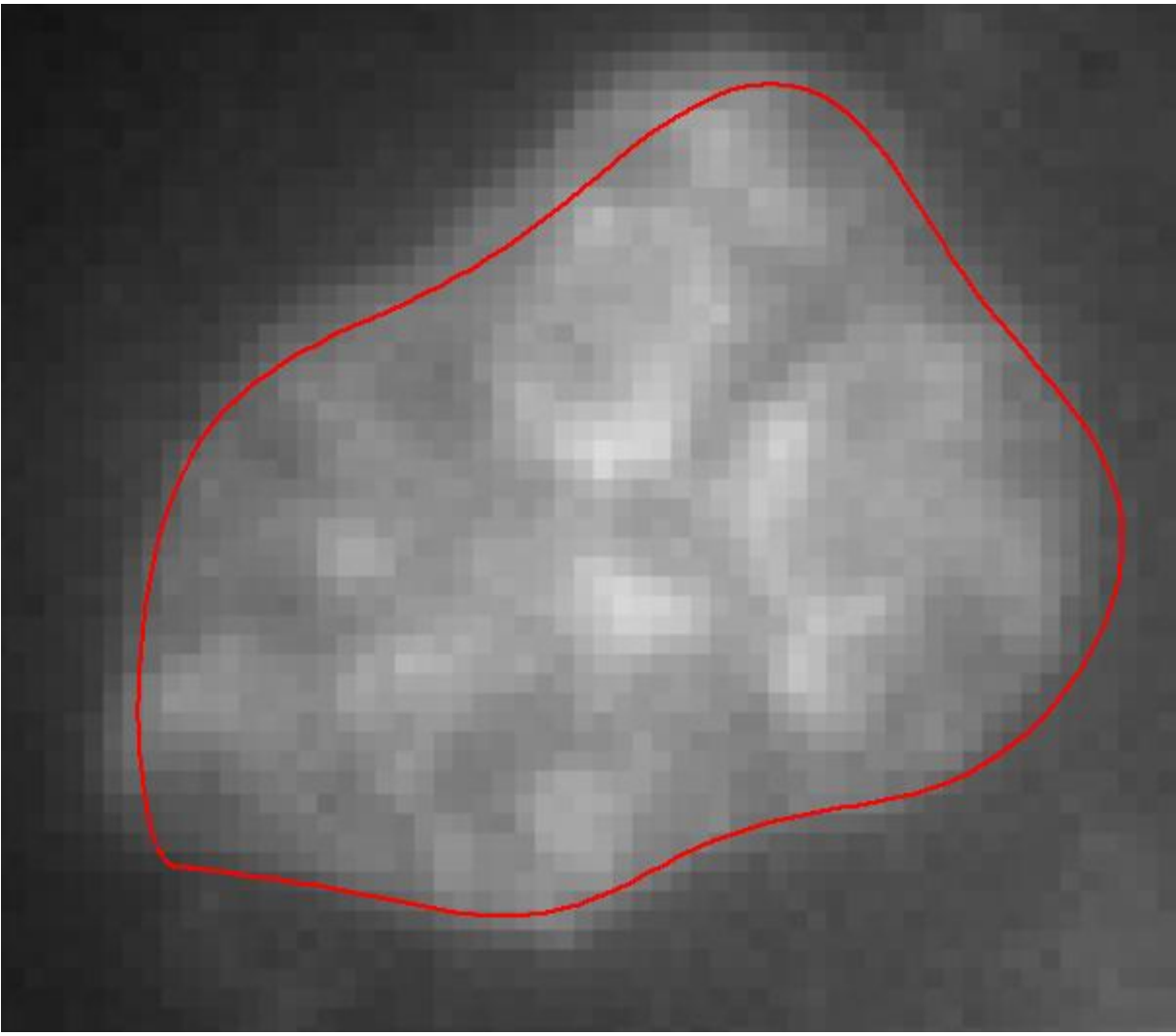


External energy,
image dependant

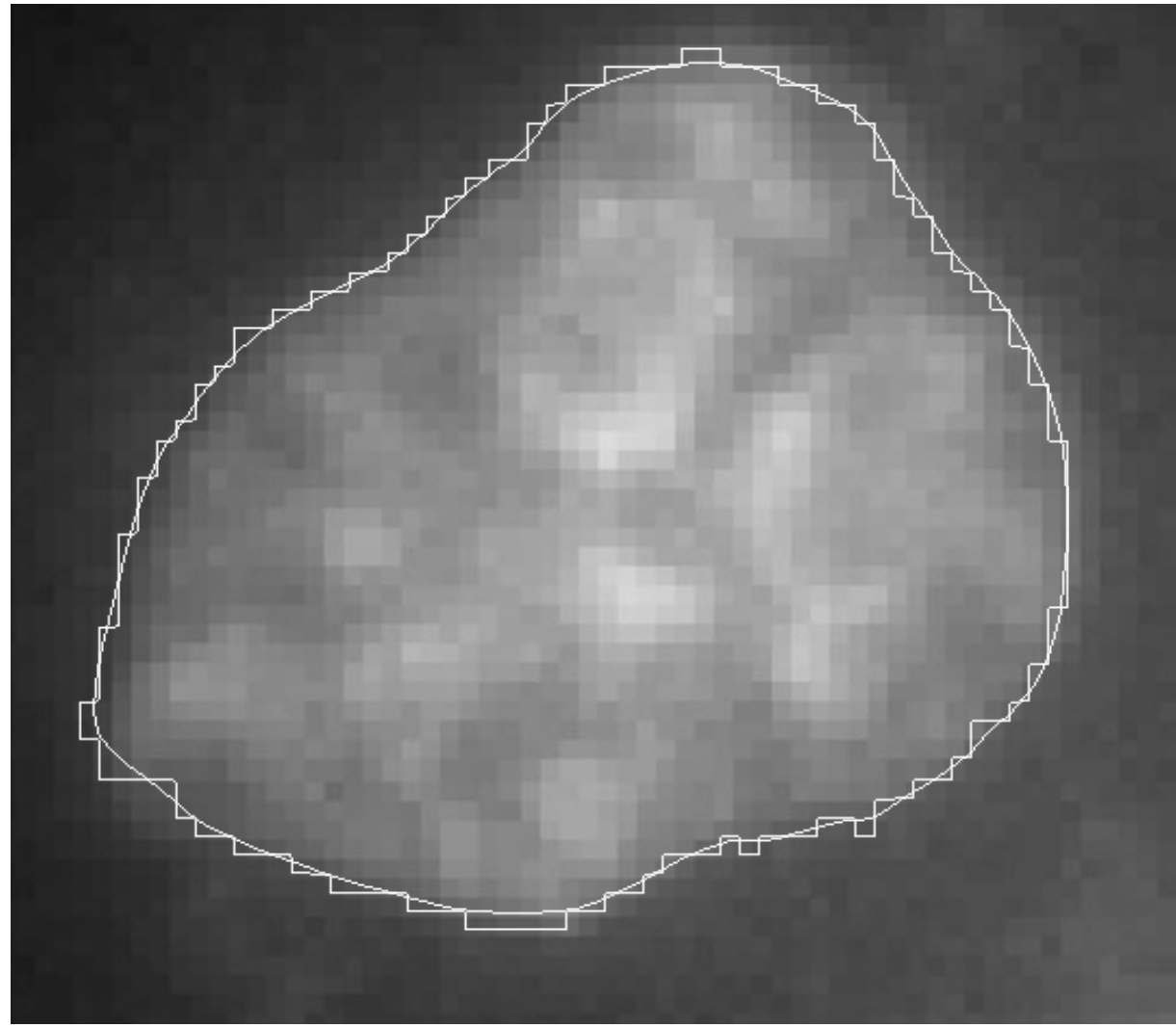
Elasticity - α



Rigidity - β

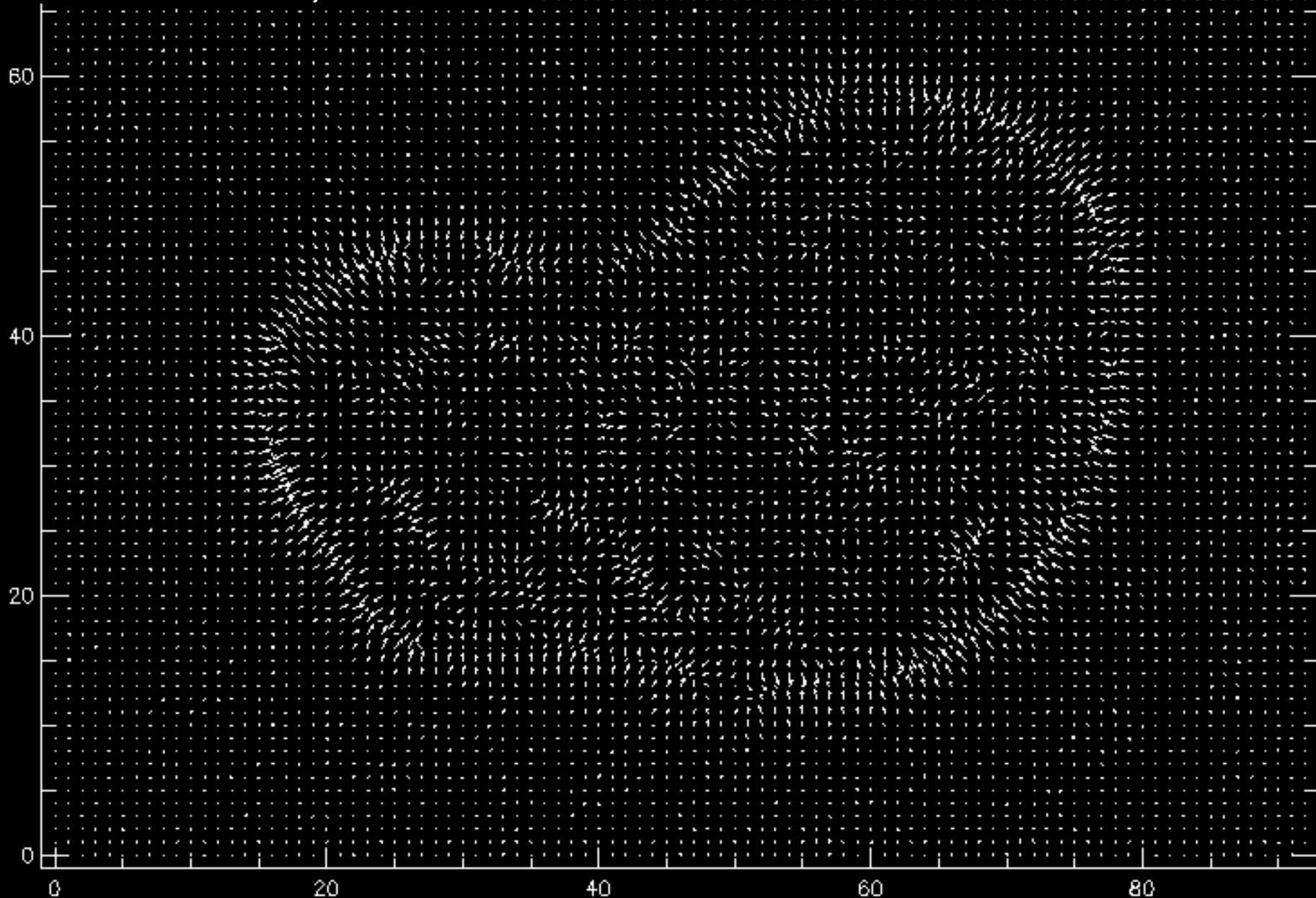


Resolution (f)

