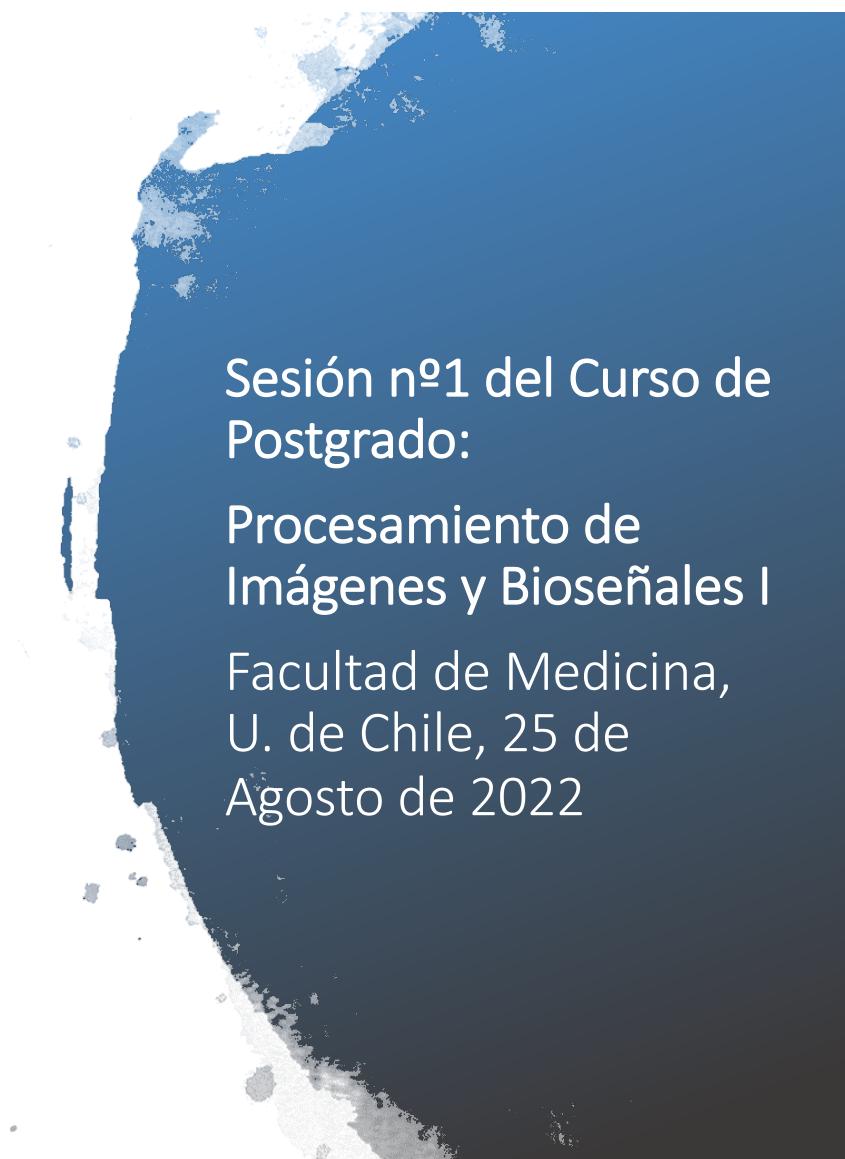


Adquisición de Imágenes biológicas y biomédicas

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Sesión nº1 del Curso de
Postgrado:
Procesamiento de
Imágenes y Bioseñales I
Facultad de Medicina,
U. de Chile, 25 de
Agosto de 2022

inverse problems and crimes

- **detective novels:** the scene of the crime, clues and a mystery to discover.
- more than one suspect, the facts and observations, the logical deductions led to a single and surprising explanation.



CRIME? → CLUES



- one goes back in time, connecting the available evidence to **reconstitute** the hidden and most plausible truth.
- in mathematical language, this is an **inverse problem**, instead of going from the cause to the effect, it is the contrary: going from the effect to the cause.

$A \rightarrow ?$
direct

$? \rightarrow B$
inverse

Guía de re-lectura: a conocer/entender/retener

- Cuáles son y qué distingue las distintas modalidades de adquisición de imágenes biomédicas según emisión v/s transmisión, ionizante v/s no ionizante, resolución v/s sensitividad o proceso físico: transporte de fotones v/s ondas.
- Conocer aspectos de la génesis e historia de las distintas modalidades ¿Por qué cada una de estas modalidades se considera un problema inverso?
- Para CT, entender cuál es el proceso de adquisición de las imágenes (rayos X, ley de Beer y sinograma) y cómo se recupera la imagen de atenuaciones (etapas de retroproyección y filtrado de la transformada de Radón inversa).
- Para MRI, entender las etapas del proceso de adquisición de imágenes distinguiendo primero que nada la presencia de un campo magnético general que alinea los espines de los protones y la presencia de otros 3 gradientes magnéticos asociados a las diferentes direcciones espaciales.
- Para MRI, entender que la frecuencia de giro de precesión de los protones es proporcional a la intensidad del campo magnético aplicado (ley de Larmor) por lo que si el campo magnético varía (gradientes) la frecuencia de precesión varía y este es el fenómeno que se explota para obtener imágenes con MRI.
- Para MRI distinguir las etapas de: 1) el pulso de radiofrecuencias y la selección de una sección usando un primer gradiente, 2) que los distintos tejidos se diferencian por los tiempos de relajación de los espines de sus protones una vez que el pulso de radiofrecuencia es apagado y que la codificación de esta información en la sección seleccionada se puede hacer usando los otros dos gradientes, 3) la adquisición de señales se hace en el tiempo de eco TE o tiempo en que los protones oscilan de nuevo en fase, 4) finalmente la construcción de la imagen de frecuencias línea a línea usando tiempos de repetición TR.
- Para MRI: conocer que es la transformada de Fourier Inversa (IFT) la que convierte la imagen de frecuencias en una imagen visible de la sección.
- Para MRI: Conocer que las distintas modalidades de una imagen MRI llamadas T1,T2 y PD dependen de las combinaciones de tiempo TE y TR. Saber que también es posible medir desplazamientos y velocidades (de la sangre por ejemplo técnica llamada 4DFlow) usando MRI.

motivation: medical diagnosis, imaging the human body

- focus: medical **diagnosis** is an inverse problem *par excellence*, and one of the oldest that has confronted humanity: we are faced with the problem of determining the cause of the symptoms of an illness.
- we are often asked for **images** of some kind: **x-rays, ultrasounds, resonances, scanners, electrocardiograms**, etc.
- behind each of these images there is an **inverse problem** and involve revolutionary physics, mathematics and technology to build the electronic devices that make a reality to see images of our **inner world**.



illness → symptoms

body composition and living tissues...

- the human body is more than **50% water**. The rest of their mass are **living tissues**, softer and harder: fat, muscles, bones.

	water	fat (storage)	fat (essential)	muscle	bone	remainder
women	52%	15%	12%	36%	12%	25%
men	57%	12%	3%	45%	15%	25%

Behnke's model of reference man and woman 1978

- waves or particles can **travel through** the body: **sound or elastic waves, electricity, radiation, light**.
- we can register their effect to see the inner body → **inverse problem**

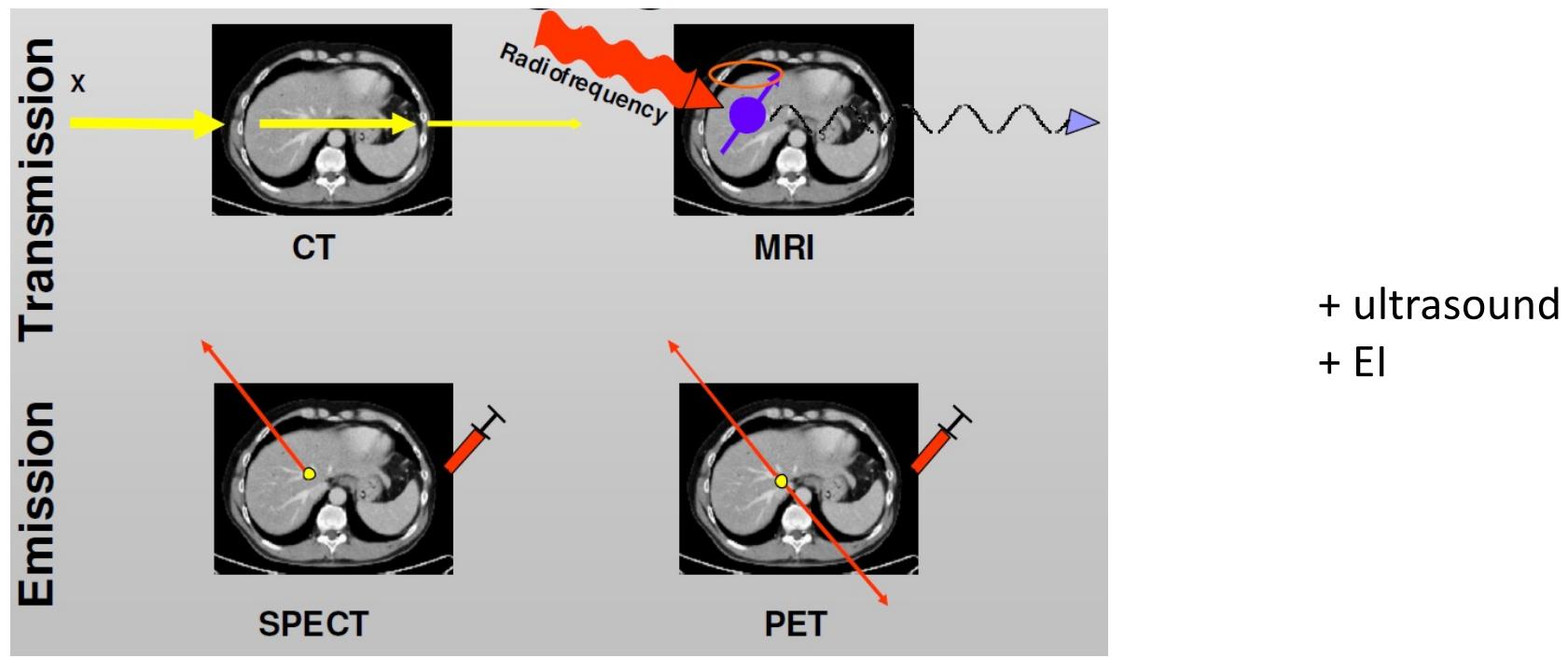


photon transport/scattering



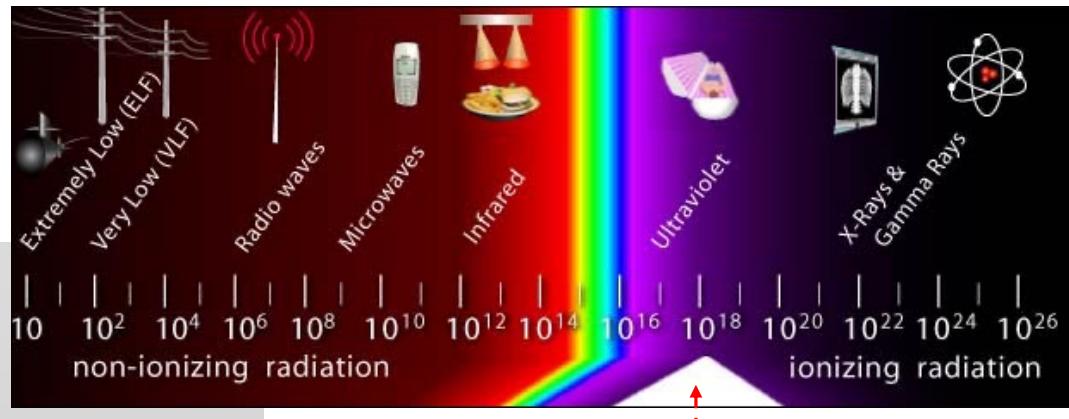
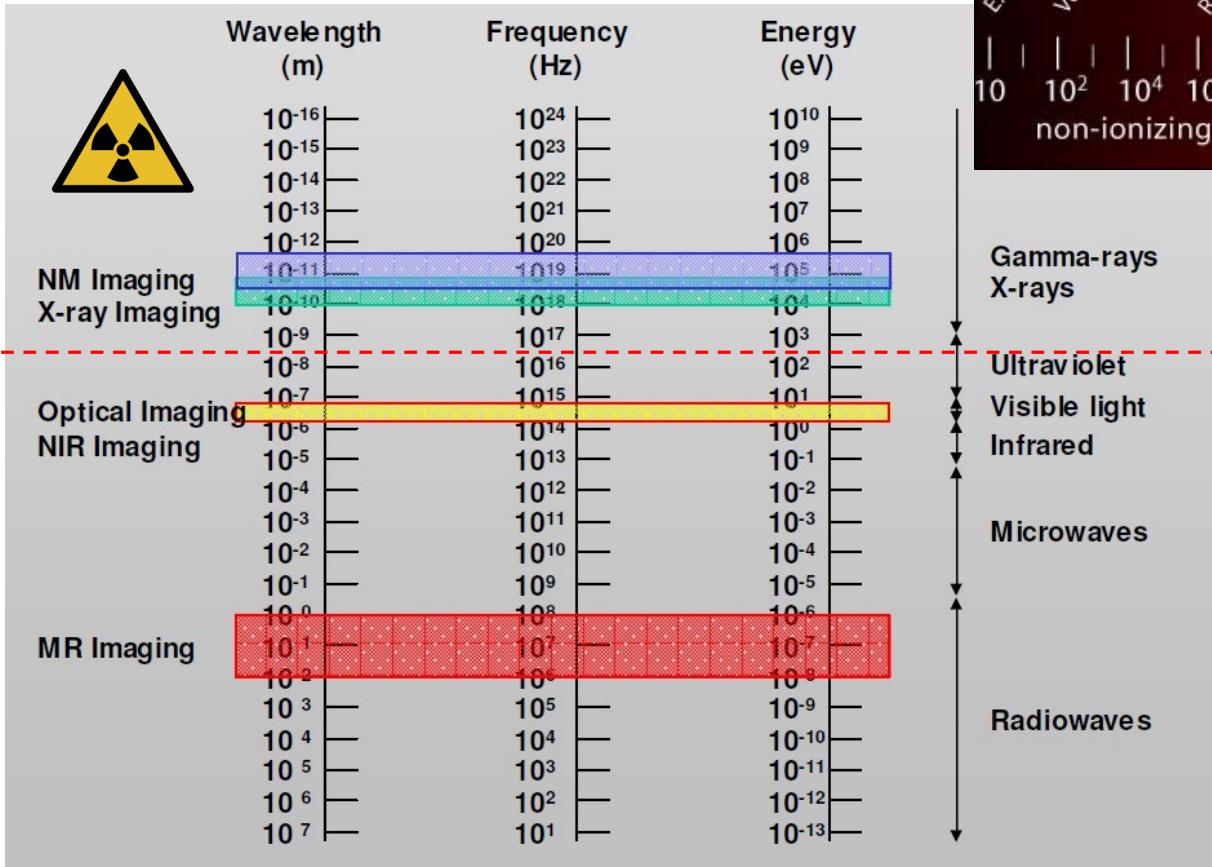
wave propagation

Biomedical imaging techniques: emission v/s transmission...



