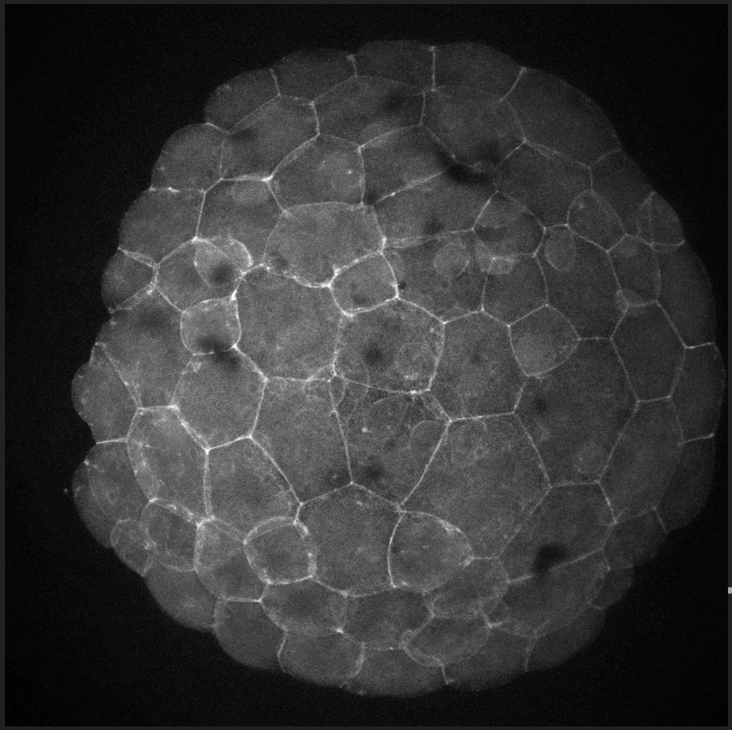
PRINCIPLES OF MECHANICS IN BIOLOGY



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UNIVERSIDAD DE CHILE

Optics, Forces & Development 2024

ACTIVE MATTER
Núcleo Milenio Física de la Materia Activa

Ministerio de Economía, Fomento y Turismo
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MOLECULAR MOTORS

F=m a

VECTORS

FORCES

TENSORS

NEWTON

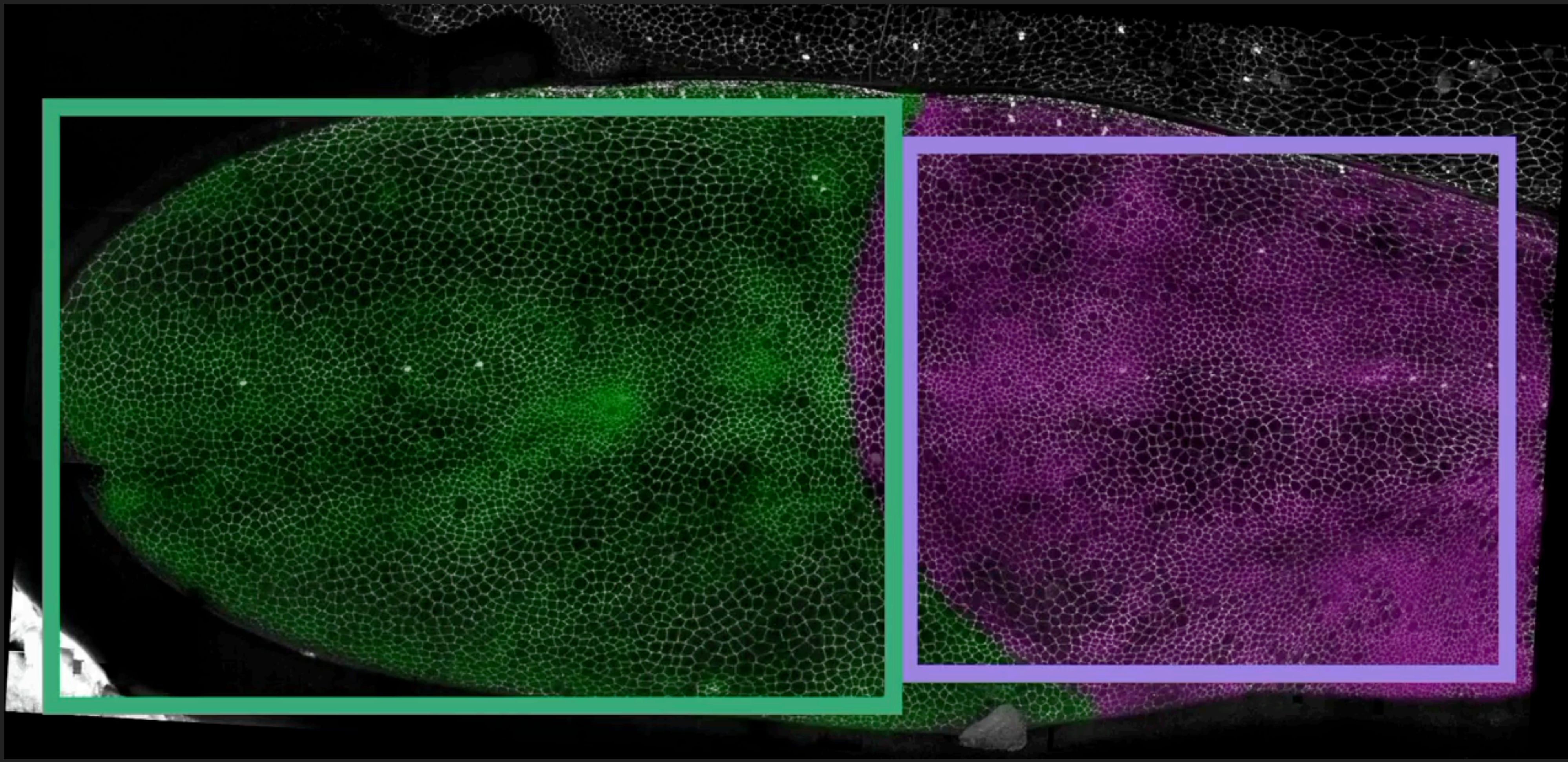
TENSION

STRESSES

ACTION-REACTION

MECHANICS?

FORCES ACTING ON DEVELOPMENT



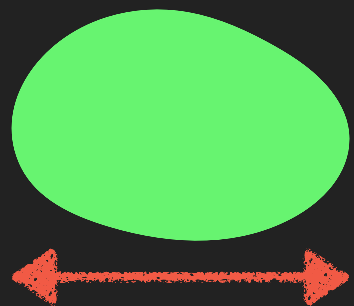
Drosophila pupal wing
R. Etournay et al, eLife 2015

IN THE GREEN DOMAIN, IN WHICH DIRECTION DO THE FORCES ACT?