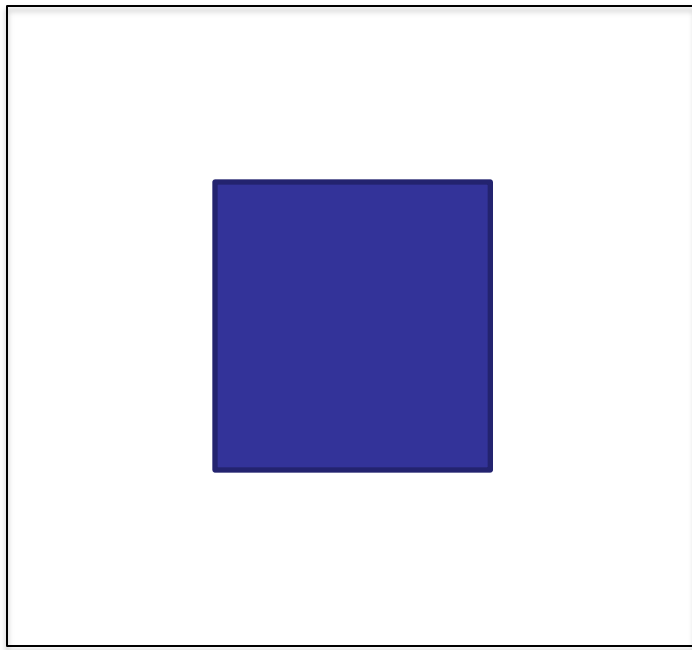


Intensity gradient vectors

$$V_0 = [I_x, I_y]$$

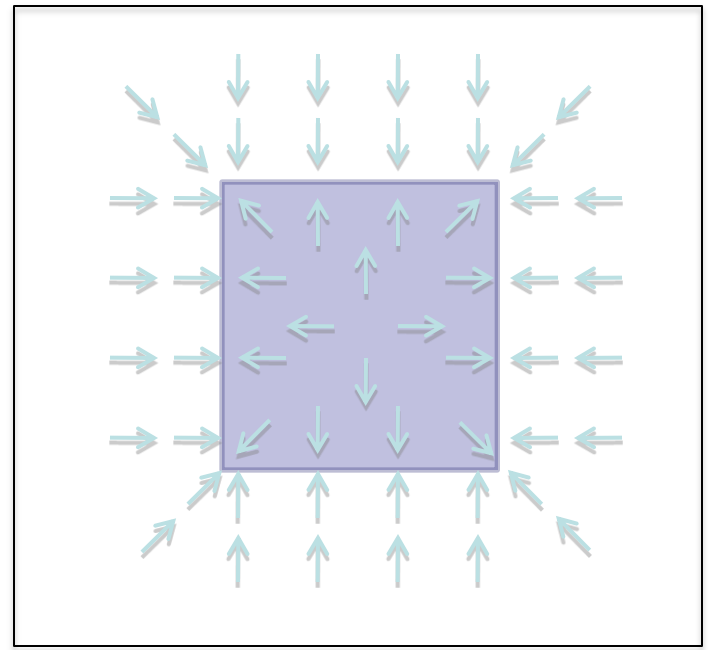


Attraction fields can be constructed from the intensity gradients



$$I(x, y)$$

Image intensity



Edgemap image

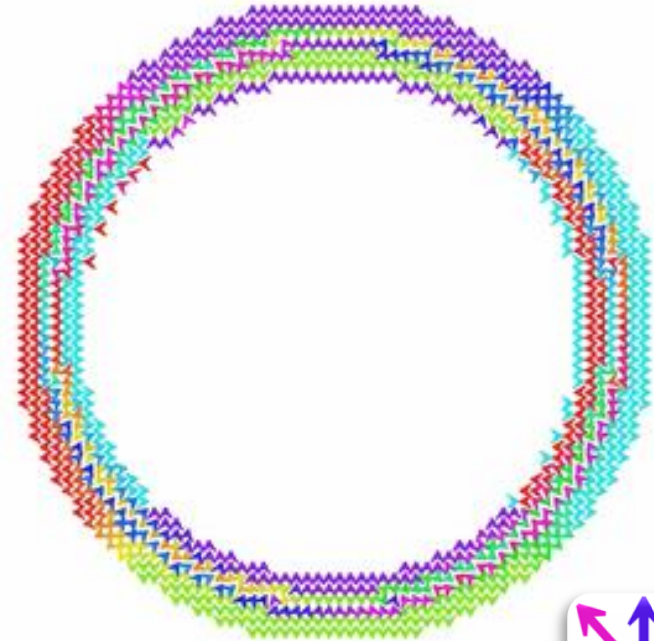
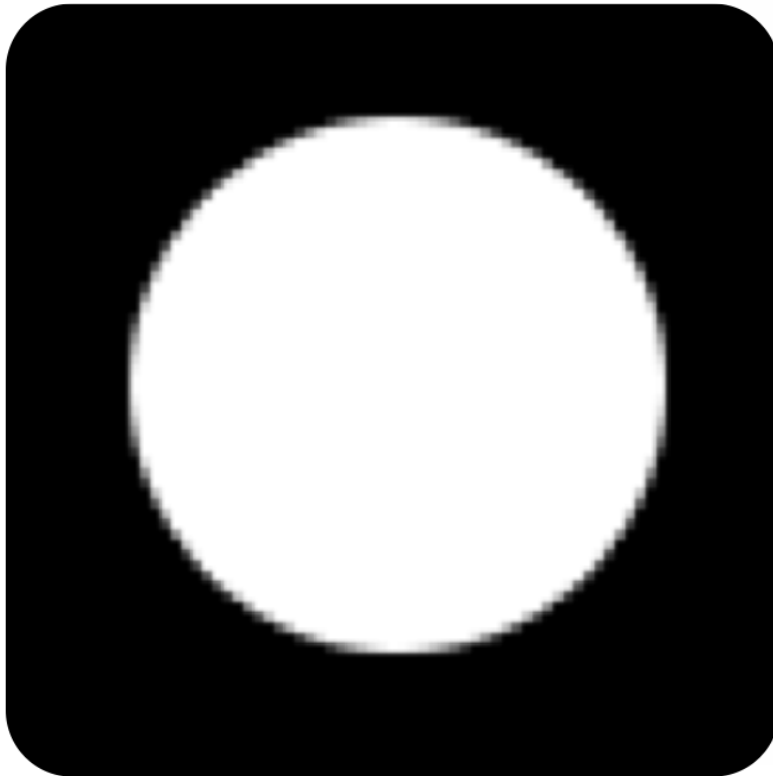
$$\nabla |\nabla I(x, y)|^2$$

Or a gaussian-smoothed version...

$$\nabla |G_\sigma * \nabla I(x, y)|^2$$

Image force fields (external forces in active contour models)

- Gradient vector flow (GVF)
- Generalized GVF (GGVF)



Xu & Prince (1998) Active contours and gradient vector flow <http://iac1.ece.jhu.edu/projects/gvf>

Xu & Prince (1998) Generalized gradient vector flow external forces for active contours

GVF image force field

Defined by an energy functional to be minimized

Let

$$V(x, y) = [u, v] \quad f = |G_\sigma * \nabla I|$$

$$u = u(x, y)$$

$$v = v(x, y)$$

$$\min_{(u,v) \in C^2(\Omega; \mathbb{R}^2)} \int_{\Omega} \underbrace{g(|\nabla f|) (|\nabla u|^2 + |\nabla v|^2)}_{\text{Diffusion (global smoothness) term}} + \underbrace{h(|\nabla f|) (|u - f_x|^2 + |v - f_y|^2)}_{\text{Edge similarity term}} dx$$

$$g\Delta u - h(u - f_x) = \frac{\partial u}{\partial t} \longrightarrow g\Delta u - h(u - f_x) = 0$$

$$g\Delta v - h(v - f_y) = \frac{\partial v}{\partial t} \longrightarrow g\Delta v - h(v - f_y) = 0$$