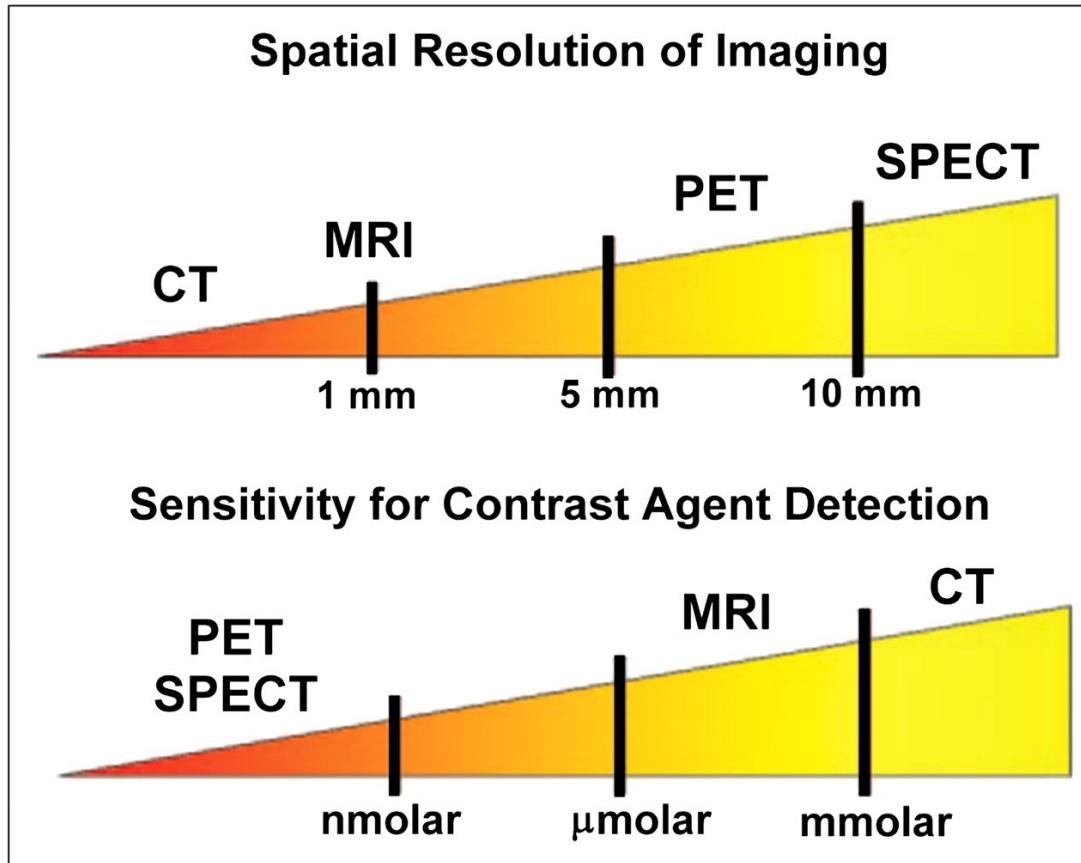
credit: Dr. Castañeda

credit: Dr.
Castañeda



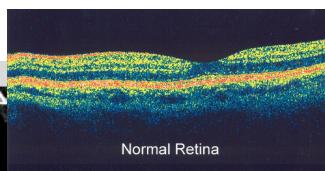
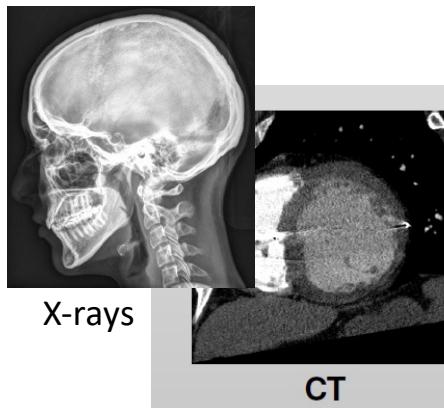
Biomedical imaging
techniques:
ionizing v/s non-ionizing

Biomedical imaging techniques: resolution v/s sensitivity...

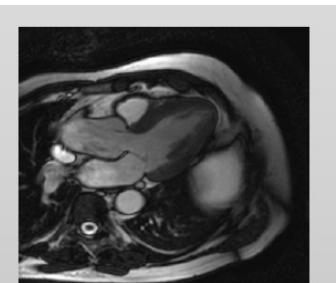
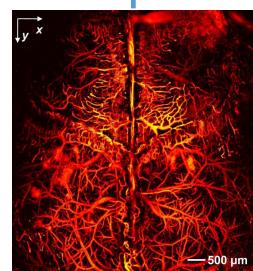


credit: web

biomedical imaging techniques: physics

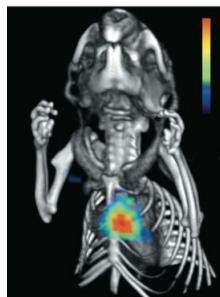
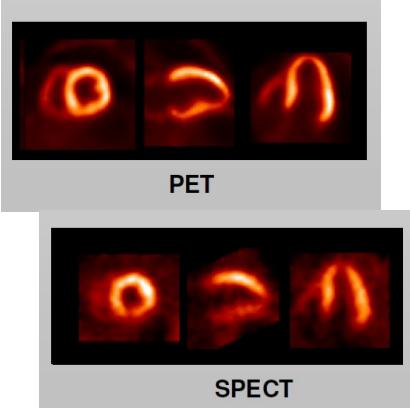


optical coherence
tomography
OCT



Magnetic resonance
modalities: dMRI,
MRE, 4DFlow, etc.

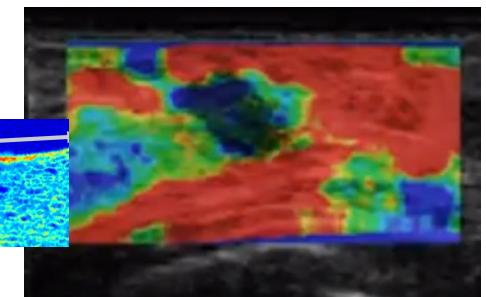
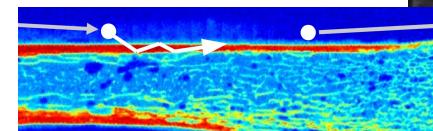
PHOTON TRANSPORT



Nuclear medicine

HYBRID

photo-acoustic
tomography



WAVE PROPAGATION

First part (photon transport): 20:00 – 20:30

- 1.- X-rays, CT, nuclear medicine, PET, SPECT:
 - *current research topic: simultaneous source and attenuation in SPECT*
- 2.- Fluorescence molecular tomography FMT :
 - *current research topic: mathematics of light-sheet microscopy*

Second part (wave phenomena): 20:30 – 21:00

- 3.- Magnetic resonance MRI:
 - *current research topic: non-invasive pressure and blood flow estimation*
- 4.- Ultrasound, elasticity imaging, MRE:
 - *current research topic: bone porosity estimation by ultrasound*

First part (photon transport)

1.- X-rays, CT, nuclear medicine, PET, SPECT

*correct research topic: simultaneous source-
attenuation identification in SPECT*

first the basis... X-rays... the scanner

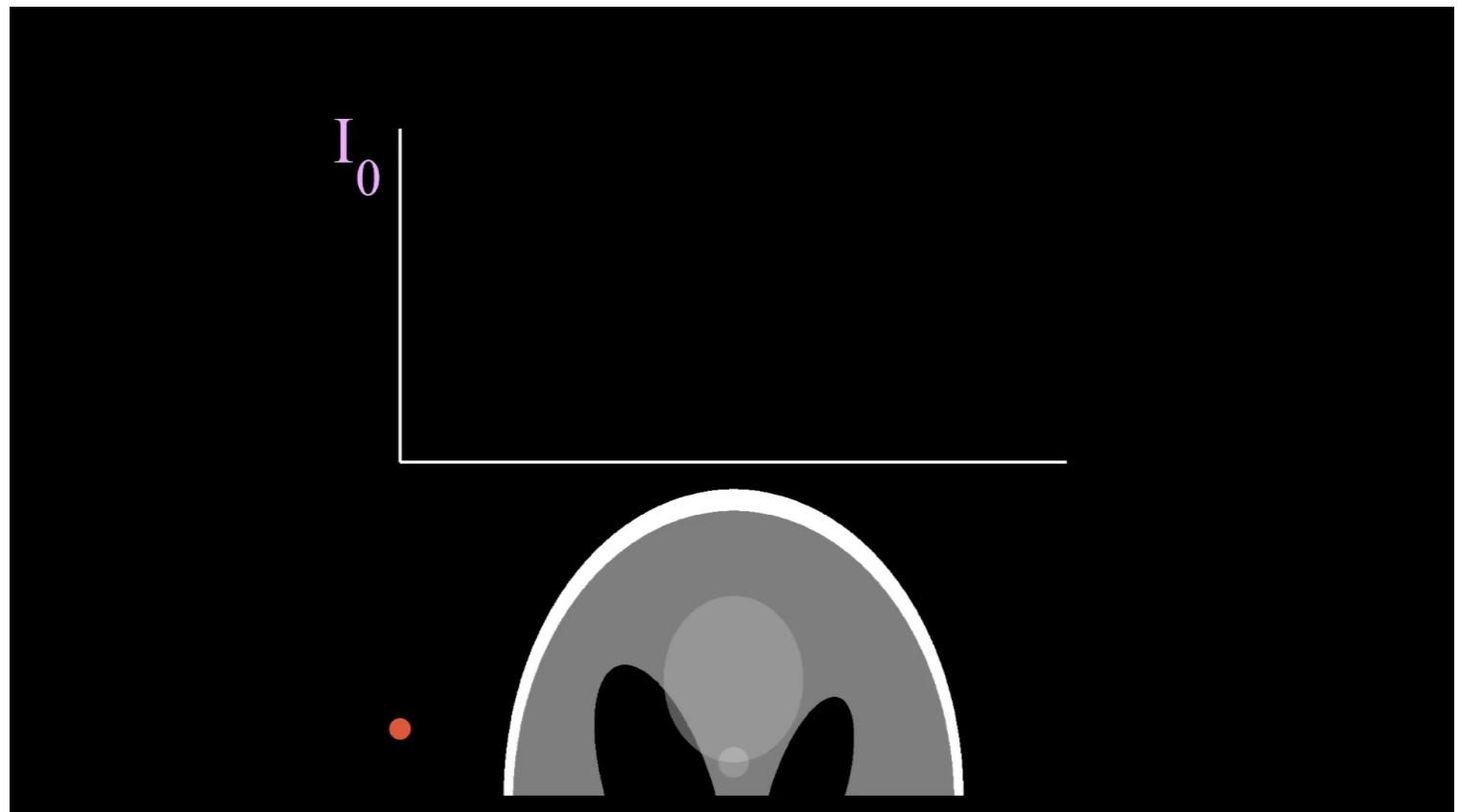
- Wilhelm Röntgen, a German engineer, obtained the Nobel in 1901 for detecting and producing **X-rays** for the first time.
- X-rays: invisible high energy electromagnetic waves that cross the body almost as if it were transparent, **attenuated** principally by bones.
- In 1917, Johann Radon, an Austrian mathematician, developed the mathematical basis for combining X-rays at different angles to obtain **three-dimensional images**.
- The British engineer Godfrey Hounsfield build the **first scanner** in 1971 and obtained the Nobel for this in 1979.
- Before that, in 1953, diffraction X-ray images were used by the crystallographer Rosalind Franklin to discover the form of the **ADN double helix** (1962 Nobel)



X-ray attenuation

credit: Samuli Siltanen

the
measurent
 $\log(I_0/I_1)$
is the integral
of the
attenuation
along the
line



The Shepp-Logan phantom: Larry Shepp and Benjamin F. Logan for their 1974 paper *The Fourier Reconstruction of a Head Section*

Intensity decay...

is proportional to attenuation, thickness and intensity...