AT THE FINAL EQUILIBRIUM STATE, ARE THERE FORCES ACTING?

IS THE TISSUE UNDER STRESS?

FORCES IN THE MICROSCOPIC WORLD



$$\ell \sim 1 - 10\mu\text{m}$$

$$F = ma$$

$$m \sim \rho \times \text{vol}$$

$$m \sim \ell^3$$

SCALING OF DIFFERENT FORCES

$$\begin{aligned}
\text{Adhesion:} \quad & F \sim \text{area} \sim \ell^2 \\
\text{Weight:} \quad & F \sim \text{mass} \sim \ell^3 \\
\text{Friction by weight:} \quad & F \sim \text{mass} \sim \ell^3 \\
\text{Friction by compression:} \quad & F \sim \text{area} \sim \ell^2 \\
\text{Viscous friction:} \quad & F \sim \text{area} \sim \ell^{1-2} \\
\text{Elastic:} \quad & F \sim \text{area} \sim \ell^{0-2}
\end{aligned}$$

AT THE MICROSCOPIC SCALE, MASS AND WEIGHT ARE NEGLIGIBLE.

NO INERTIA

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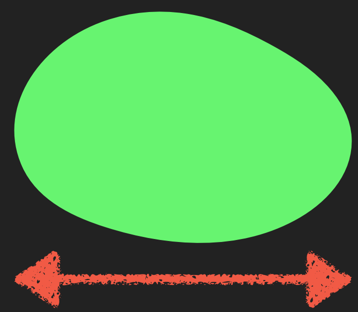
FORCES IN THE MICROSCOPIC WORLD



$\ell \sim 1 - 10\mu\text{m}$

AT THE MICROSCOPIC SCALE, MASS AND WEIGHT ARE NEGLIGIBLE.

NO INERTIA

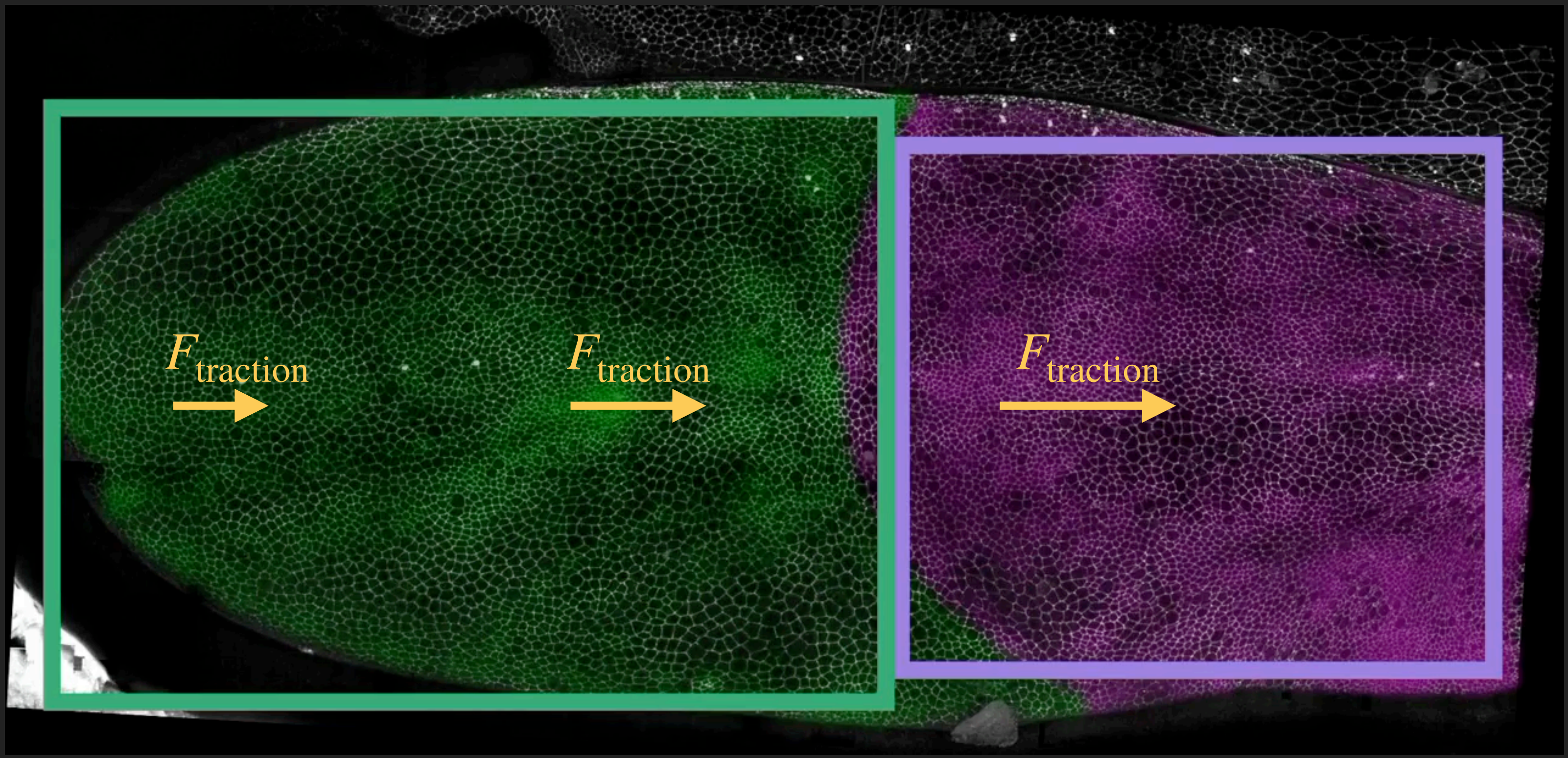
Newton's law reduces to

$$F_1 + F_2 + F_3 + \ldots F_n = 0$$

At every instant, not only on average

How is it possible that F=0 gives rise to motion?