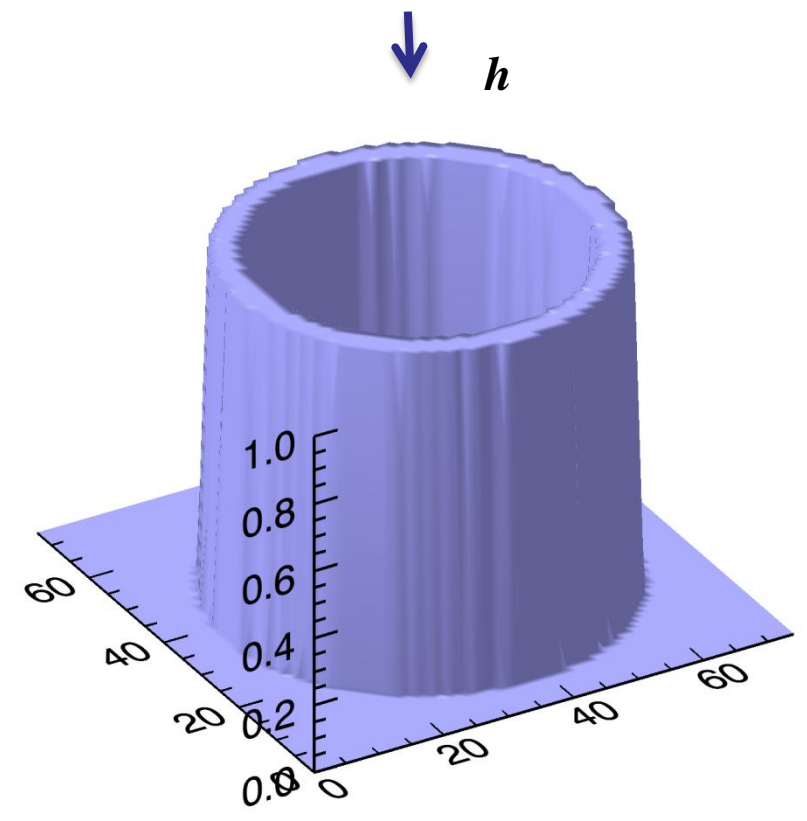
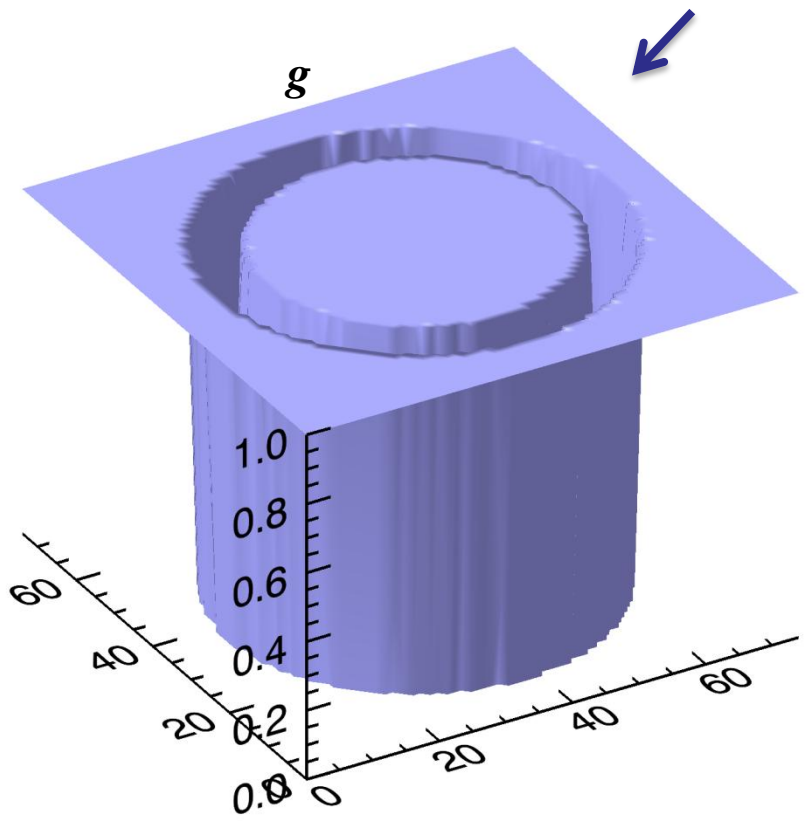
$$\nabla g \cdot \nabla u + g\Delta u - h(u - f_x) = 0$$

$$\nabla g \cdot \nabla v + g\Delta v - h(v - f_y) = 0$$

The GVF (GGVF) energy minimization

$$\min_{(u,v) \in C^2(\Omega; \mathbb{R}^2)} \int_{\Omega} \underbrace{g(|\nabla f|) (|\nabla u|^2 + |\nabla v|^2)}_{\text{Diffusion term}} + \underbrace{h(|\nabla f|) (|u - f_x|^2 + |v - f_y|^2)}_{\text{Edge term}} dx$$

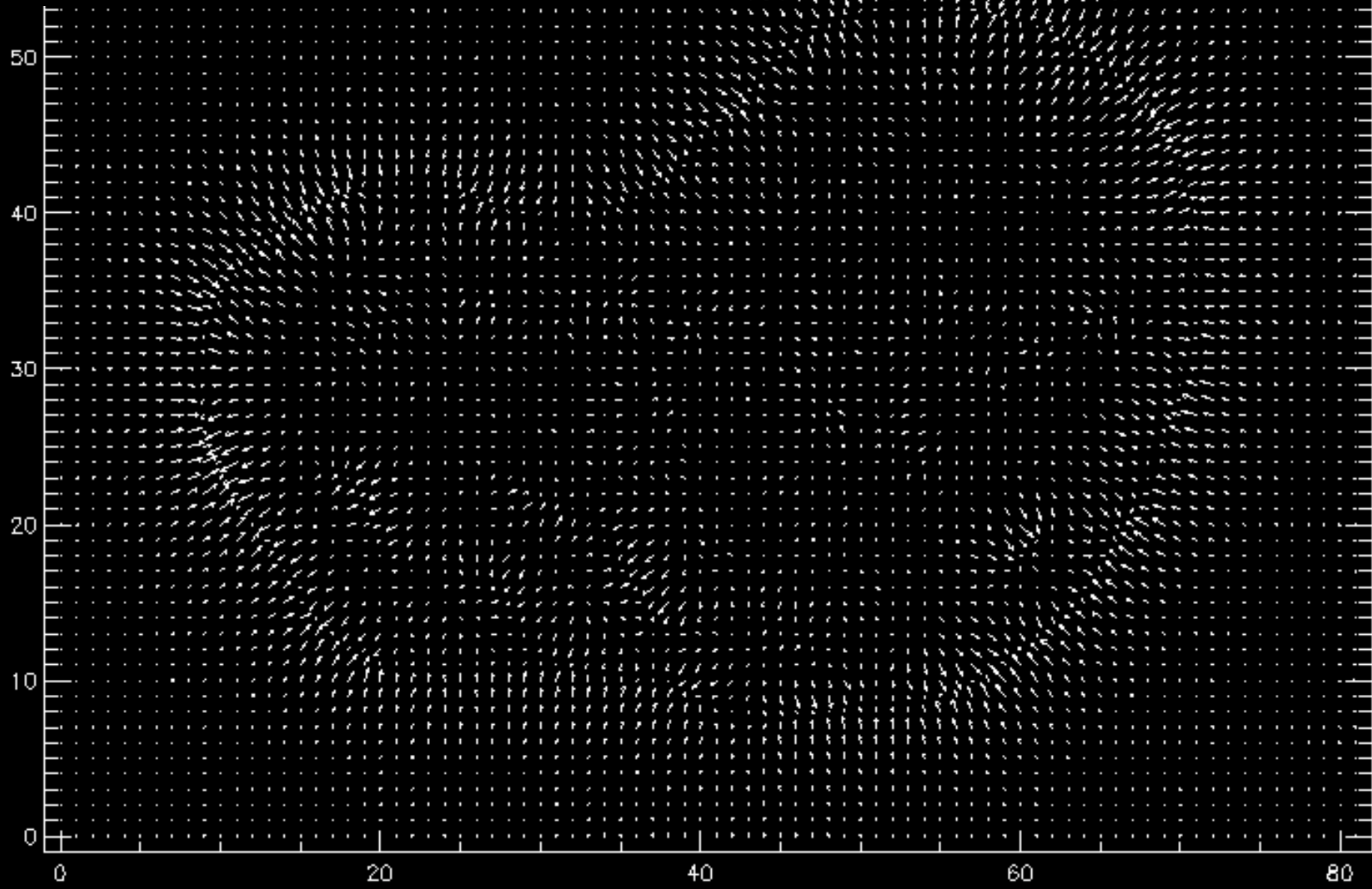
$f = |G_{\sigma} * \nabla I|$



GVF

$$g(|\nabla I|) = \mu \quad \mu = 0.05 > 0$$

$$h(|\nabla I|) = |\nabla I|^2$$

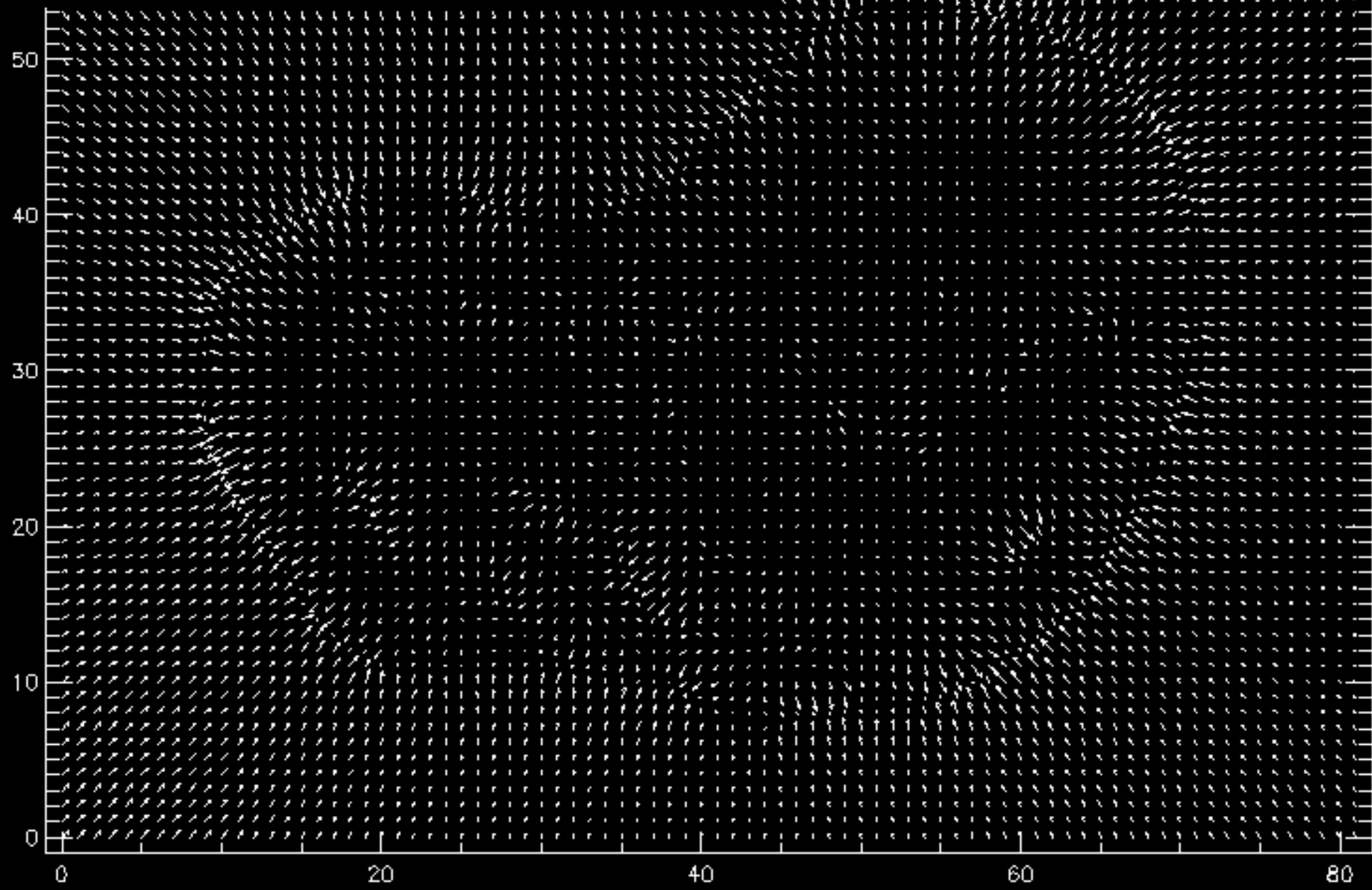


Generalized GVF (GGVF)