The infinitesimal change... gives by integration on lines the solution:

$$I = I_0 \exp \left(- \int_L a \right)$$

L : line from x_0 to x

so the sum over lines can be measured

$$\int_L a = - \ln \frac{I}{I_0}$$

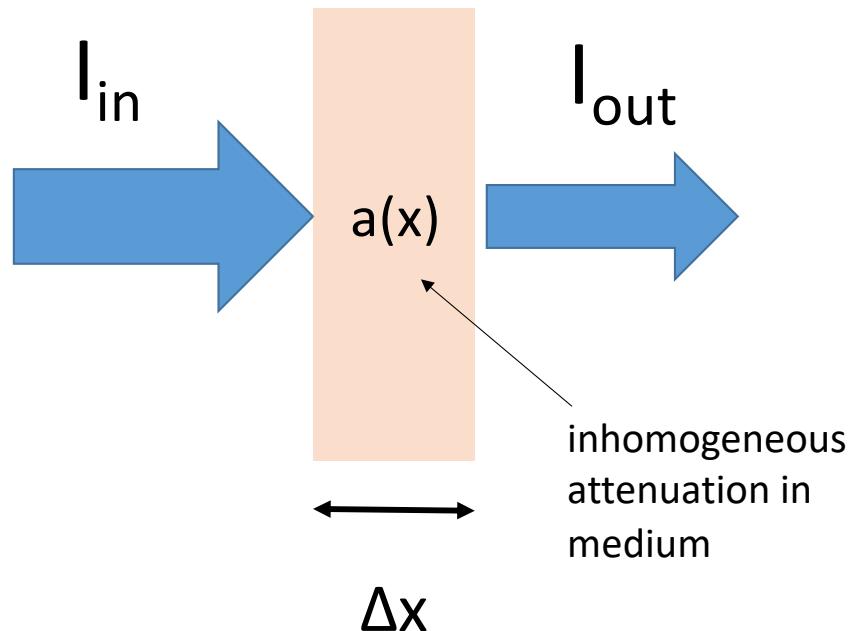
$$\Delta I = I_{out} - I_{in}$$

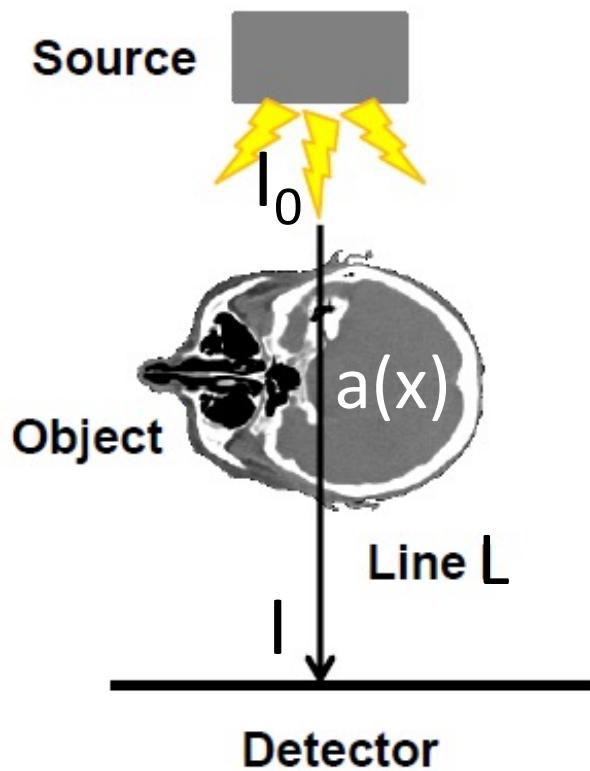
$$\Delta I = -a(x)I\Delta x$$

$$\frac{dI}{dx} = -a(x)I$$

Beer's law

(monochromatic, X-ray beam no refraction or diffraction)





attenuation integral

$$\int_L a = - \ln \frac{I}{I_0}$$



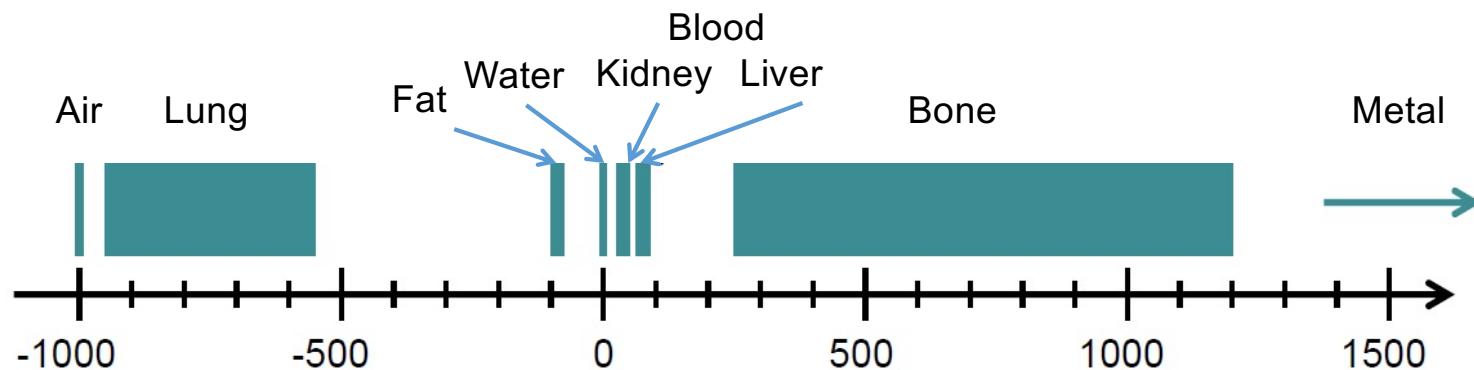
- Photons with energies 50-120 keV are emitted by an X-Ray source
- Interaction with biological tissue absorbs & scatters some of the photons (photo-electric and Compton effect)

credit: Dr. Castañeda

Typical X-ray data: Hounsfield Units (HU)

CT-scanner calibration Air=-1000, Water=0

$$HU = \frac{\mu - \mu_{Water}}{\mu_{Water}} \cdot 1000$$

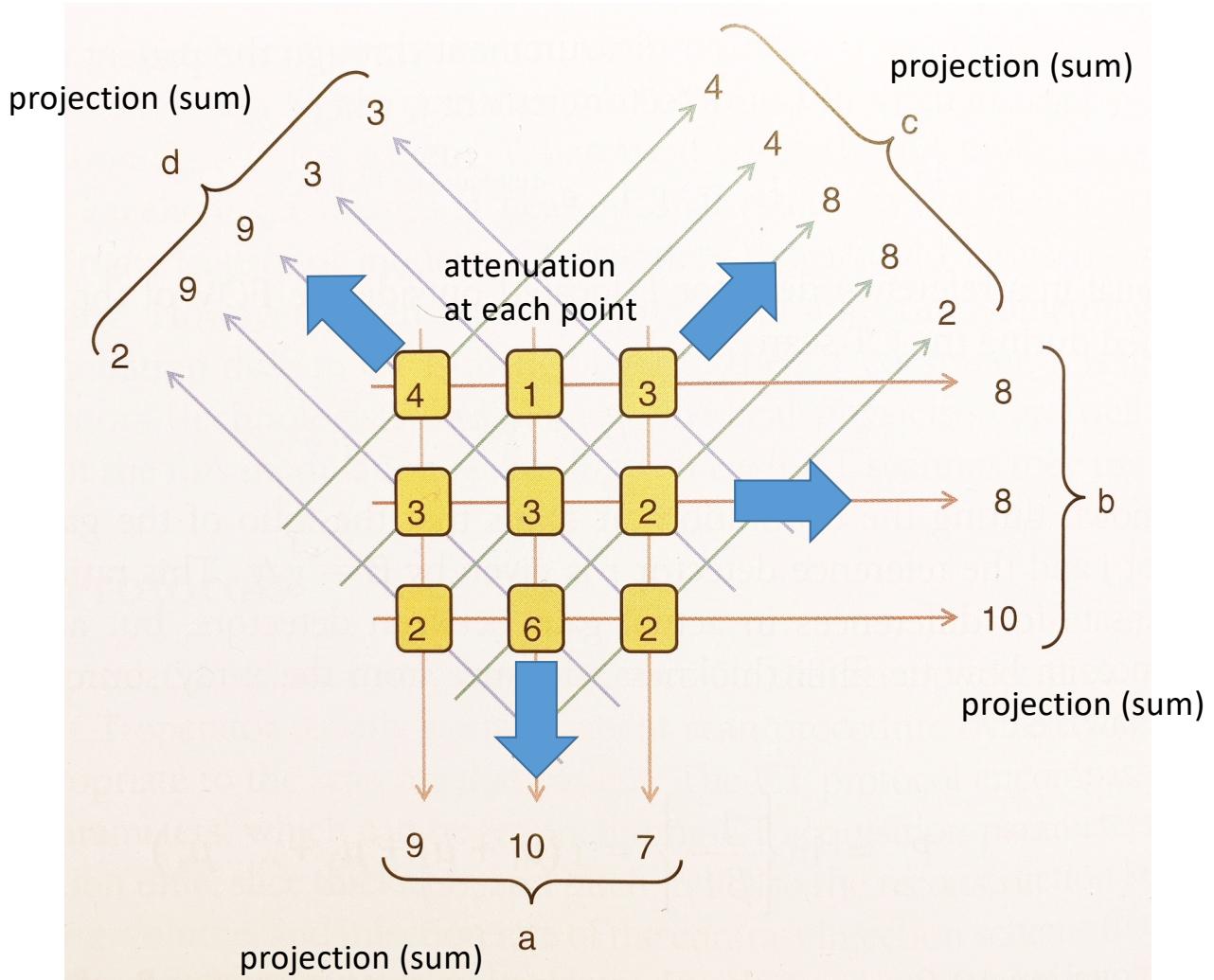


credit: Dr. Castañeda

Principle of Computed Tomography:

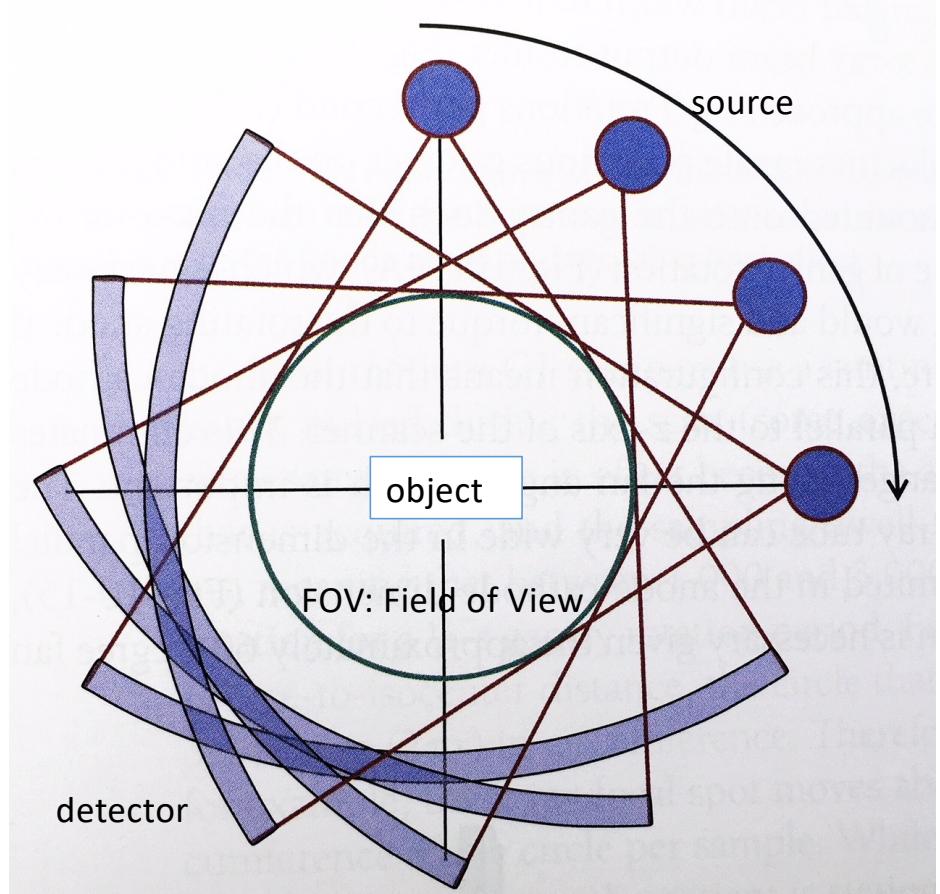
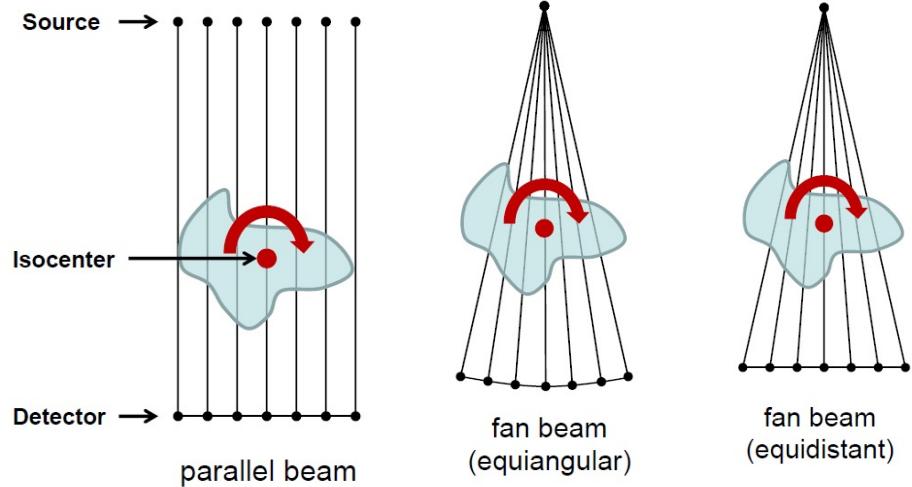
Projections