FORCES ACTING ON DEVELOPMENT



Drosophila pupal wing
R. Etournay et al, eLife 2015

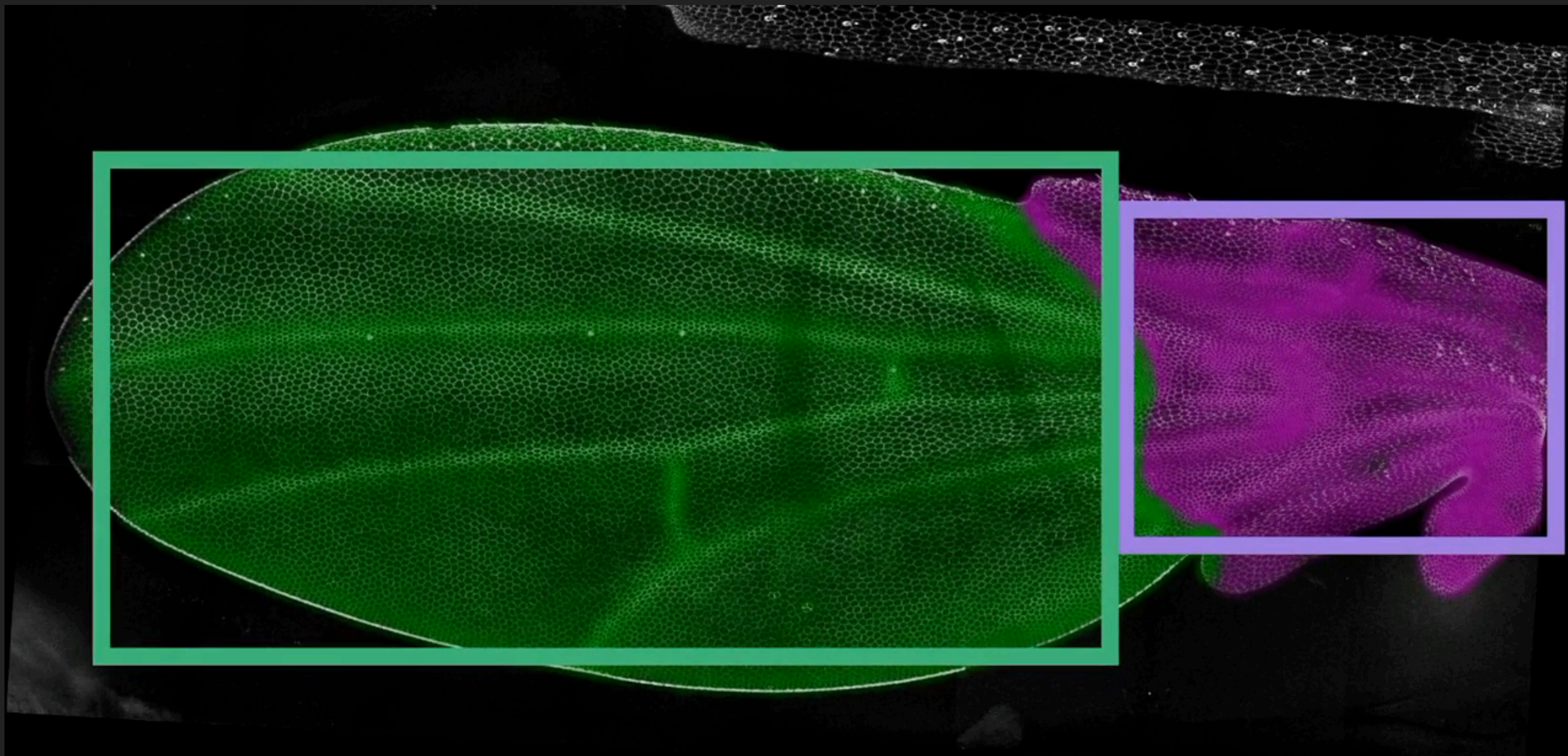
On each cell, $F_{total} = F_{friction} + F_{traction} = 0$

$F_{friction} = -\gamma V$

Then, $F_{traction} = \gamma V$

THE TRACTION FORCES PRESENT A GRADIENT

FORCES ACTING ON DEVELOPMENT



Drosophila pupal wing
R. Etournay et al, eLife 2015

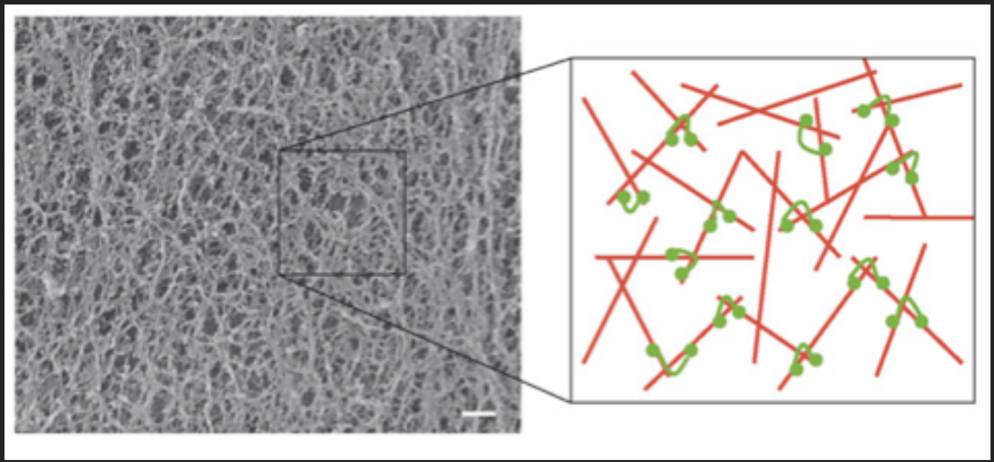
At the final state, there is no motion (V=0).