$$g(|\nabla I|) = e^{-(|\nabla I|/\mu)} \quad \mu = 0.05$$

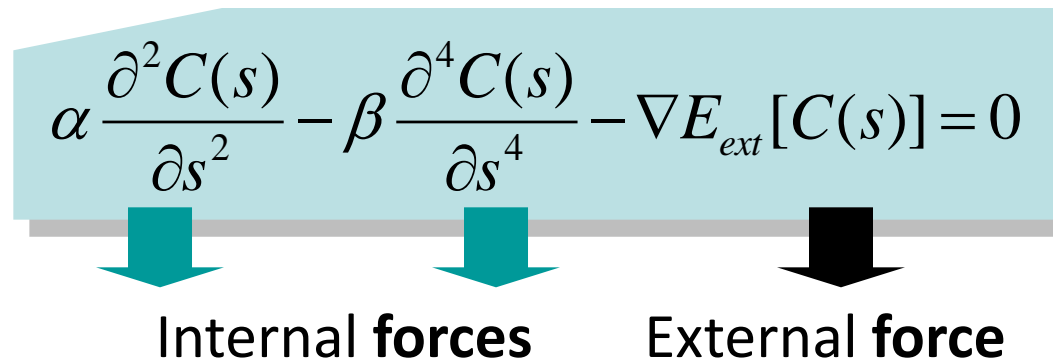
> 0

$$h(|\nabla I|) = 1 - g(|\nabla I|)$$



Snakes – formulation

- Energy minimization condition: Euler-Lagrange equation

$$\alpha \frac{\partial^2 C(s)}{\partial s^2} - \beta \frac{\partial^4 C(s)}{\partial s^4} - \nabla E_{ext} [C(s)] = 0$$


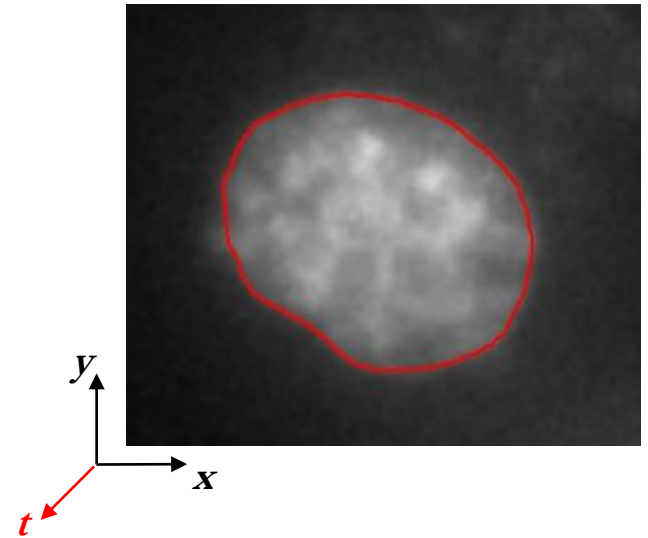
Internal **forces** External **force**

Snakes – numerical solution

– Dynamic formulation

- $C(s) \dots C(s, t)$
- Time equilibrium leads to solution

$$\frac{\partial C(s, t)}{\partial t} = 0$$



$$\frac{\partial C(s, t)}{\partial t} = \alpha \frac{\partial^2 C(s, t)}{\partial s^2} - \beta \frac{\partial^4 C(s, t)}{\partial s^4} - \nabla E_{ext} [C(s, t)]$$

Time evolution

Internal forces

External force

Snakes – numerical solution

- Parameters

- α (elasticity), β (rigidity)

$$\alpha \frac{\partial^2 C}{\partial s^2} - \beta \frac{\partial^4 C}{\partial s^4} - \nabla E_{ext} [C] = 0$$

A is a 5-diagonal matrix,
suitable for numeric solution

$$A_{n*n} C - \nabla E_{ext} [C] = 0$$

- κ (added as the external force weight),
 γ (contour viscosity)

$$\frac{\partial C(s, t)}{\partial t} = 0$$

$$AC_t - \kappa \nabla E_{ext} (C_{t-1}) = -\gamma (C_t - C_{t-1})$$

Iterative scheme, solved computationally

$$C_t = (A + \gamma I)^{-1} - (C_{t-1} + \kappa \nabla E_{ext} (C_{t-1}))$$

In order to computationally solve this system:
matrix inversion, array data structures, adaptive algorithms, ...

Segmentation - Advanced

s_Seg | -> hoechs...