How to recover
the attenuation
from sum of
lines?



credits figure: ref 2

CT-scanner: fan/parallel beam



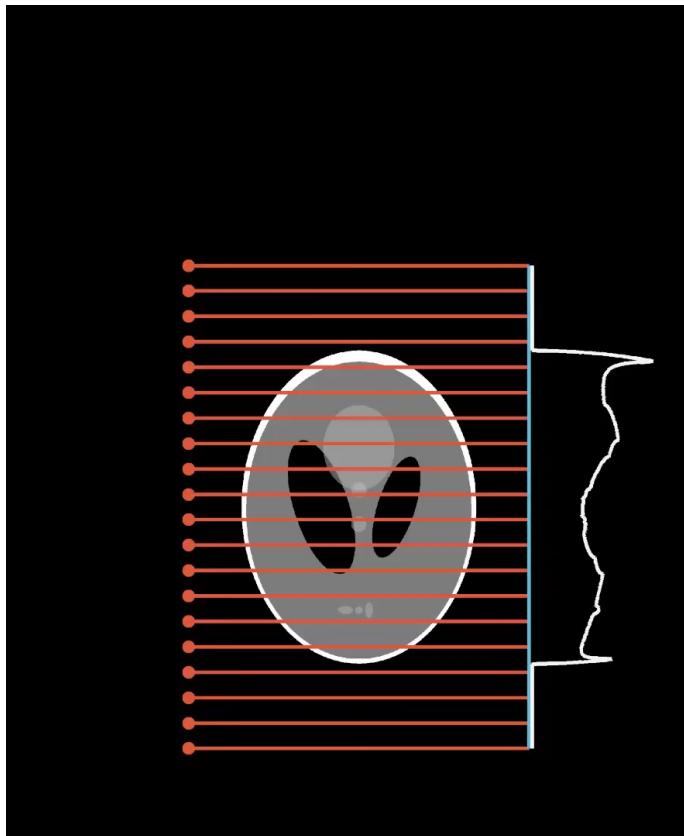
credits figure: ref 2

CT - scanner

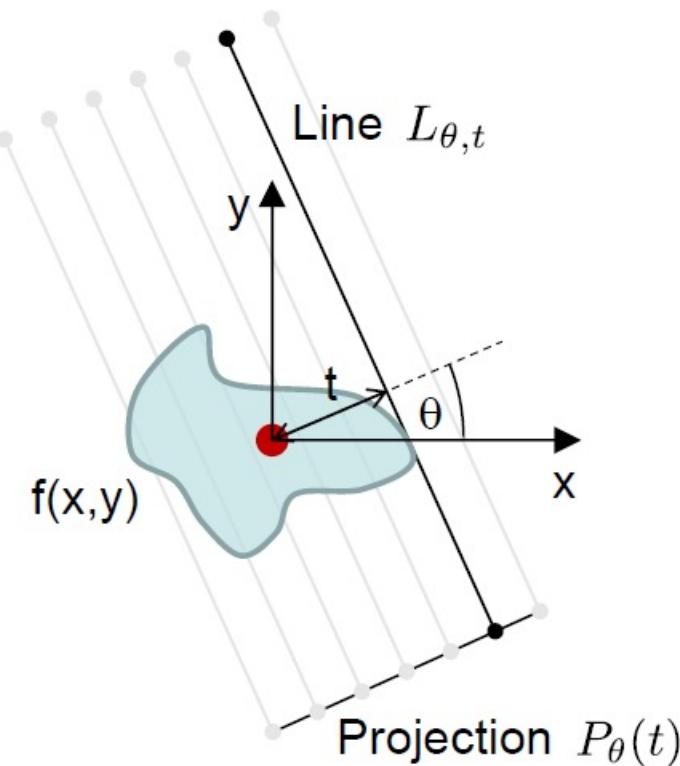
image? → sinogram (Radon transform)

Mathematical
tool:
the inverse
Radon transform

credit: Samuli Siltanen



The Radon transform



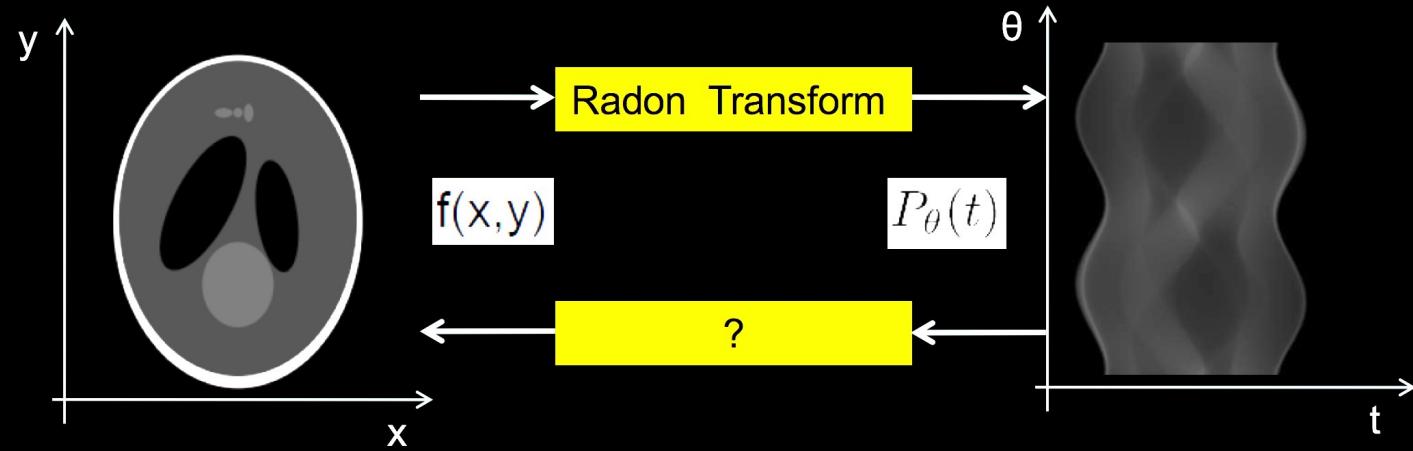
Integral of a function over
lines

$$P_\theta(t) = (R \circ f)(\theta, t) = \int_{L_{\theta,t}} f(\vec{x}) ds$$

credit: Dr. Castañeda

The inverse problem (Johann Radon, 1917)

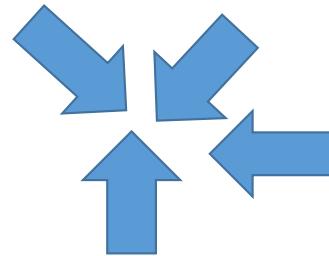
- Original image and sinogram of the Shepp-Logan head phantom



- We have the sinogram – how can we invert the Radon transform to yield the reconstructed image?

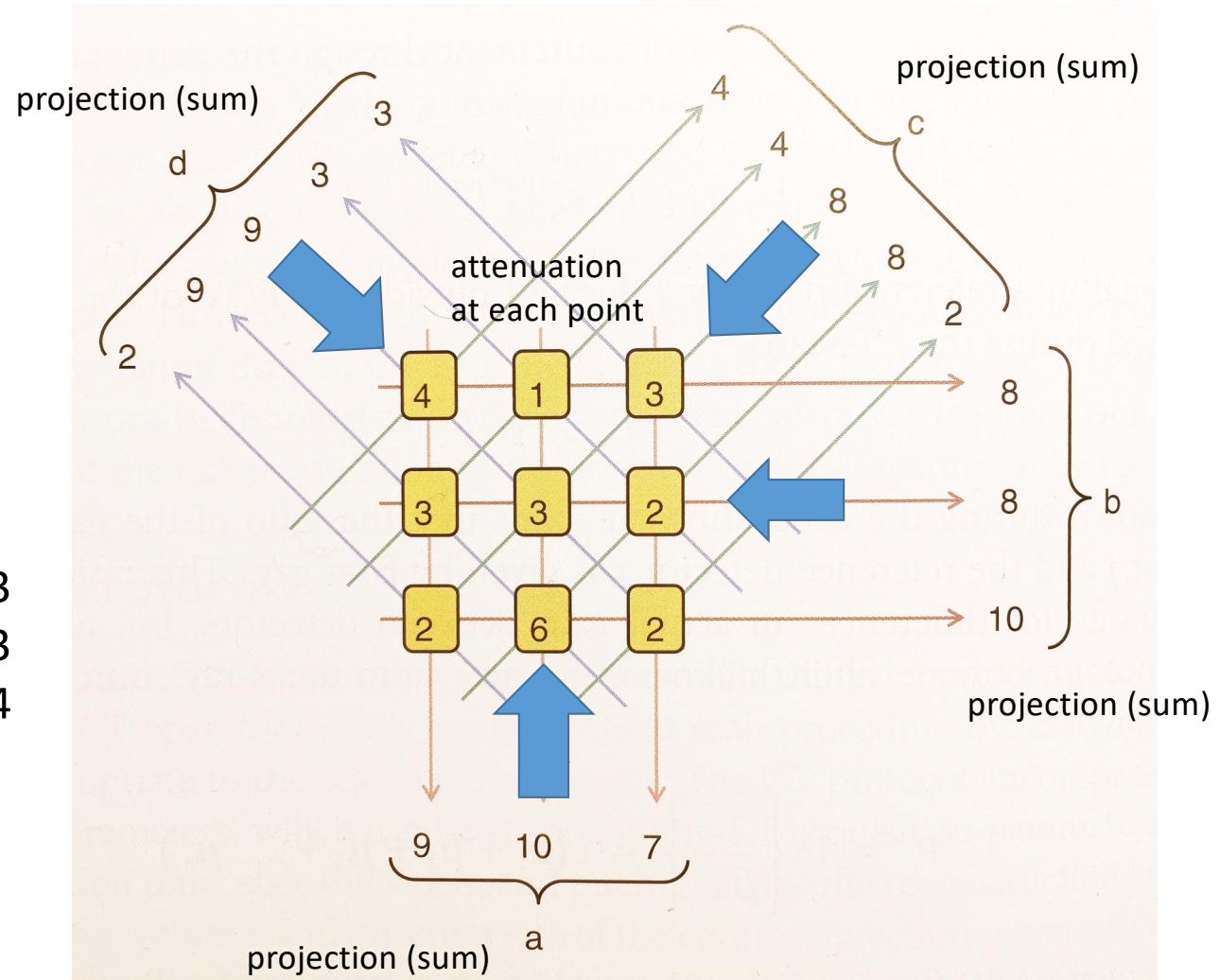
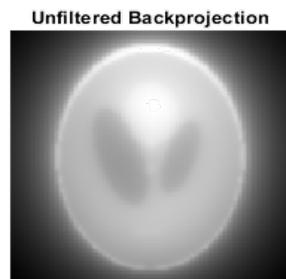
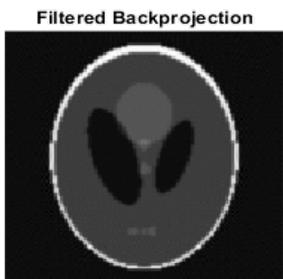
credit: Dr. Castañeda