Then $F_{\text{traction}} = 0$

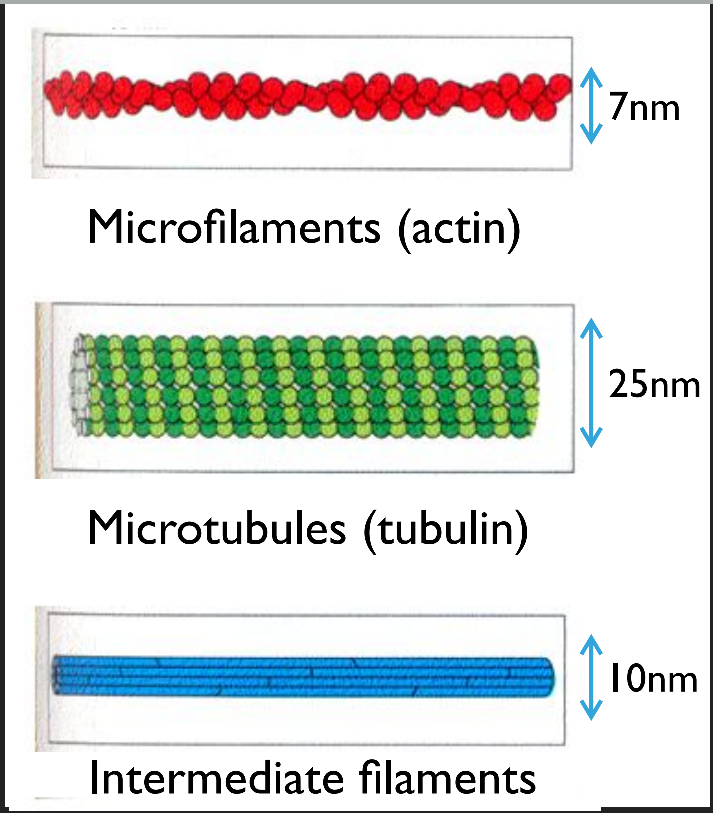
How is that compatible with the idea of the **tissue being under stress?**

CELULAR STRESSES

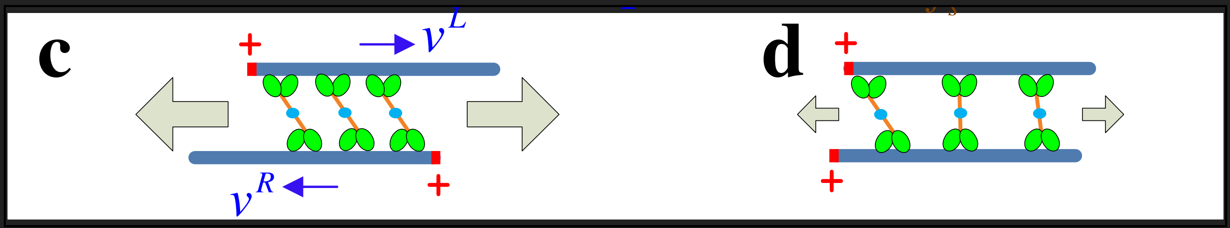
(DETAILS IN ANDREA RAVASIO'S AND CRISTINA BERTOCCHI'S LECTURES)



Kasza et al. Curr Opin Cell Biol (2007)

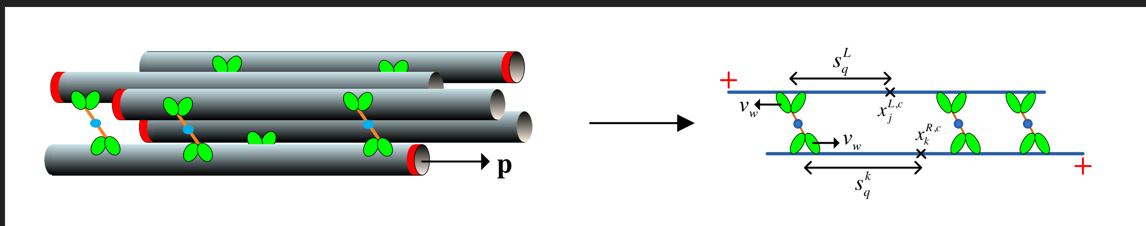


The filaments are polar and the molecular motors can "walk" in a specified direction

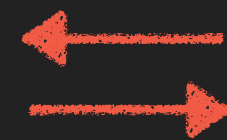


Gao et al. (2015)

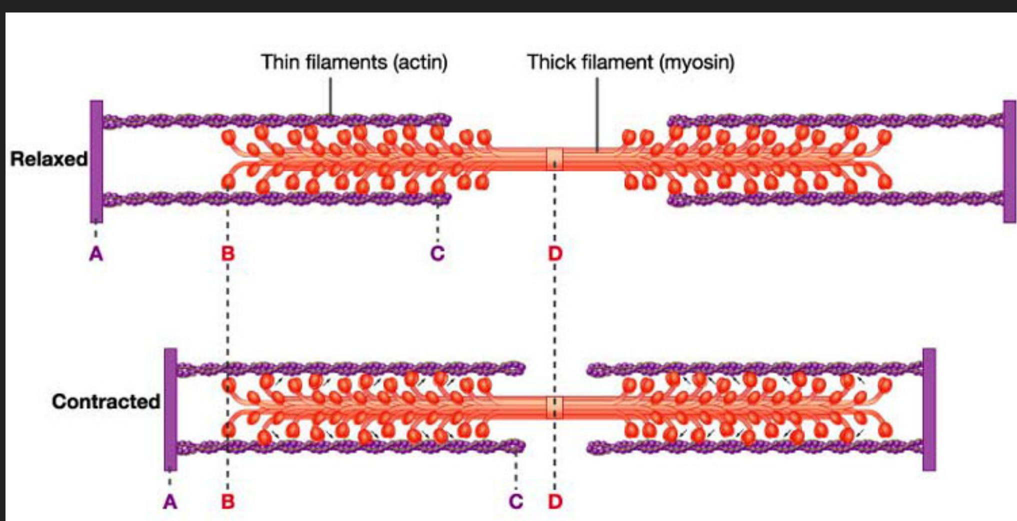
CELULAR STRESSES



Gao et al. (2015)