Options Analyse

Segmentation Cluster | ->

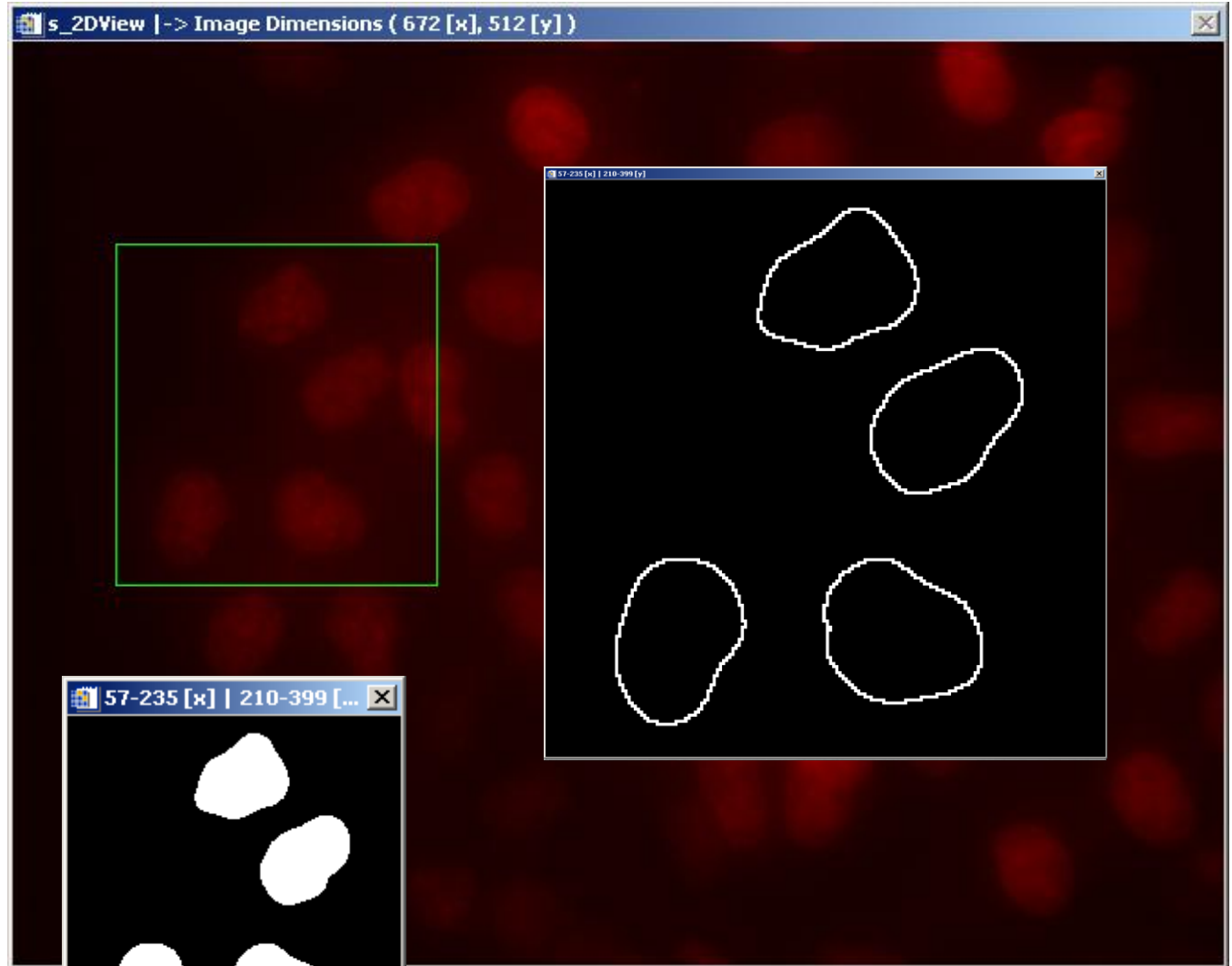
Cluster_0

Add Segmentation Method | ->

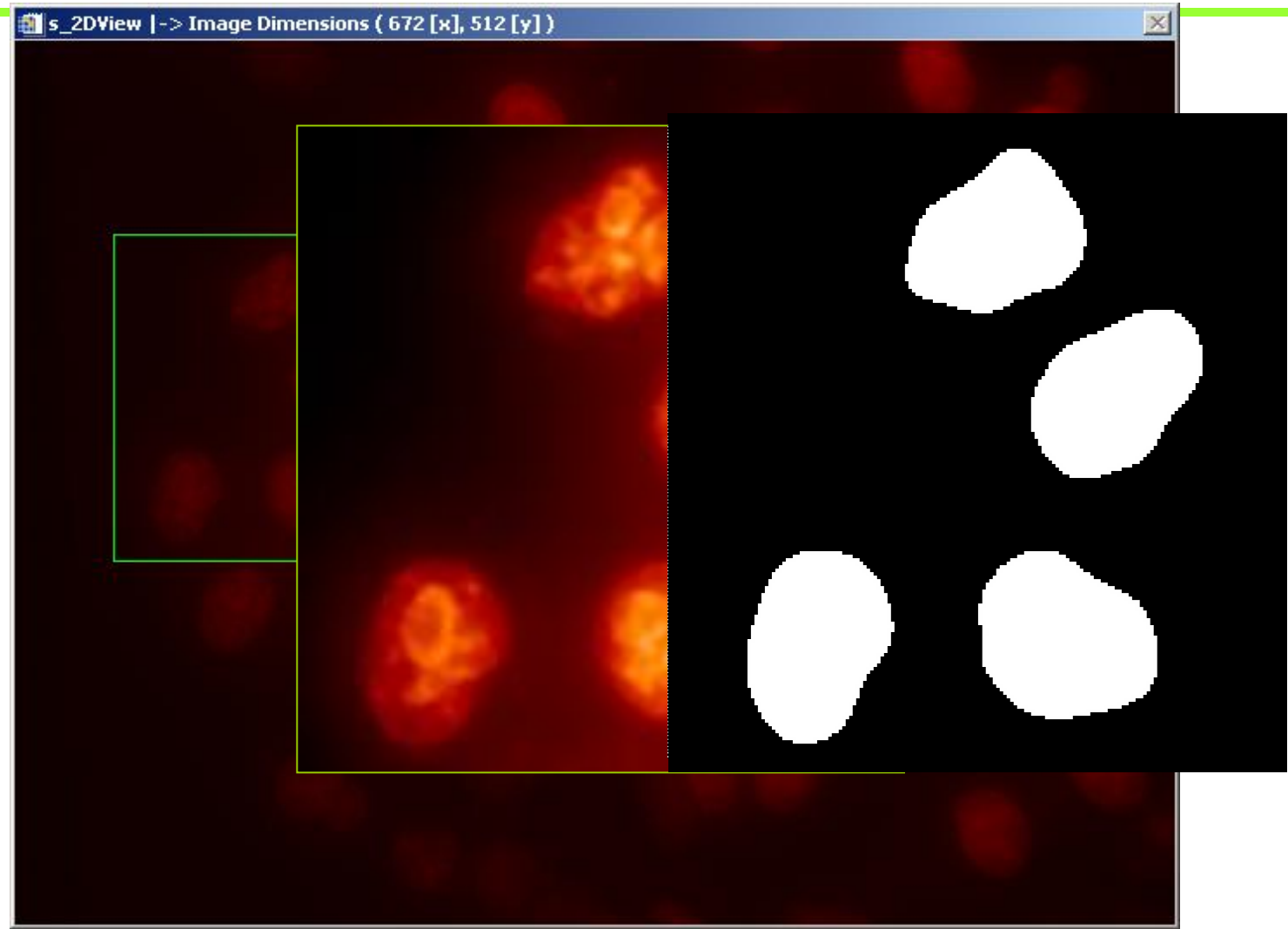
- C_Median
- C_1stDeviation
- C_Threshold
- C_FillRemove
- C_TouchBorder
- C_ActiveContours

Del

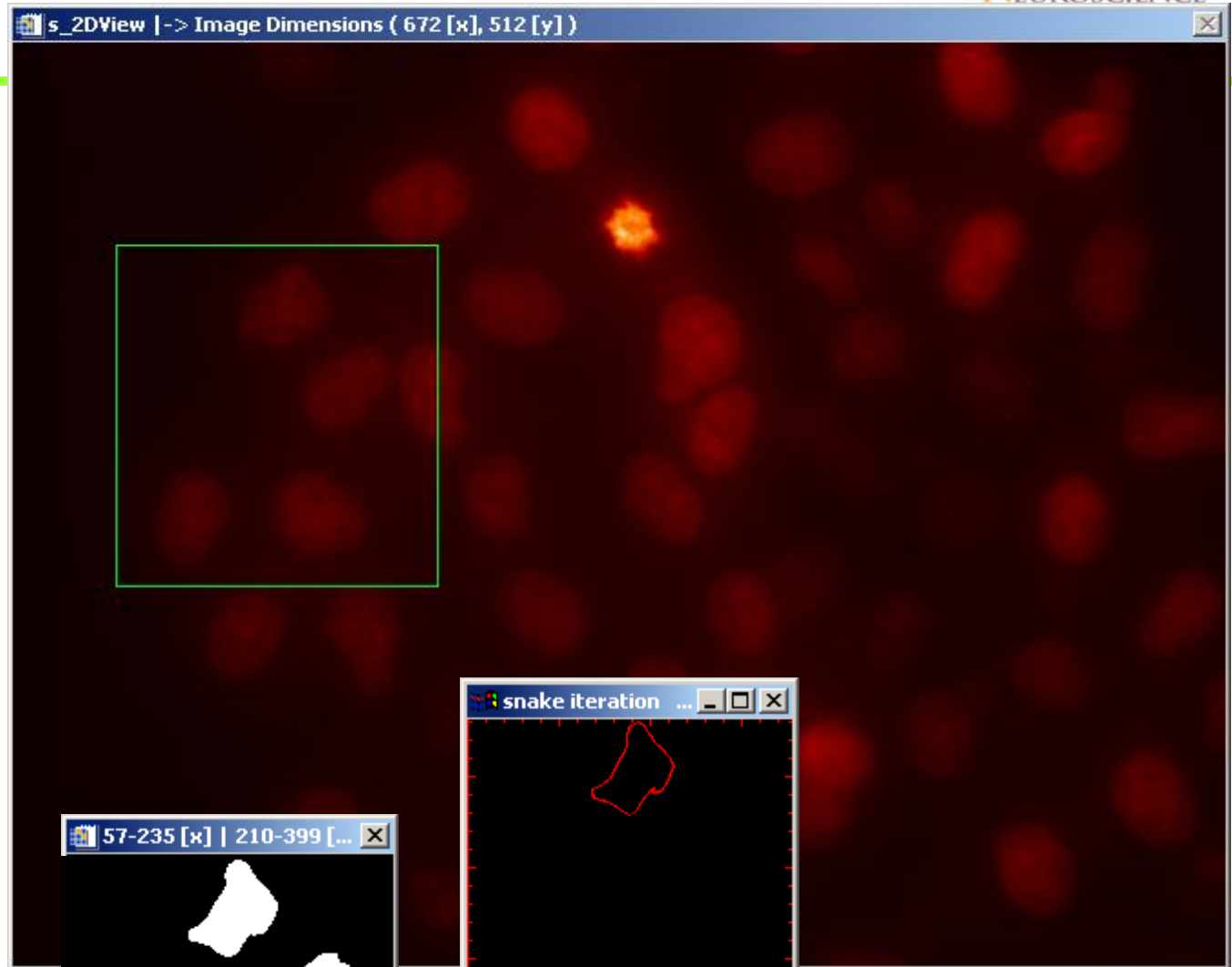
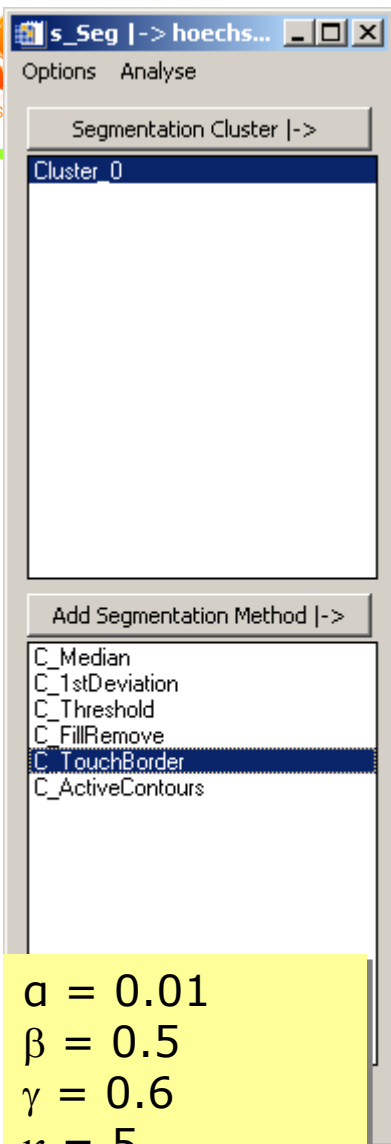
$\alpha = 0$
 $\beta = 0.01$
 $\gamma = 3$
 $\kappa = 0.01$
 $\mu = 0.05$
Iterations = 5
 $f = 0.15$