Backprojections



30 25 26
 30 35 26 *9/26/4
 29 37 28

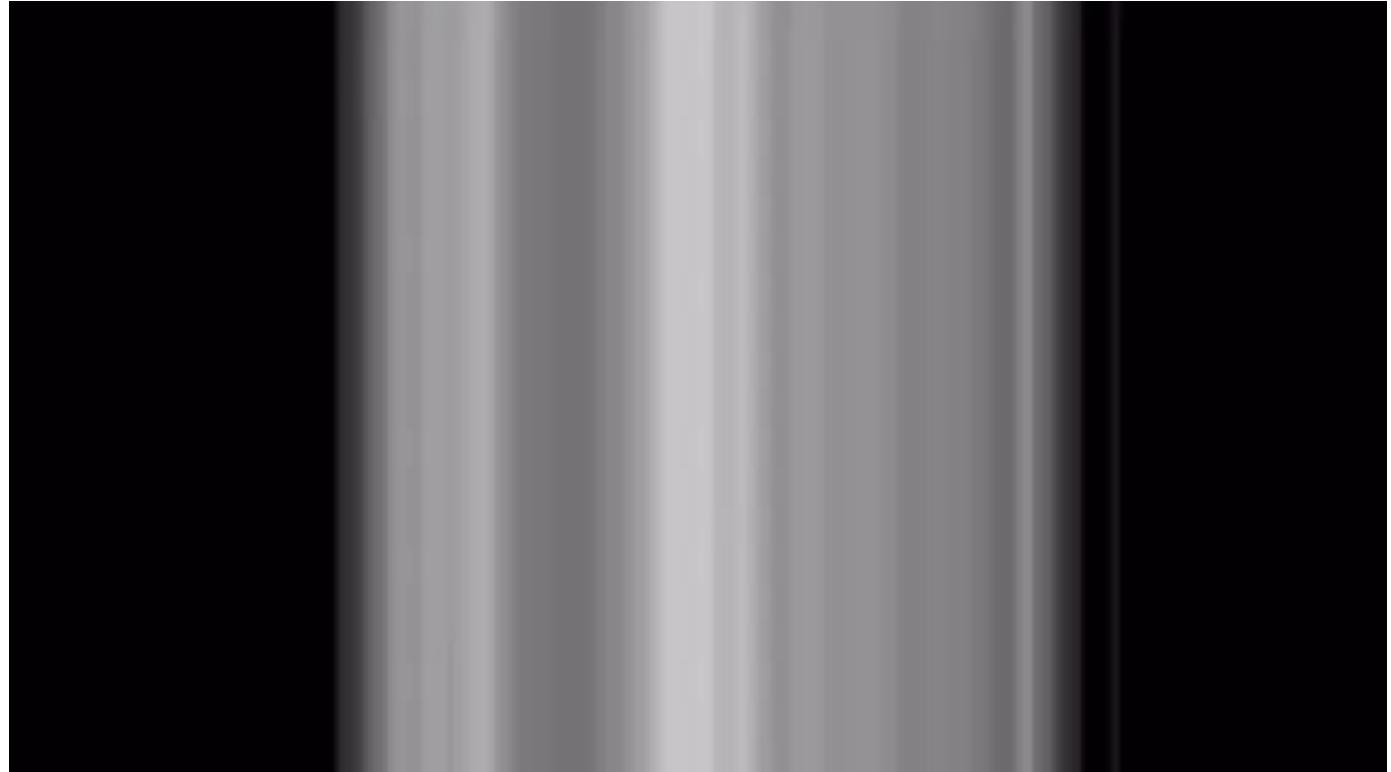
2.6 2.2 2.3
 2.6 3.0 2.3
 2.5 3.2 2.4



credits figure: ref 2

Backprojection:
effect of an increasing
number of projections

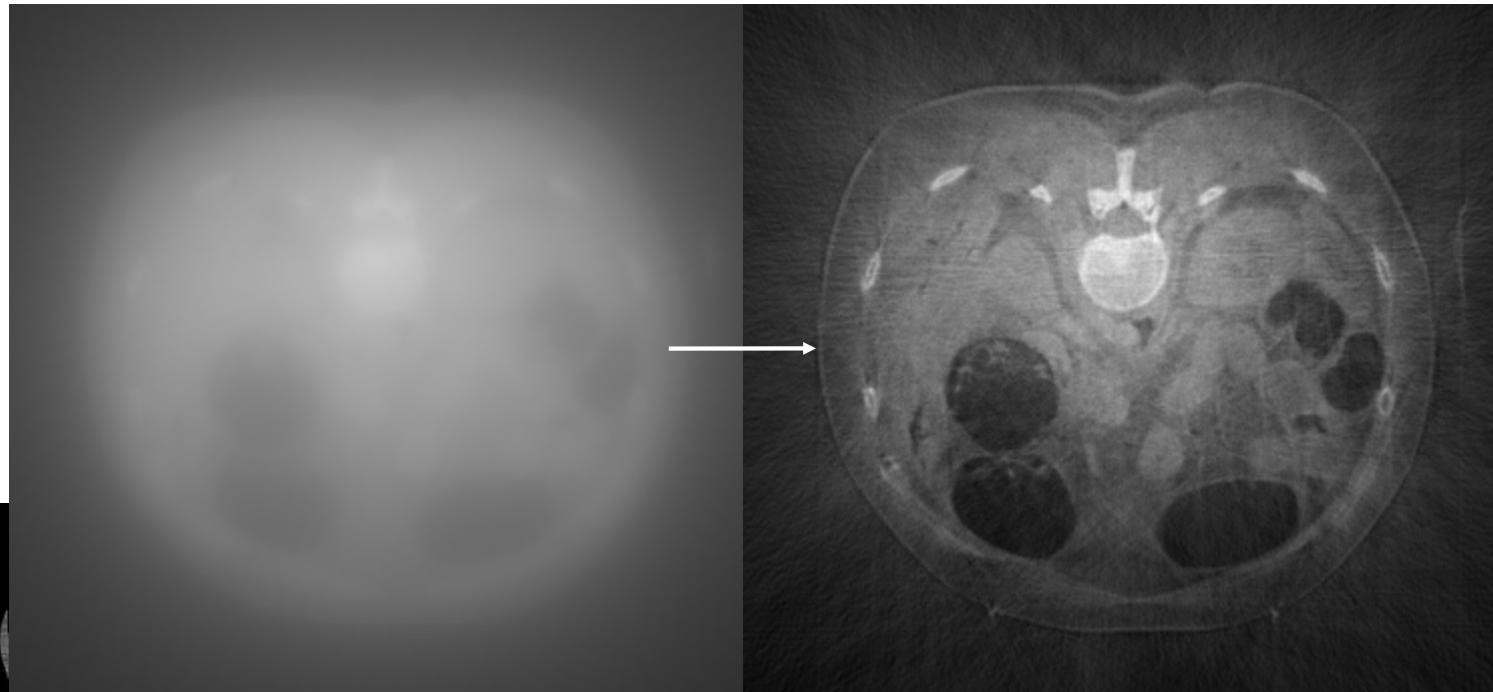
original (axial view)



magic!

Deconvolution Filter

original (axial view)



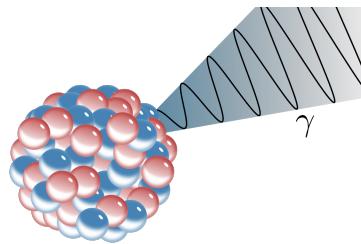
backprojection

filtered backprojection

Source: <http://www.perlproductions.at/index.php?choice=referenz&lang=en&id=15>

2nd... gamma rays

- Paul Villard discovered the **gamma rays** in 1900 in his laboratory of rue d'Ulm in Paris.



- 3 years later Marie and Pierre Curie, together with Henri Becquerel, would receive the Nobel Prize for the discovery of **radioactivity**.
- Nowadays, in **nuclear medicine**, radio-pharmaceuticals are used to emit gamma rays into the body.
- The idea is to mark certain key molecules that reflect the activity in the study of diseases such as **Alzheimer's or epilepsy**, or evidence of the metabolism when evaluating an **oncological** treatment.

Nuclear Medicine

- Radioisotop

What do we want:

- not too short and not too long half life time
- only γ radiation (α & β would increase patient dose without gain for diagnosis)

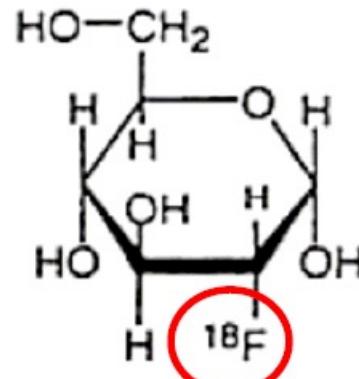
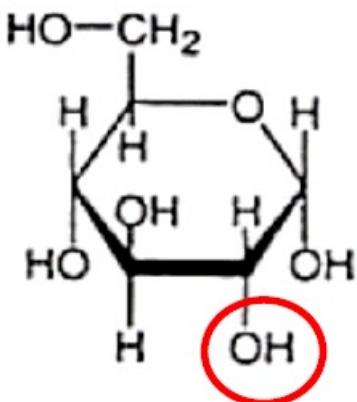
Nuclide	Half life time	Decay	Energy
^{99m}Tc	6 h	γ	140 keV
^{201}TI	73 h	γ	70 keV
^{123}I	13 h	γ	159 keV
^{18}F	110 min	$e^+, \gamma\gamma$	511 keV
^{11}C	20 min	$e^+, \gamma\gamma$	511 keV
^{13}N	10 min	$e^+, \gamma\gamma$	511 keV

credit: Dr. Castañeda

Nuclear Medicine

- **Radiopharmaceuticals**

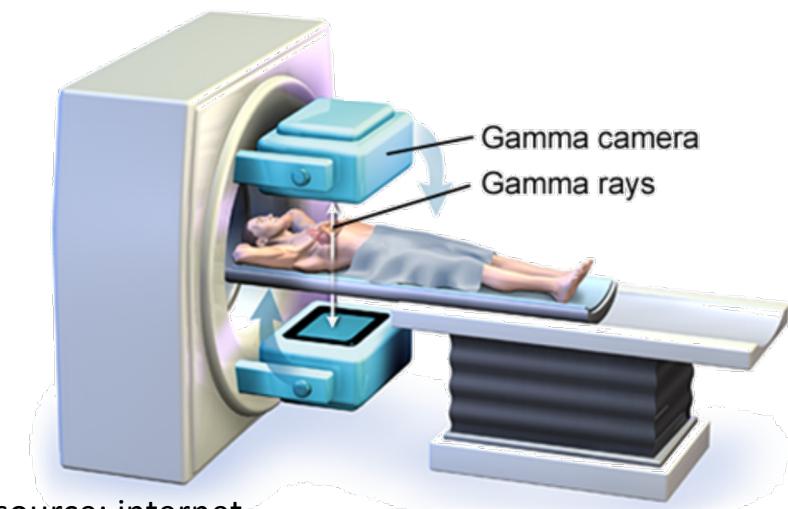
- The radioisotope has to be connected (labelling) to a pharmaceutical based on the organ specific question.
- Radiopharmaceuticals should not disturb the process under investigation.
- e.g. FDG (18F-Fluorodesoxyglucose) to analyze the Glucose metabolism



credit: Dr. Castañeda

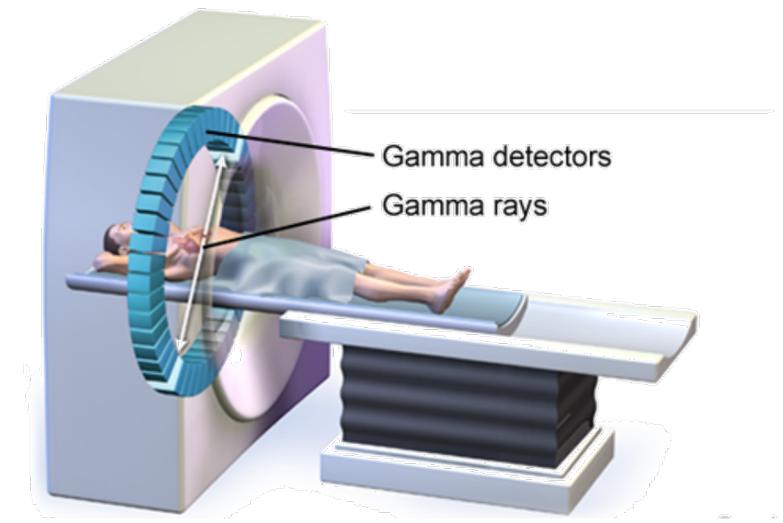
Nuclear medicine

SPECT scanner



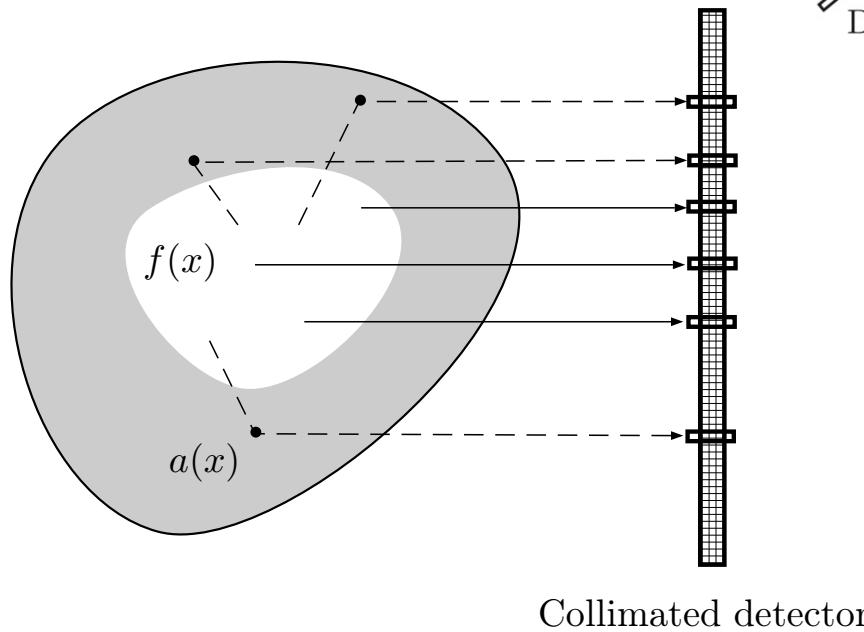
source: internet

PET scanner

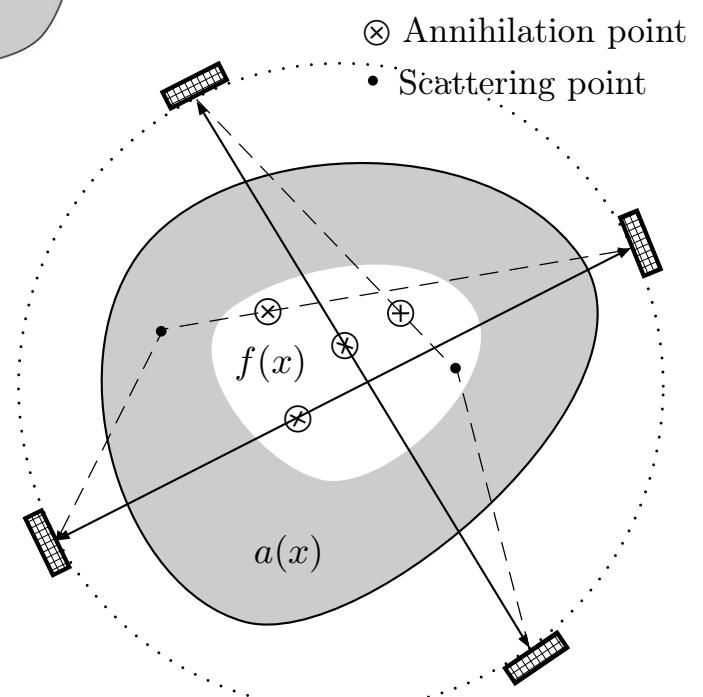
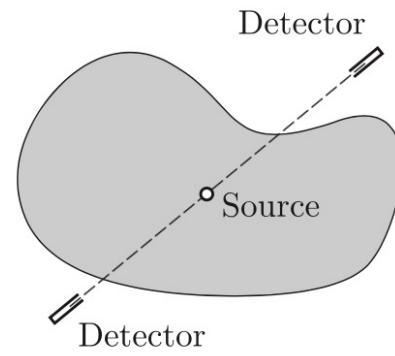