PAIR OF OPPOSITE FORCES.
SEPARATED BY FEW NANOMETERS.
A FORCE DIPOLE



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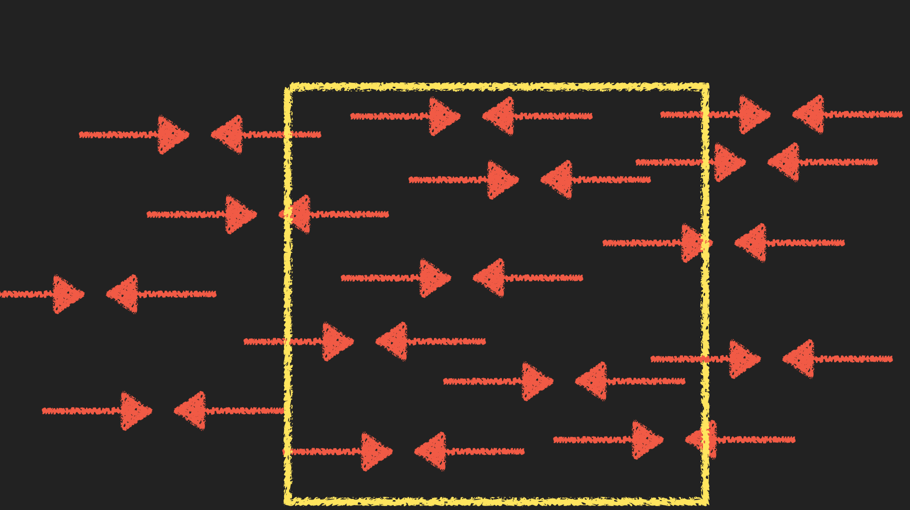
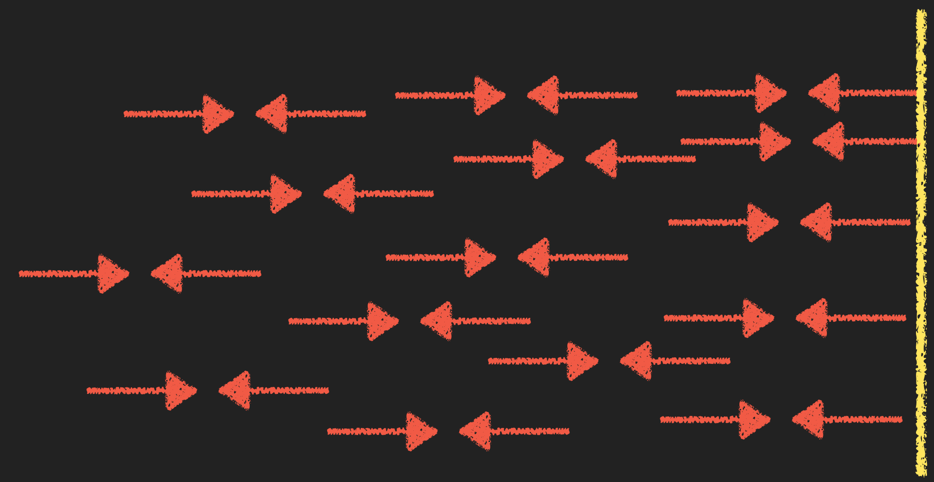
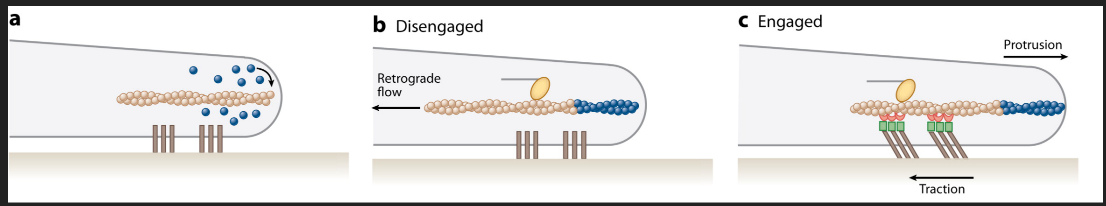


THE STRESSES ARE FINITE,
BUT THE SUMMED FORCE VANISHES



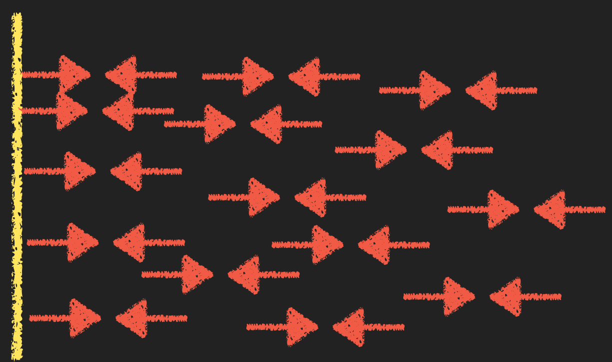
BUT ON THE SURFACE THERE IS A NET FORCE
A TENSILE OR TRACTION STRESS.

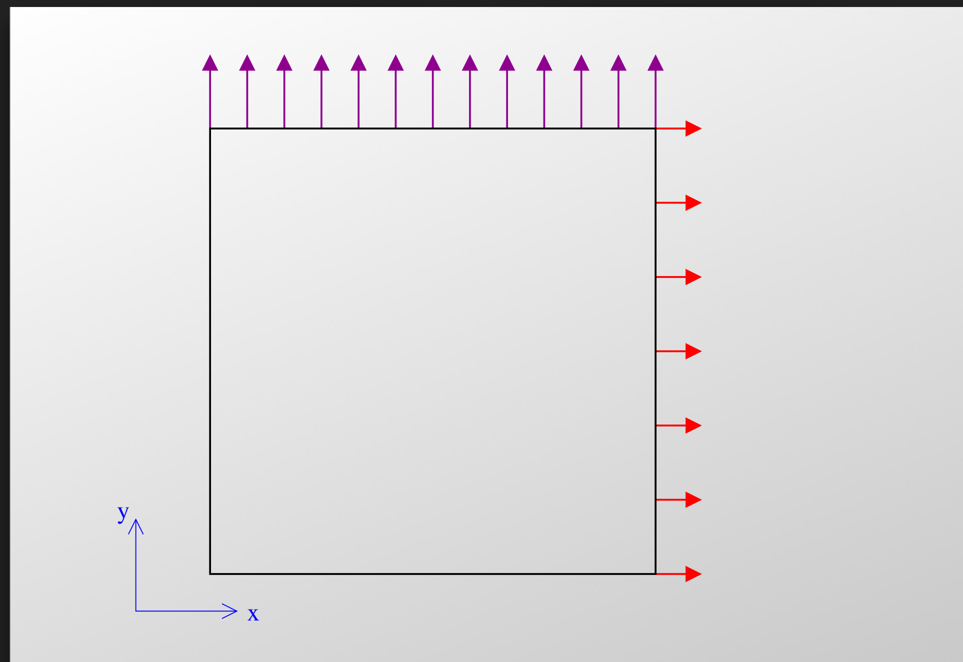
THE NET FORCE IS PROPORTIONAL TO THE AREA



Gardel et al., Annu Rev Cell Dev Biol

STRESS TENSOR *(TENSOR DE ESFUERZOS O DE TENSIONES)*





$$F_x \propto L_y$$
$$F_y \propto L_x$$

$$\sigma_x = F_x / L_y$$
$$\sigma_y = F_y / L_x$$

$$\sigma = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$$

Stress tensor:

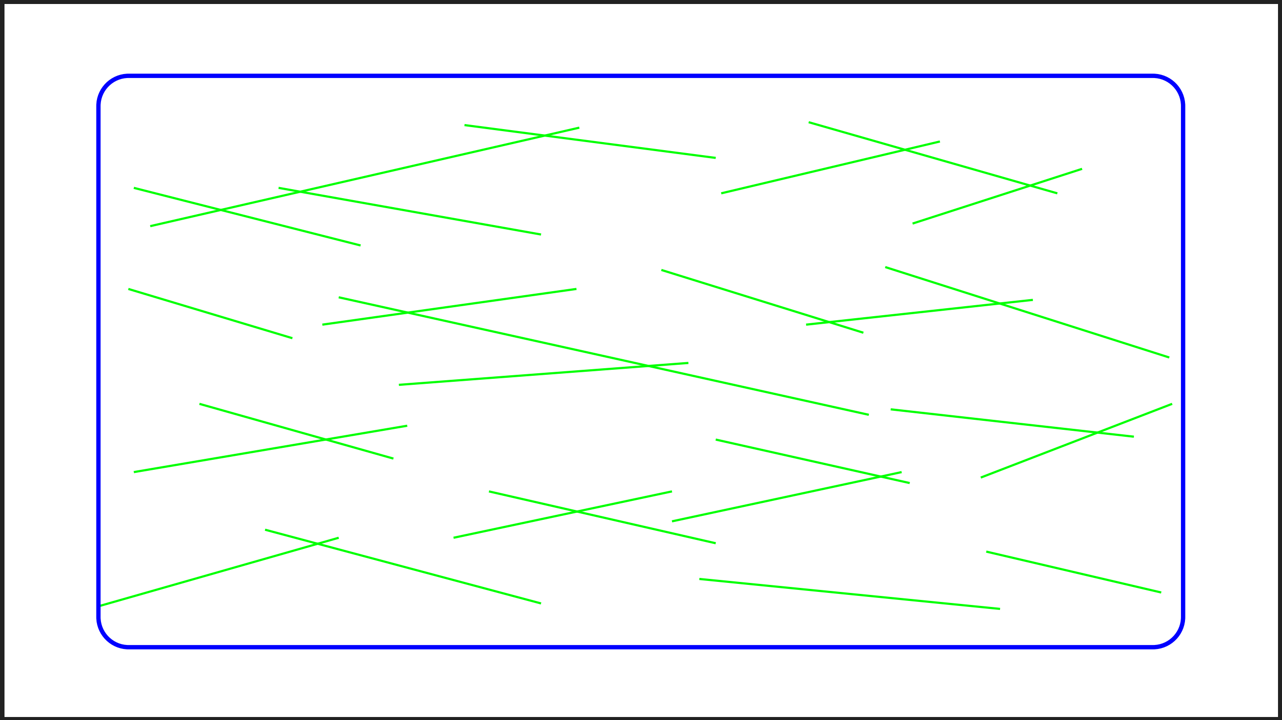
It gives the stress on each direction (principal directions), two or three

Stresses (or tensions) are forces per unit of length or surface

The direction of the force depends on the stresses and the surface

ACTIVE STRESS (CYTOSKELETON)

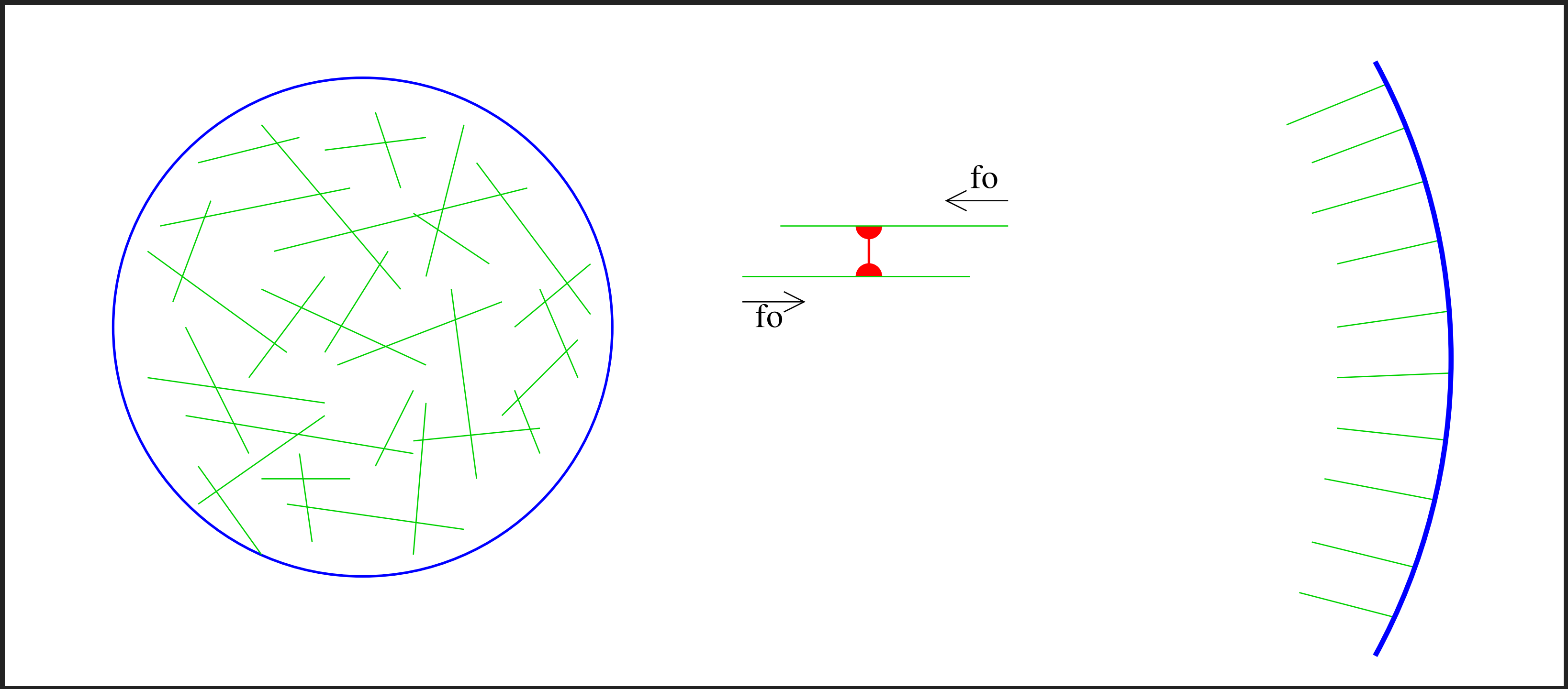
Polarized cytoskeleton



$$\sigma_x > \sigma_y$$

$$\sigma^a = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$$

ACTIVE STRESS (CYTOSKELETON)