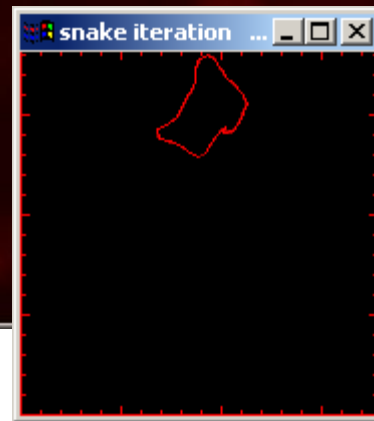
Segmentation - Advanced



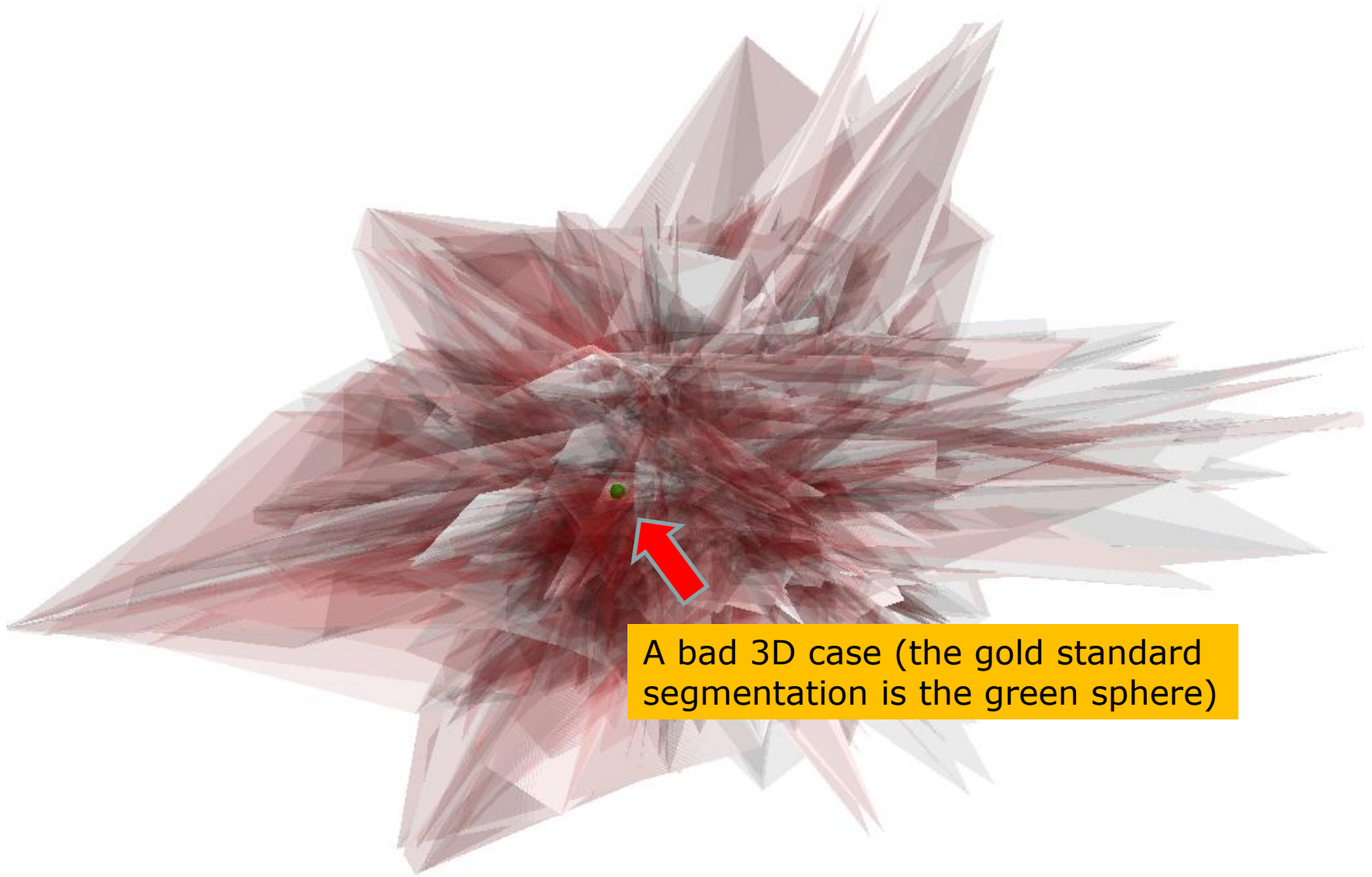
- Some problems with the snakes method
 - The quantity of contours must not change, given the 1-1 correspondence of the model
 - Dependency on the initial guess
 - Local optima for the functional minimization
 - A bad initialization of the snakes can lead to instability and undesired results
 - Control of the snake parameters is **crucial**



$\alpha = 0.01$
 $\beta = 0.5$
 $\gamma = 0.6$
 $\kappa = 5$
 $\mu = 0.05$
Iterations = 5
 $f = 0.15$



Bad parameter selection....

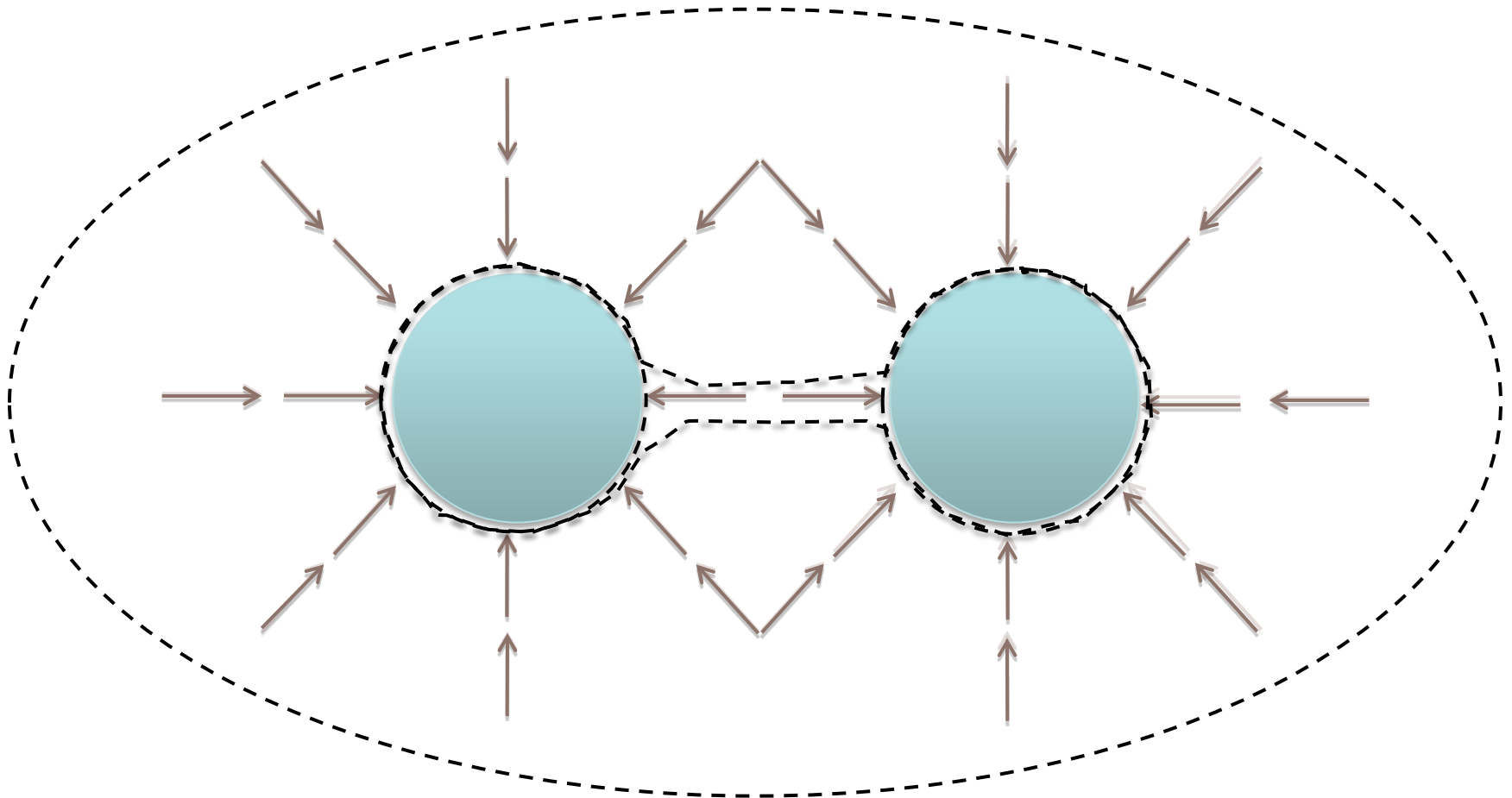


A bad 3D case (the gold standard segmentation is the green sphere)

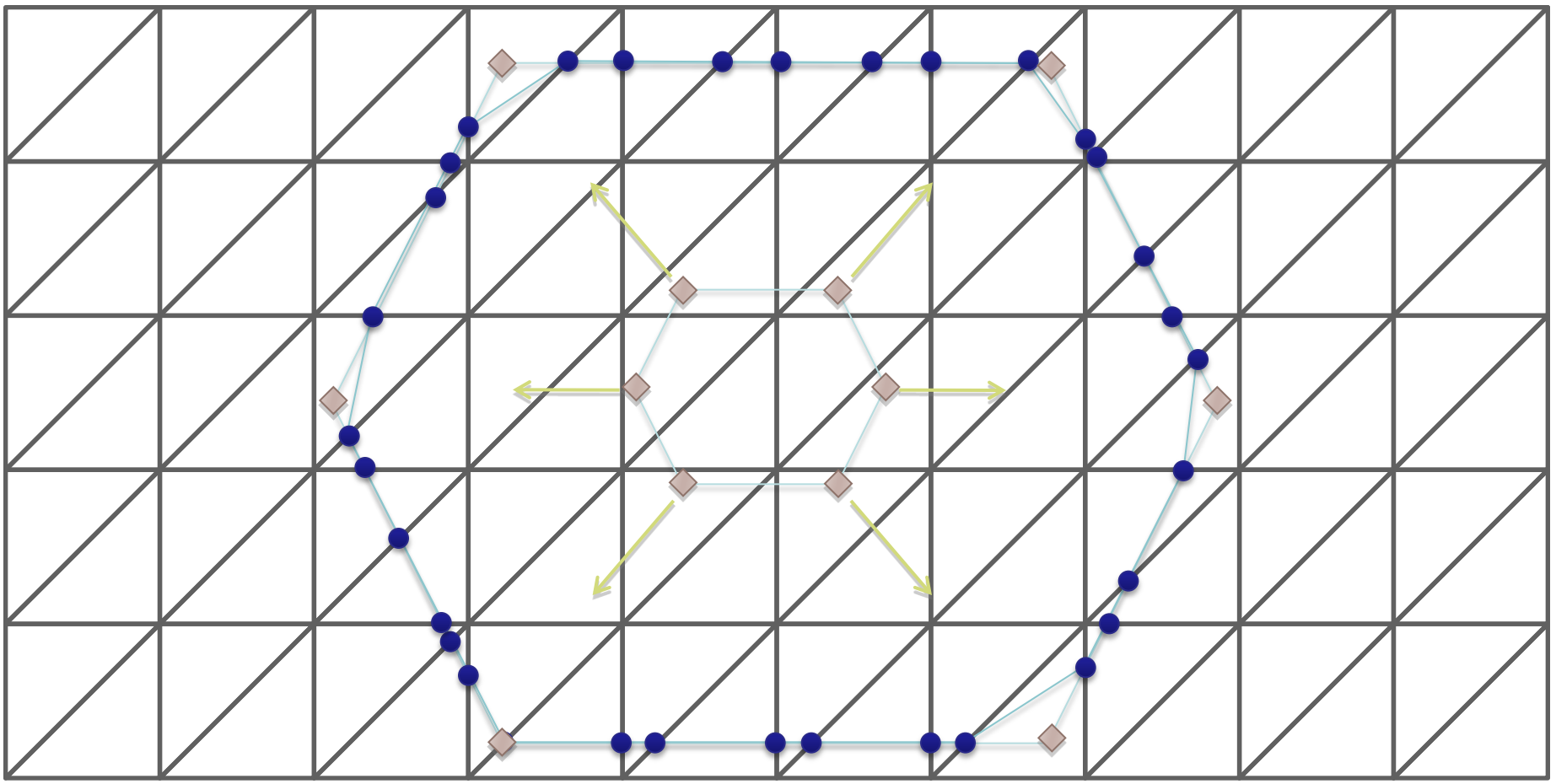
- Alternatives...
 - Arbitrary contour initialization... automation
 - Inference (machine learning approaches)