- monophotonic (SPECT) and biphotonic (PET) emission tomography the most extensively used in nuclear medicine

SPECT and PET



SPECT: Single photon
emission computed
tomography



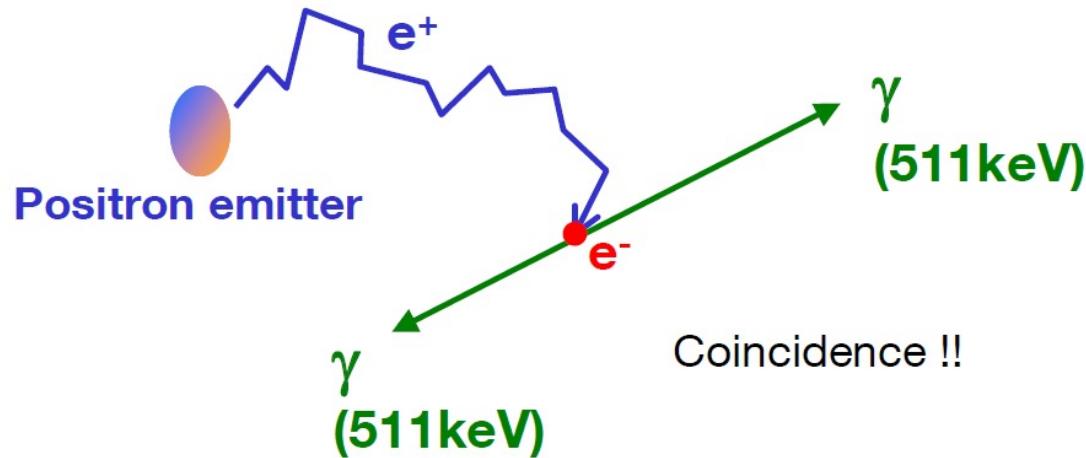
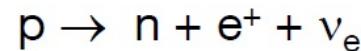
PET: Positron emission
tomography

- ⊗ Annihilation point
- Scattering point

Nuclear Medicine

- **Positron Emission Tomography – PET**

- Positron decay:

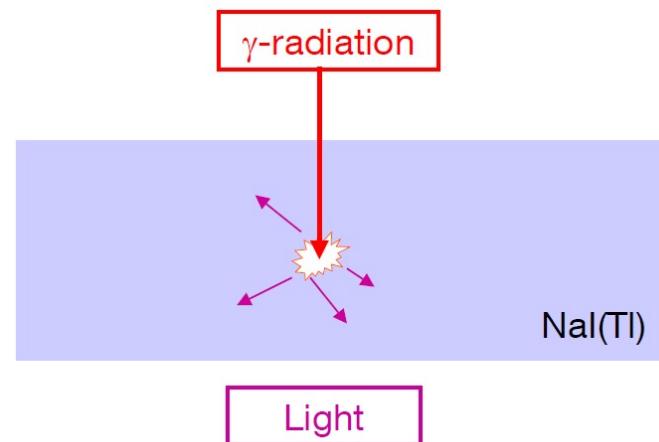


Coincidence !!

credit: Dr. Castañeda

Nuclear Medicine

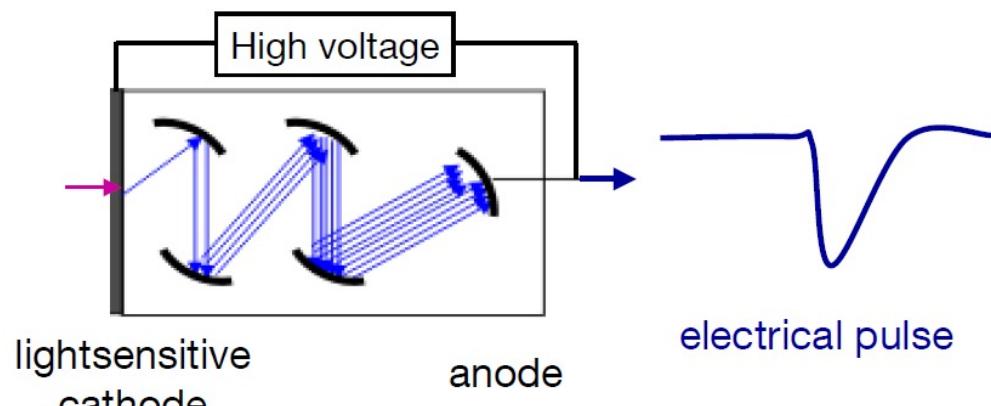
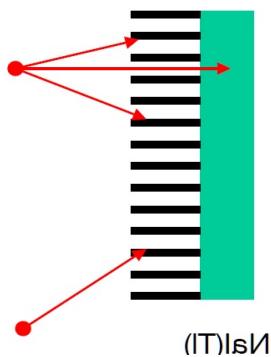
- **Radiation-Detection**
 - Scintillators, e.g. NaI(Tl) – Thallium doped Sodium Iodide



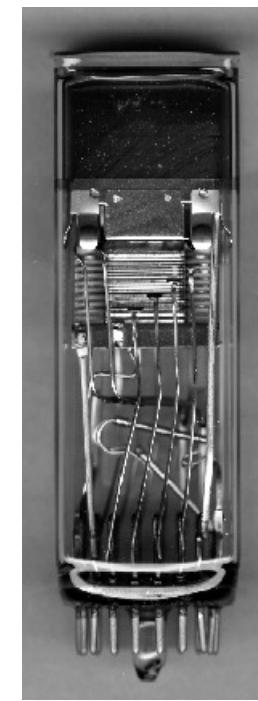
credit: Dr. Castañeda

Nuclear Medicine

- Photo multiplier
 - multiply the signal produced by incident light by as much as 100 million times
- Gamma camera



γ -energy ~ light ~ pulse height



credit: Dr. Castañeda

Example: SPECT scan of the brain

- primarily used to view how blood flows through arteries, veins and capillars;
 - can detect reduced blood flow (more sensitive than MRI or CT);
 - tracer stays in blood stream rather than being absorbed by tissues;
 - cheaper and more readily available than higher resolution PET scans.
- (source: www.mayfieldclinic.com)

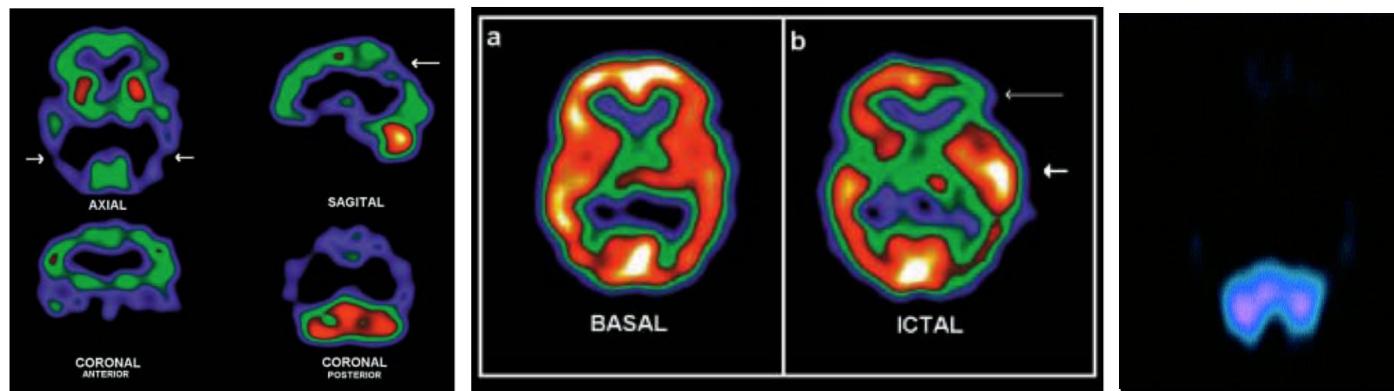
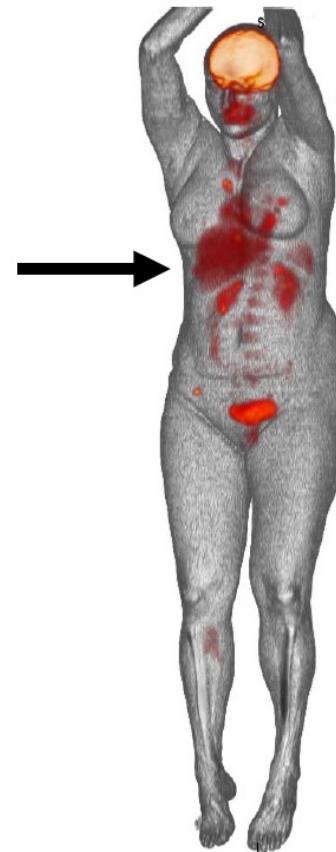
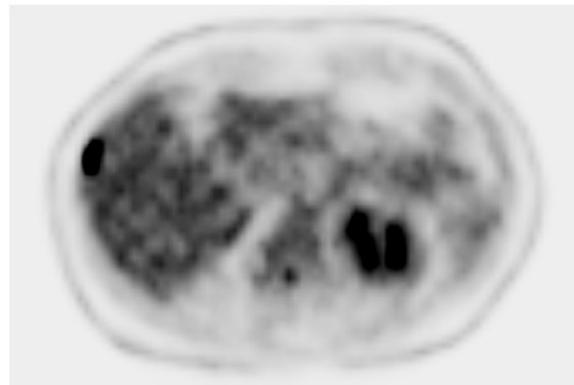


Figure: Left: Alzheimer disease patient with low blood perfusion (arrows);
Right: different blood flow before and during an epileptic seizure (arrows).
(source: Quintana, J.-C., Neuropsiquiatría: PET y SPECT . Rev. chil. radiol. 2002)

Nuclear Medicine

- Multimodality – PET/CT



credit: Dr. Castañeda

Break: application of Fourier Transform



Second part (wave phenomena)

2.- magnetic resonance

*current research topic: enhanced phase contrast
technique for MRI-4DFlow adquisition and applications
to non-invasive pressure and blood flow estimation*

imaging the body... magnetism... MRI

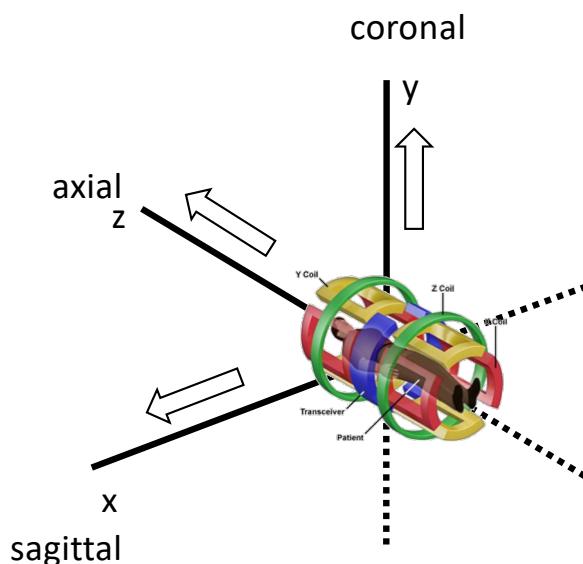
- **Magnetic Resonance Imaging (MRI).**
- Strong magnetic fields are induced in the body with suitable forms that force the **spin of water hydrogen nuclei** into the body, to obtain:
 - images of high resolution inside the body
 - the speed of blood in an artery or
 - the displacement of the heart when barking
- Discovered by Felix Bloch and Edward Mills Purcell in 1947.
- The **first MRI machine** for human patients was built and patented in 1972 by the American physician Raymond Damadian. First clinical images obtained in 1977 (Lauterbur-Mansfield-Damadian).