$$F = N f_o$$

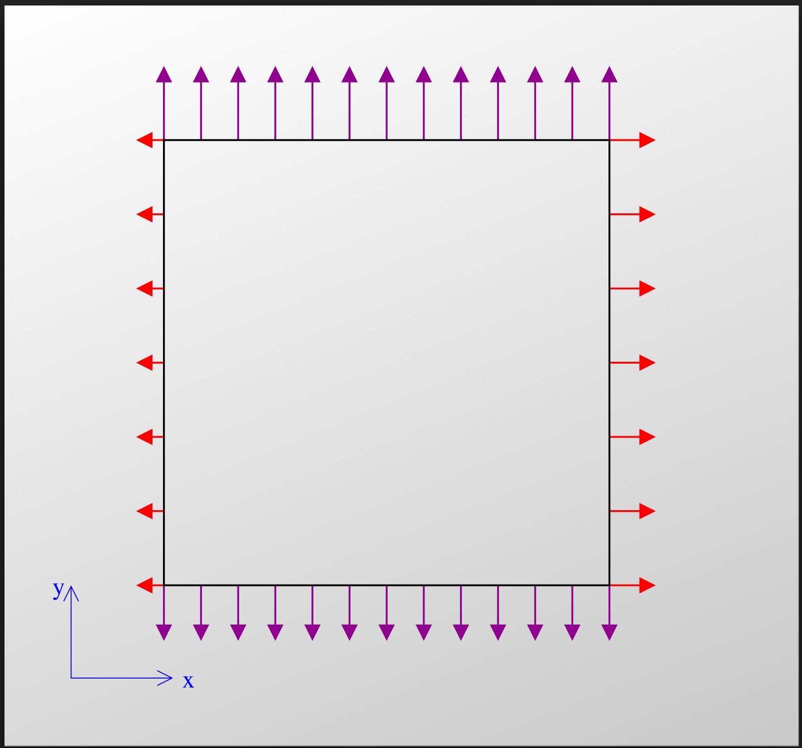
$$= \left(\frac{N f_o}{L_y} \right) L_y$$

$$= \sigma_a L_y$$

$$\sigma_a = n_{\text{filam}} f_o$$

Active stress

EQUILIBRIUM CONDITION



$$\vec{F}_{\text{total}} = [\sigma_x(x + L_x) - \sigma_x(x)]\, L_y \hat{x} + [\sigma_y(y + L_y) - \sigma_y(y)]\, L_x \hat{y}$$

Force equilibrium, F=0:

$\sigma_x \approx \text{cte.}$

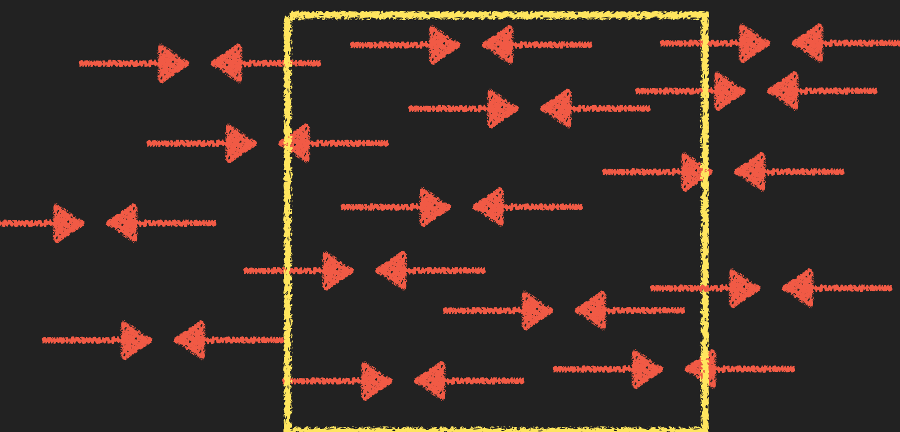
$\sigma_y \approx \text{cte.}$

But, different between directions

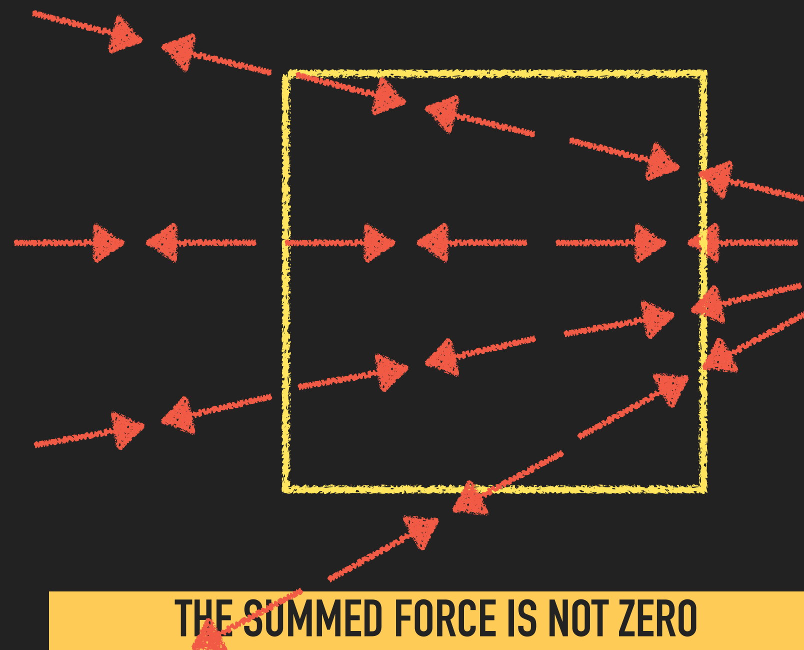
$$\sigma = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$$