- Topology adaptive snakes (or surfaces) McInerney & Terzopoulos, 1995 (2D), 2002 (3D)

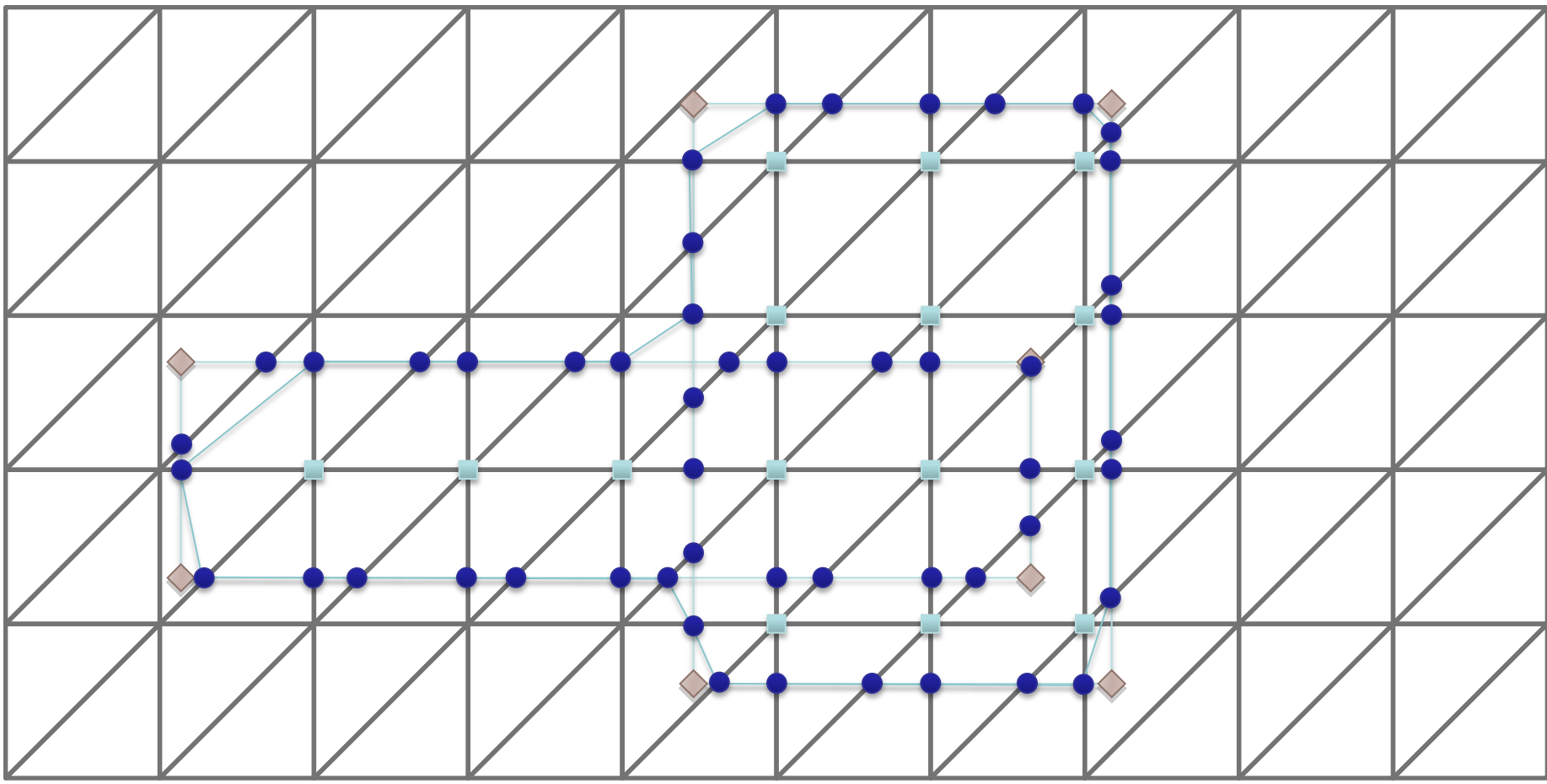
Topological changes handling: splitting, merging, collapse



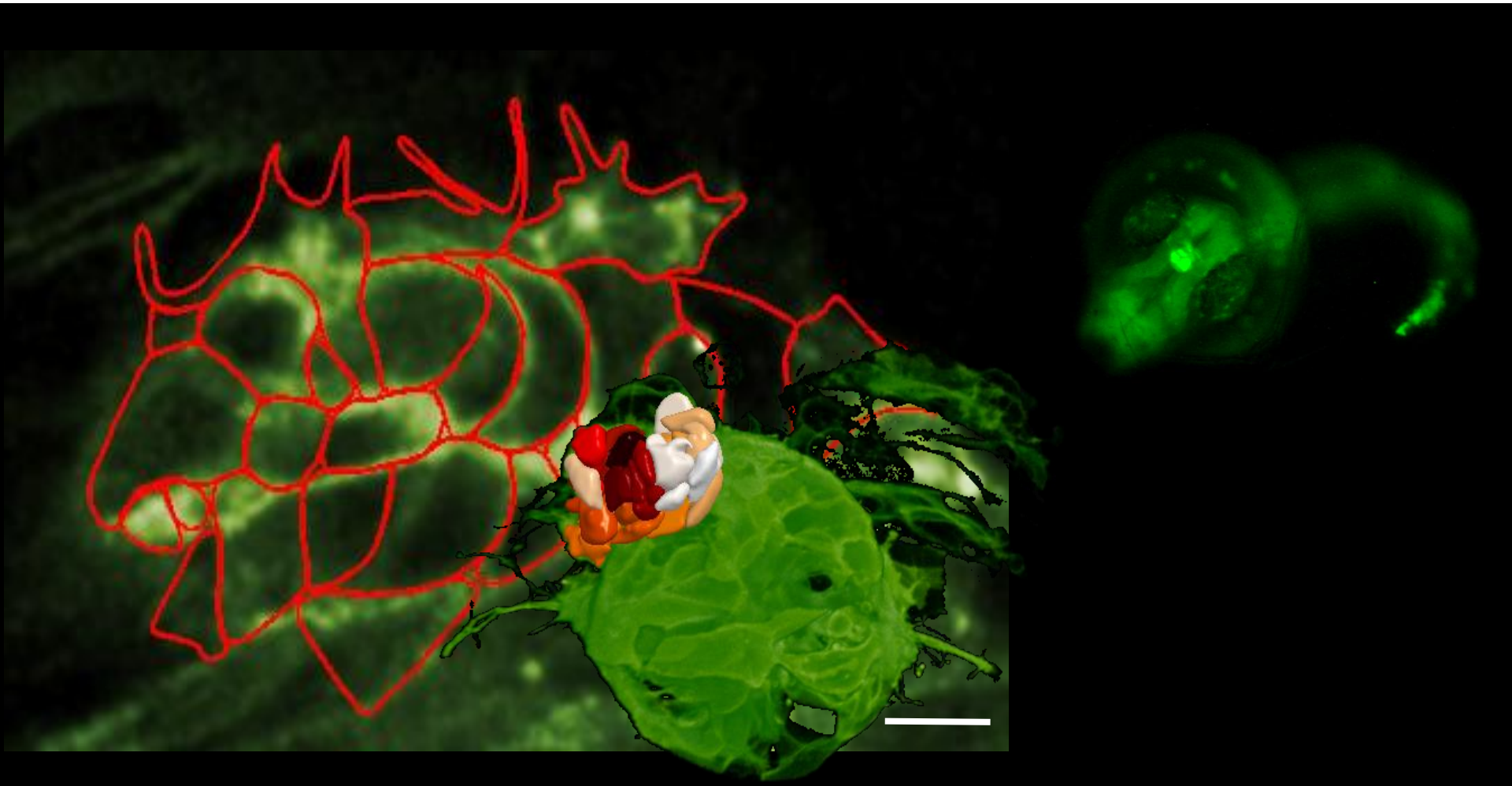
Between deformations, the contours are re-parametrized by using a grid to detect topology changes



This is an algorithmic approach, “additional” to the mathematical optimization model. The computation becomes more intensive

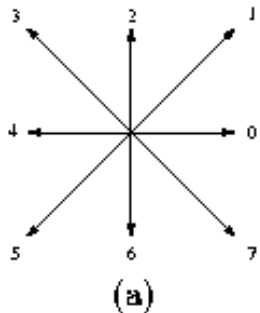


Hardest cases: initialization by manual ROI sketching

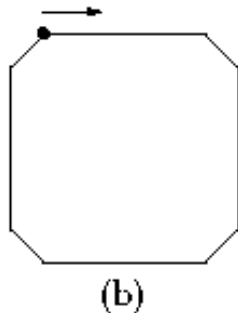


Boundary model construction

- 2D Freeman chain code
- 3D mesh models (voxel based): *marching cubes* (surface meshes), *tetraedra* (volume meshes)