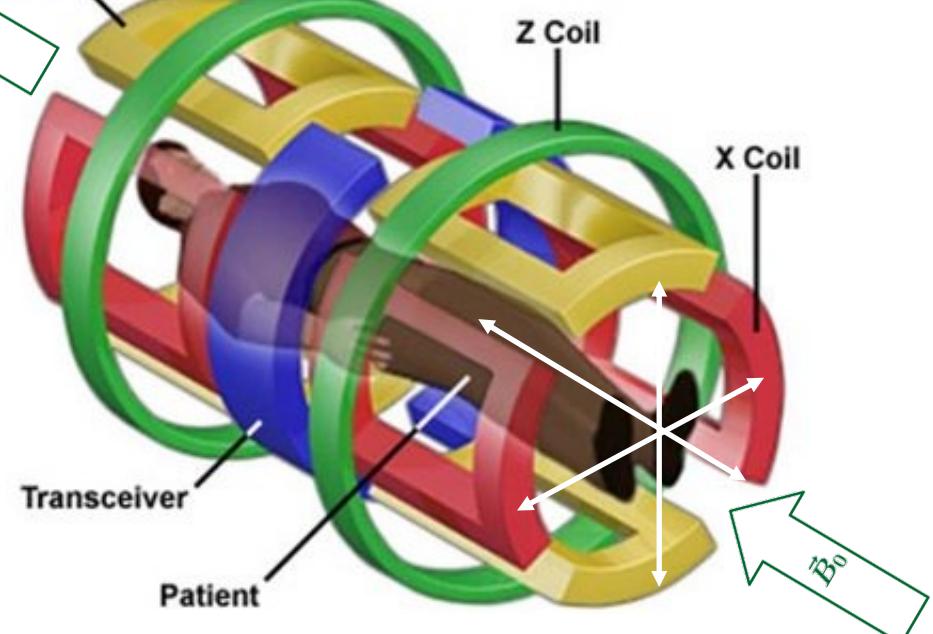
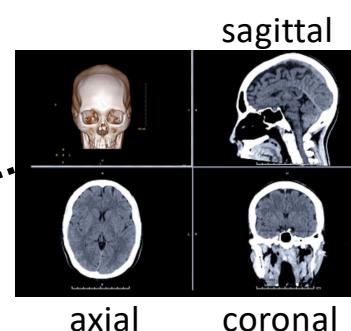
- No ionizing radiation
- Excellent image resolution
- Relatively easy to set up
- T1, T2 images, velocity, displacements
- Slow compared with physiological motion
- Medium contrast detection

Magnetic Resonance Imaging... In 4 Steps

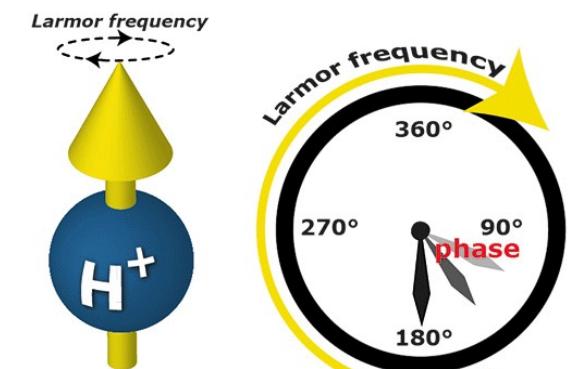
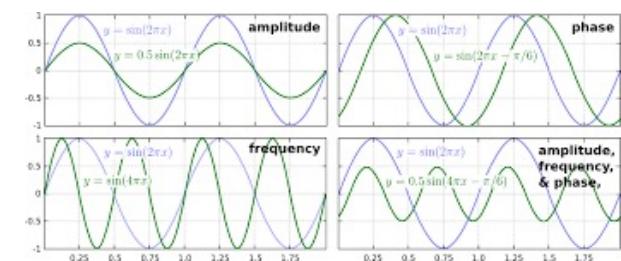
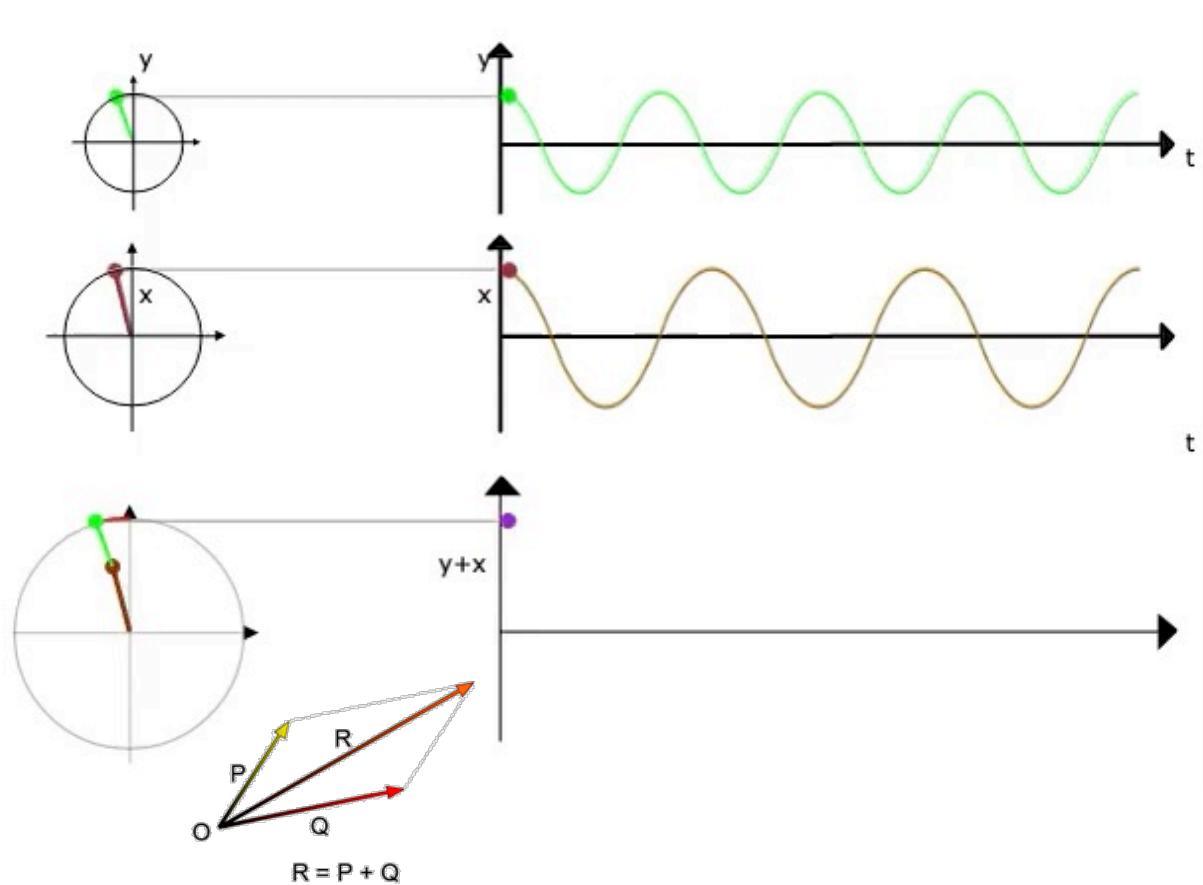
Source: Kurzhunov, 2017



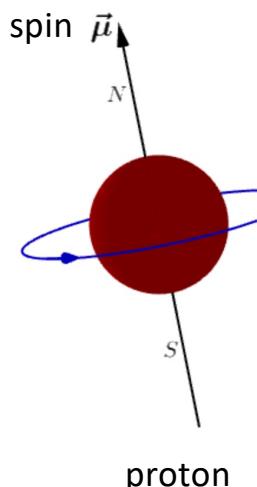
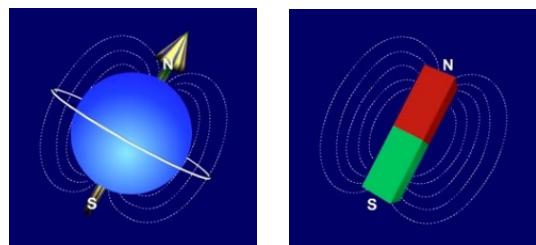
Reference frame
and gradients



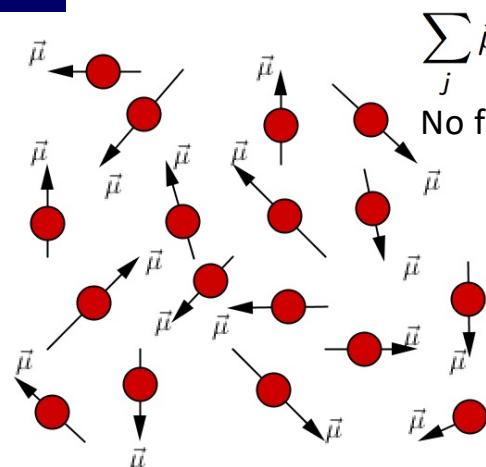
Basic concepts: amplitude, frequency, phase, sum of magnetic forces