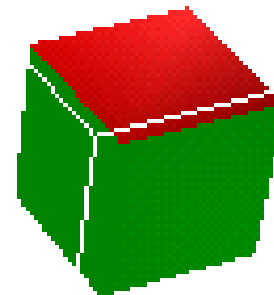


2D polygon chain code

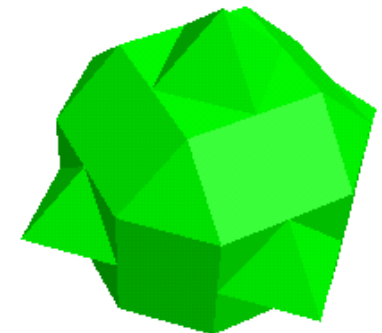


{0,0,0,0,0,7,
6,6,6,6,6,5,
4,4,4,4,4,3,
2,2,2,2,2,1}

(c)

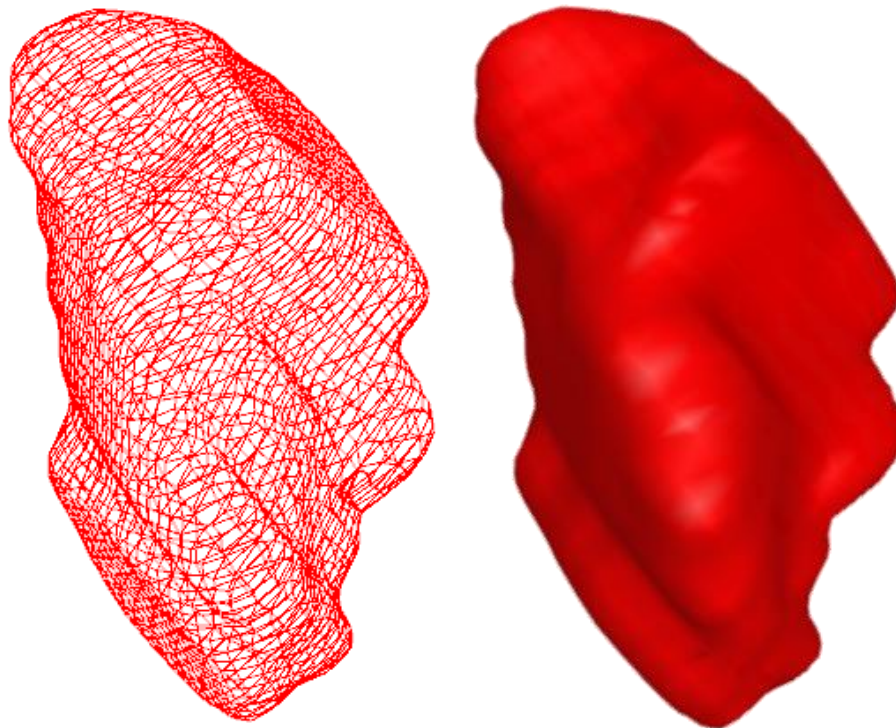


Voxel ("3D pixel") model

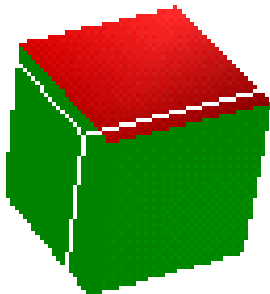
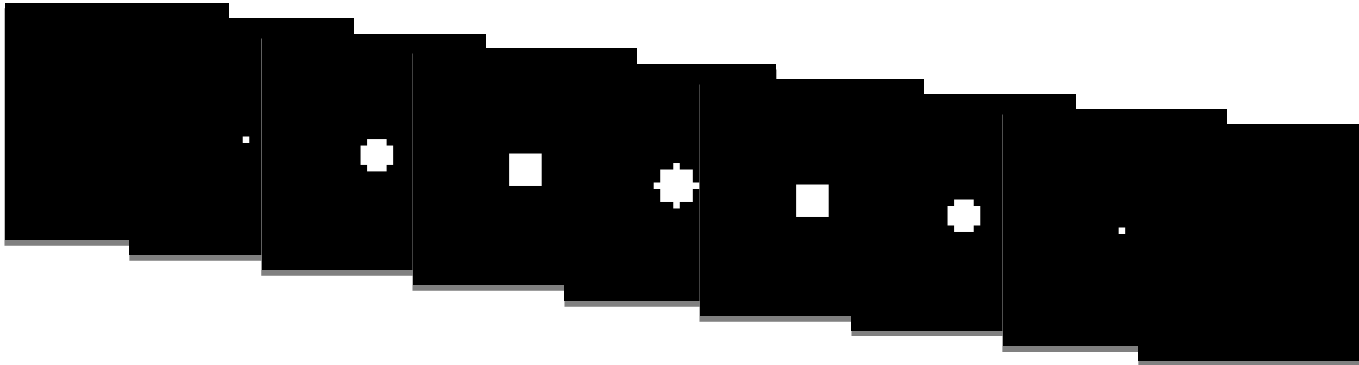


3D polygon mesh

- A typical 3D surface mesh model is formed by:
 - Nodes or vertices
 - Polygons
- Other models
 - Polynomial surfaces (splines, Bézier, NURBS, ...)
 - Primitives composition



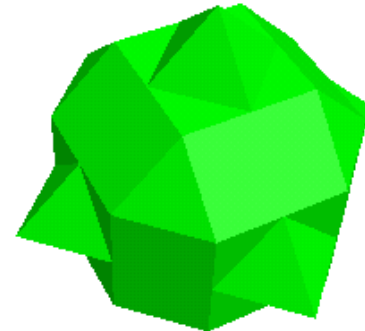
- Surface mesh model construction
 - Many approaches for construction
 - Many approaches for modeling



Voxel ("3D pixel")
model

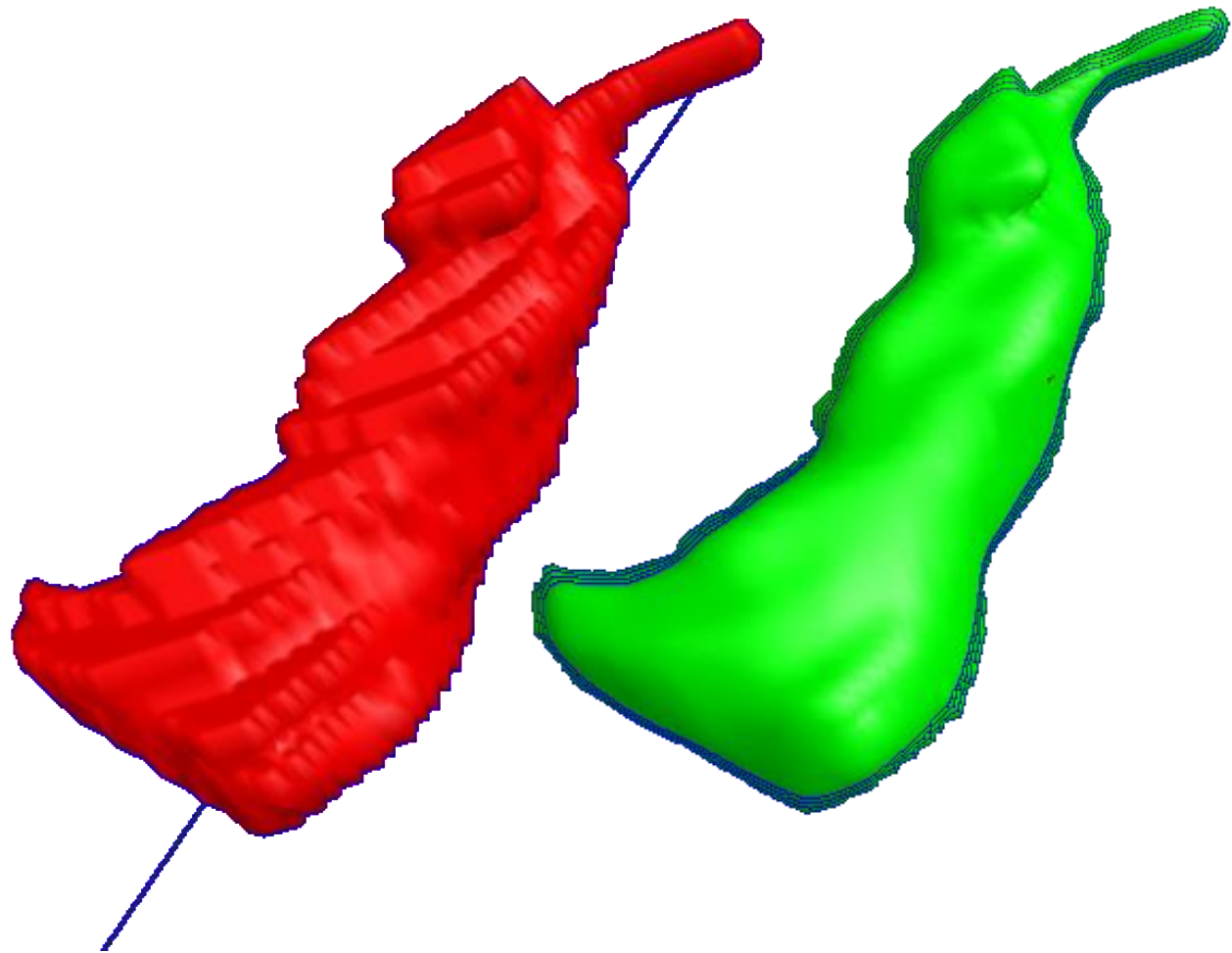


Triangle mesh



Triangular and
quarilateral mesh

- Sample 3D ROI



- David Marr. Vision
MIT press, 1982
- John Russ. The image processing handbook, 4th ed.
CRC Press, 2002
- Nixon, Aguado. Feature extraction & image processing, 1st | 2nd | 3rd ed.
Academic Press, 2002 | 2008 | 2012
- Aubert & Kornprobst. Mathematical Problems In Image Processing
Springer, 2006
- From SCIAN-Lab (see publications section on the website)
 - Jara, 2006. 2D/3D active contours
 - Olmos, 2009. 2D topology adaptive active contours (t-snakes)

Some free and open source software tools

- Java based (Java runtime required)
 - ImageJ (<http://rsbweb.nih.gov/ij/>, public domain)
 - Fiji (<http://fiji.sc>; GPL license)
 - Icy (<http://icy.bioimageanalysis.org>; GPLv3 license)
- Others
 - CellProfiler (<http://cellprofiler.org>; GPL, BSD licenses)
 - Slicer (www.slicer.org; BSD license)
 - IPOL (Image Processing Online): open access electronic journal with peer reviewed articles + code (in C language) + examples (www.ipol.im; BSD / GPL / LGPL licenses or similar)