Spin and Magnetization



1

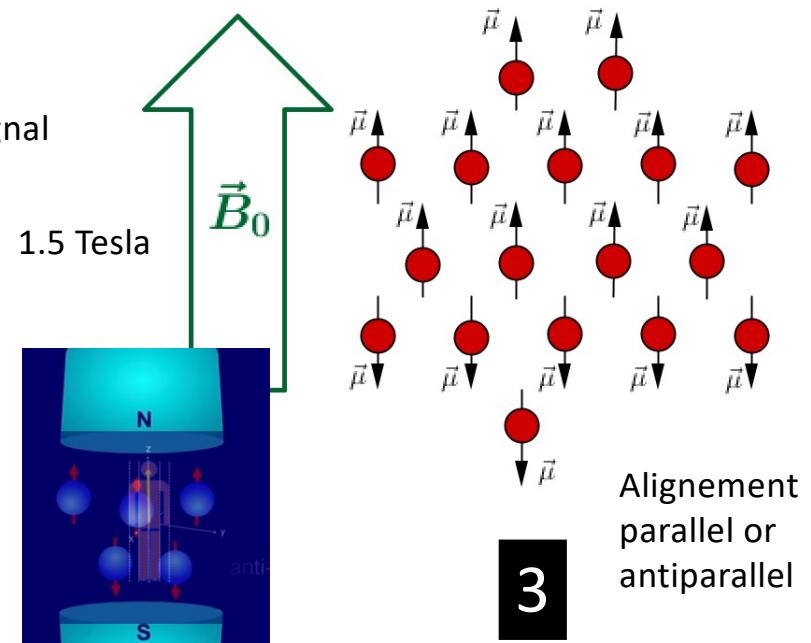


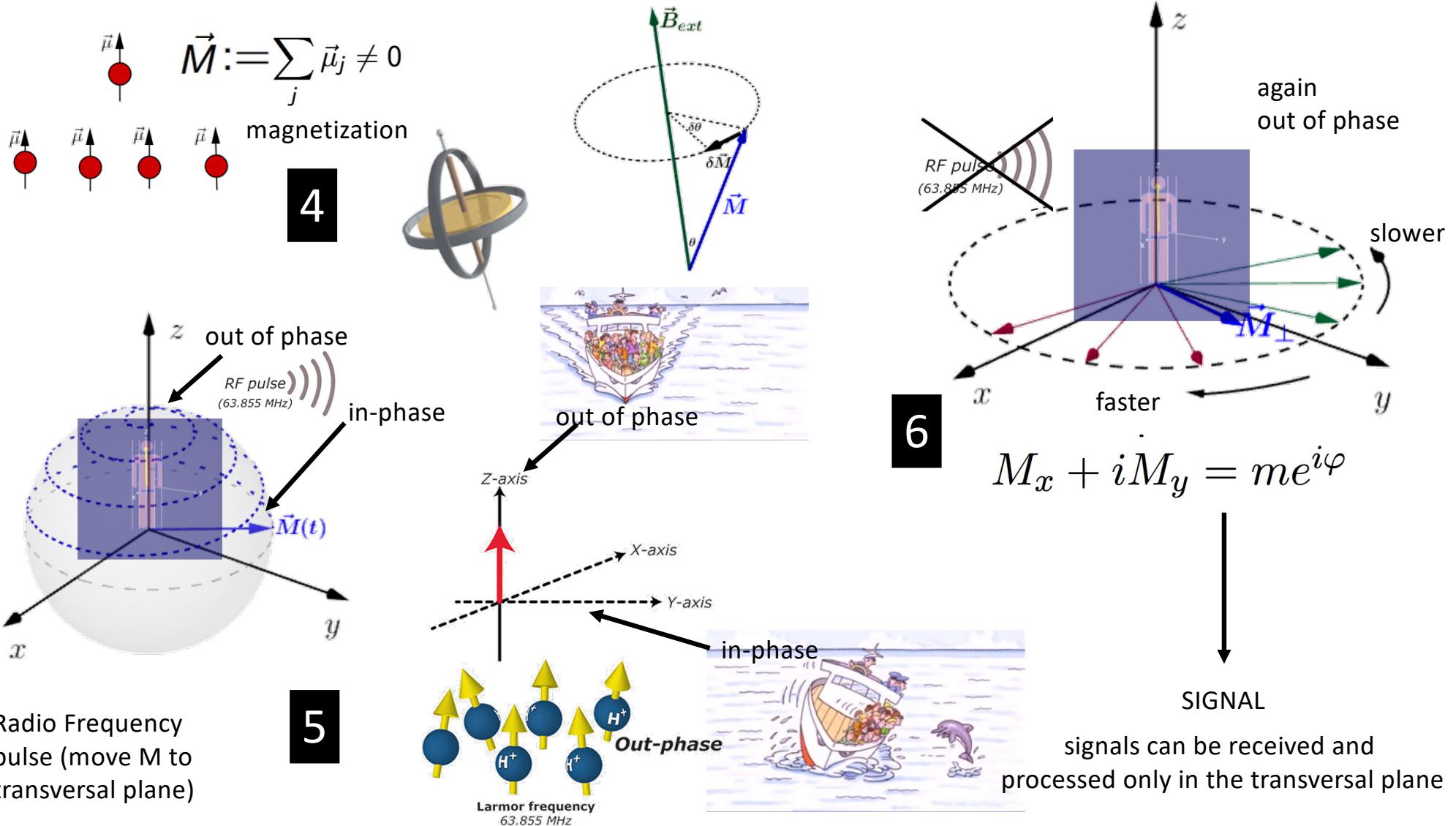
tissue protons

2

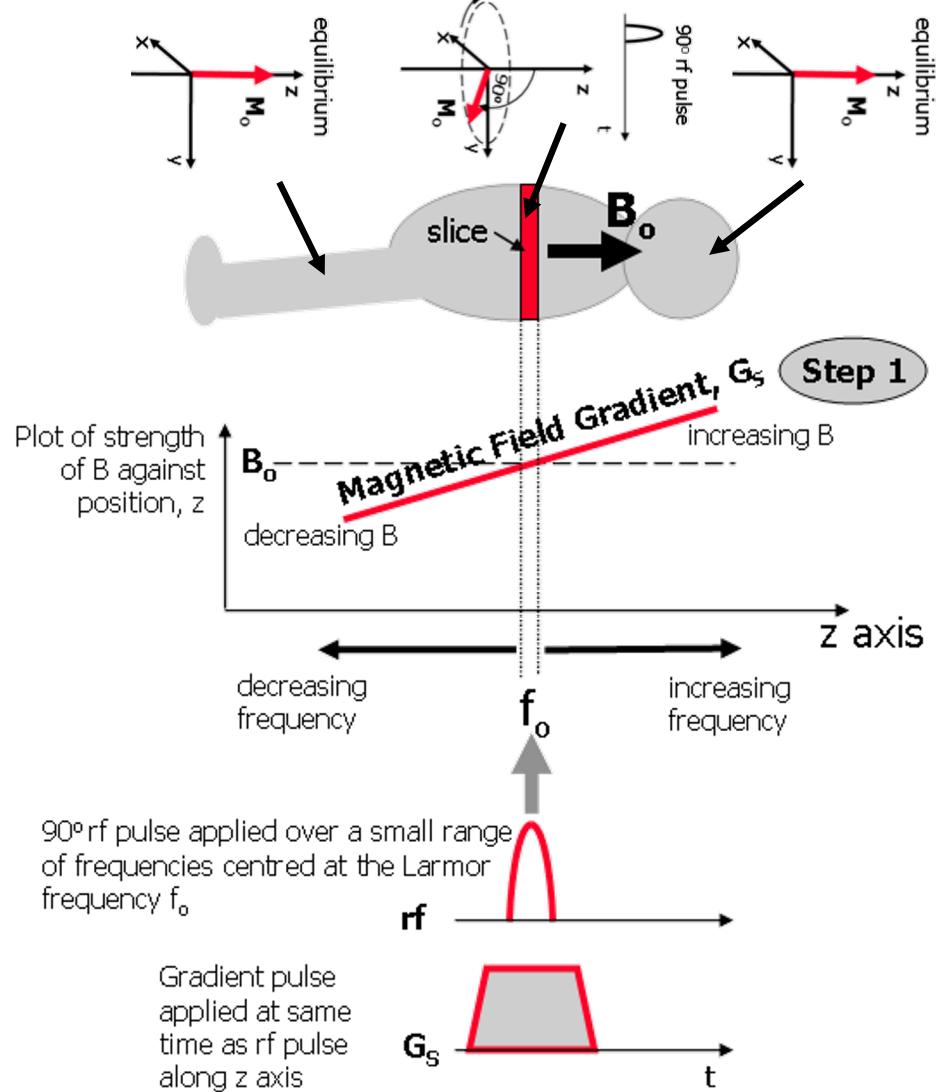
$$\sum \vec{\mu}_j = 0$$

No force-no signal



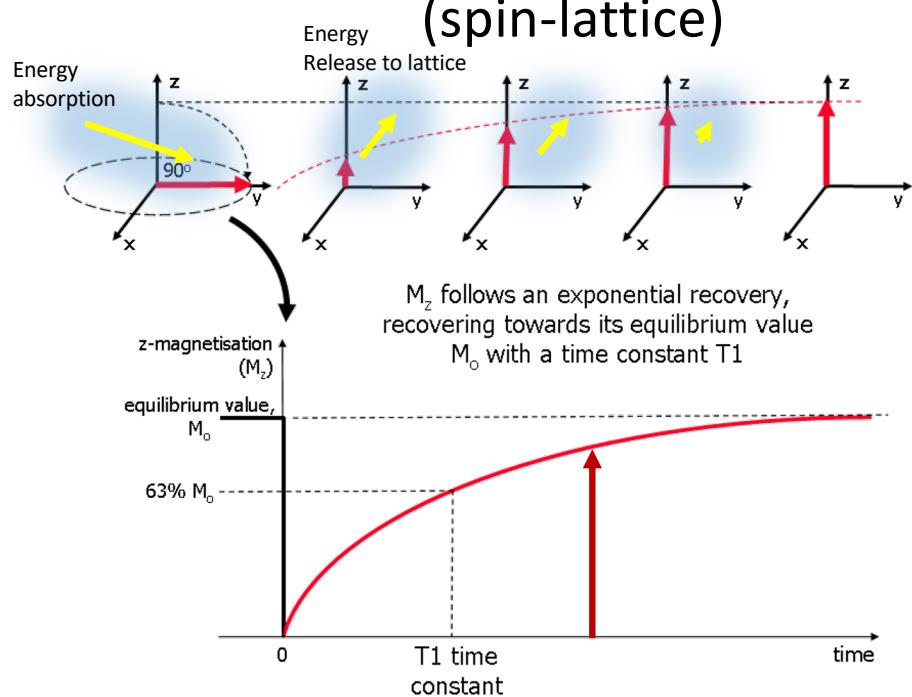


MRI Step 1: RF pulse and slice selection (axial)

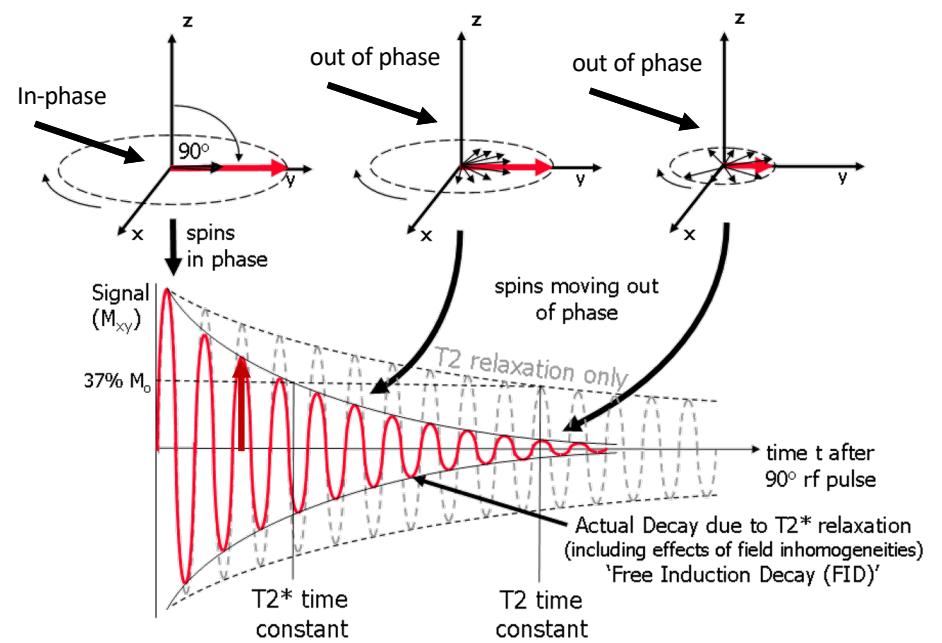


MRI relaxation times

T1: longitudinal relaxation (spin-lattice)

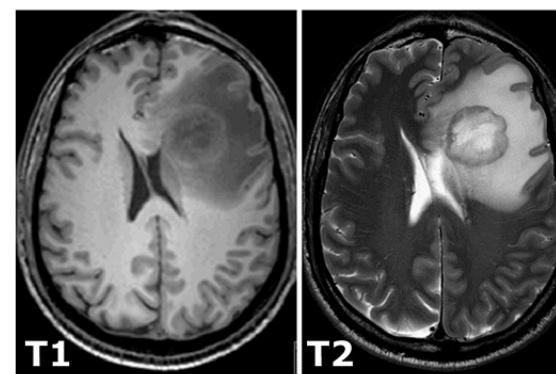
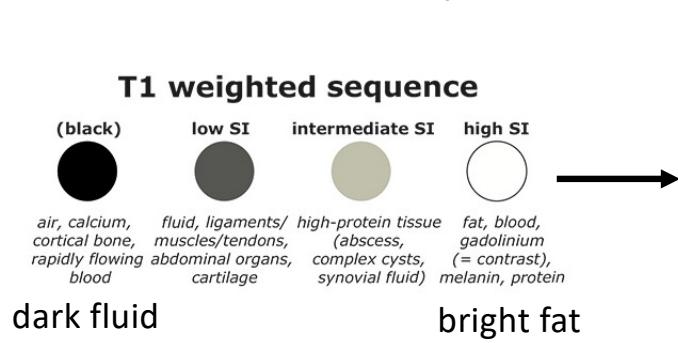


T2: transversal relaxation (spin-spin interaction)

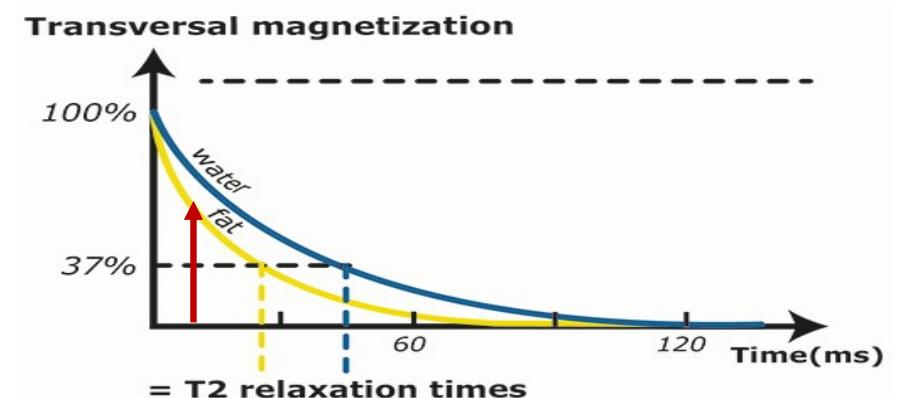
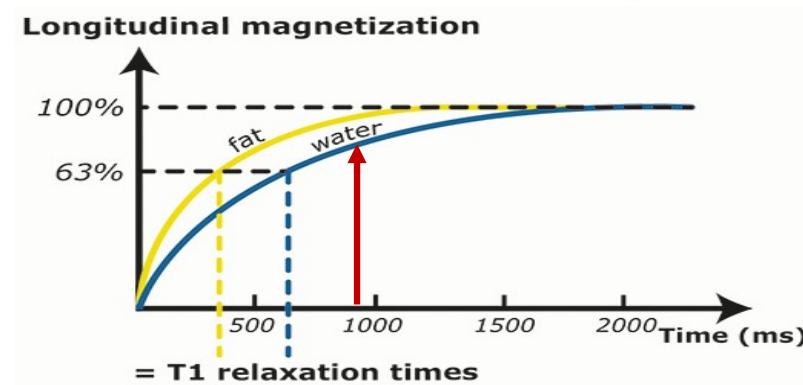
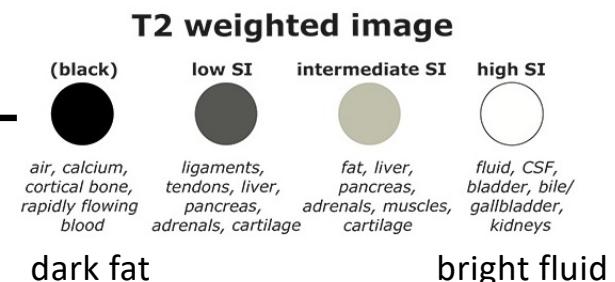


MRI relaxation times

T1: longitudinal relaxation
(spin-lattice)



T2: transversal relaxation
(spin-spin interaction)



bibliography

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 - Introduction to biomedical images course EPFL:
<https://www.youtube.com/watch?v=LdiJLkYgZ2M&list=PLTCZivgYYpFpVnxdGrxcuL5YOvPwespXy>
 - MRI questions and answers:
<http://mr/questions.com/image-contrast-trte.html>
- Other videos MRI:
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<https://www.youtube.com/watch?v=zf5oX01bRgk>
 - Lighbox Radiology, Australia:
<https://www.youtube.com/watch?v=Ok9ILIZzmaY>
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CYTA

Thanks

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