IN EQUILIBRIUM, FORCES VANISH, AND STRESSES ARE UNIFORM
CAN BE ANISOTROPIC $\sigma_x \neq \sigma_y$

STRESS TENSOR



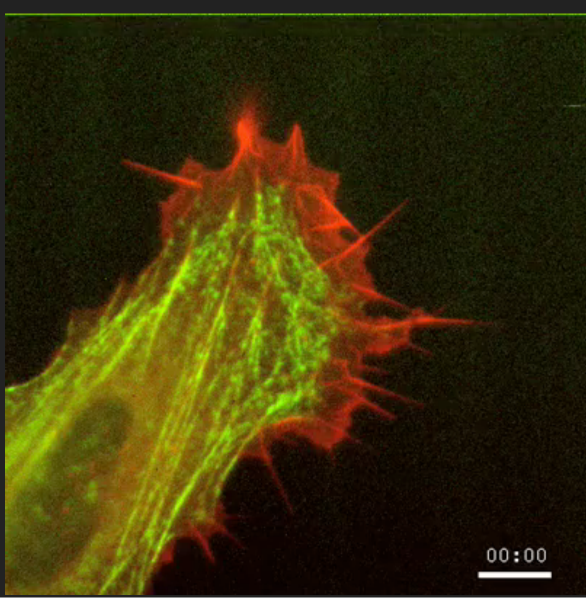
THE SUMMED FORCE VANISHES



THE SUMMED FORCE IS NOT ZERO

A NET FORCE APPEARS BECAUSE THE FILAMENT CONCENTRATION IS NOT HOMOGENEOUS

THIS FORCE MUST BE BALANCED WITH ANOTHER FORCE (E.G. FRICTION, THEN MOTION)



$$f = \nabla \cdot \sigma_A \quad \text{force density}$$

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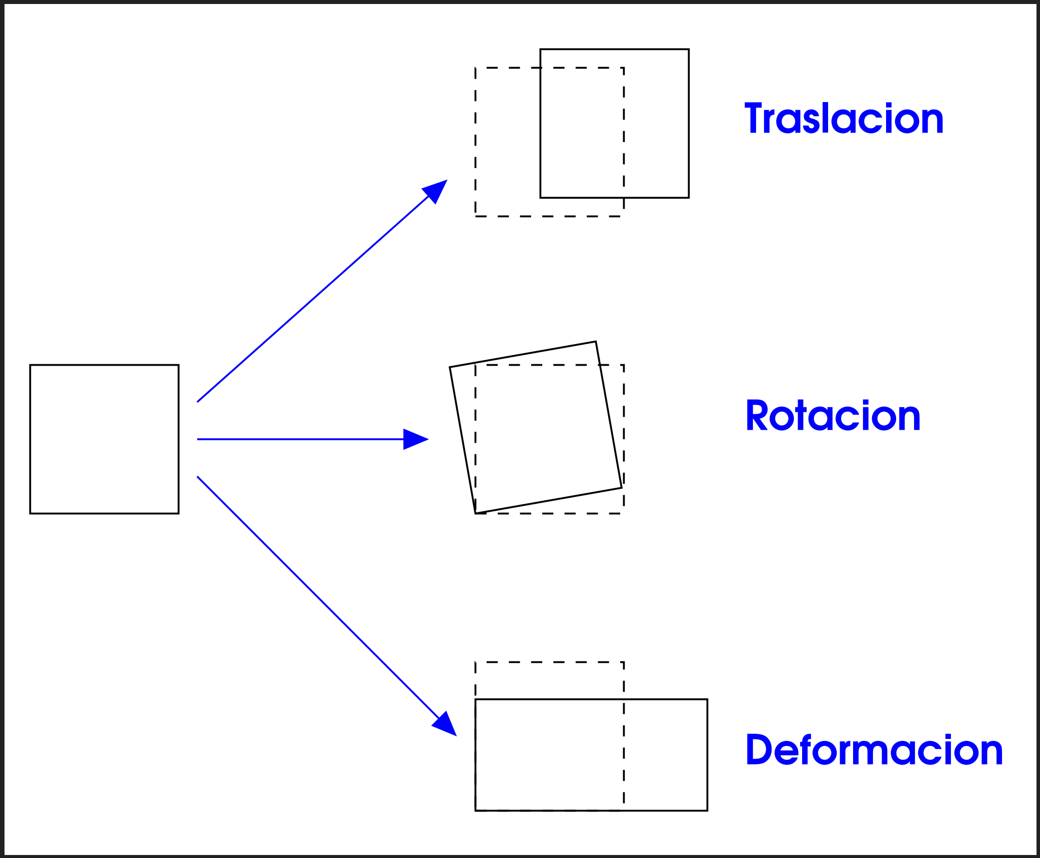
DEFORMATION (STRAIN) TENSOR



Only deformations have energy cost

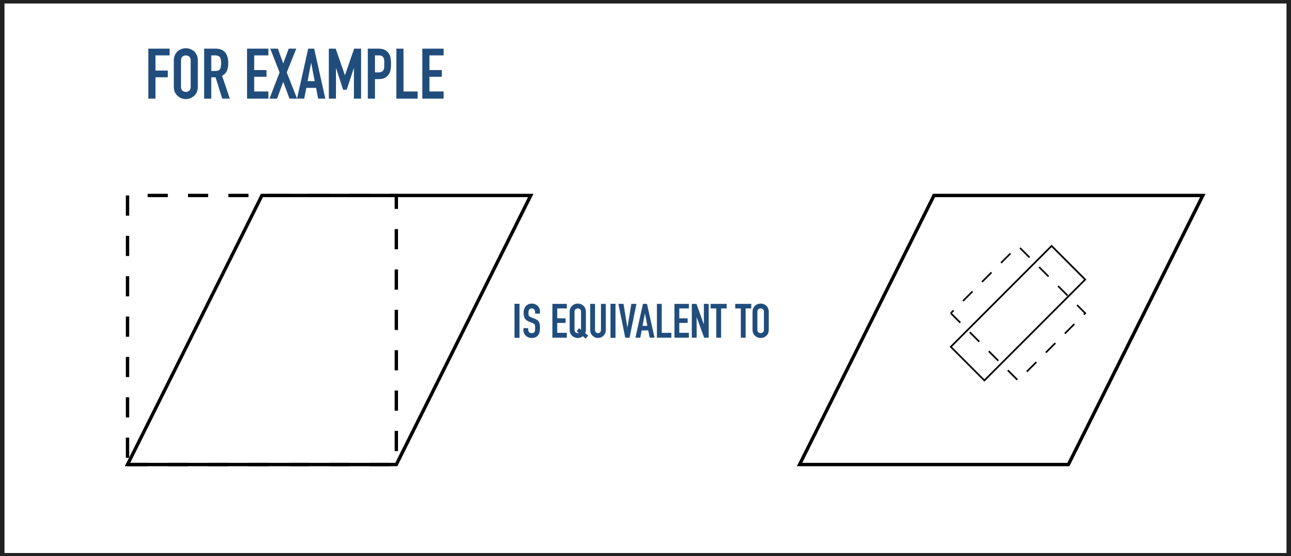
That is, require forces

DEFORMATION (STRAIN) TENSOR

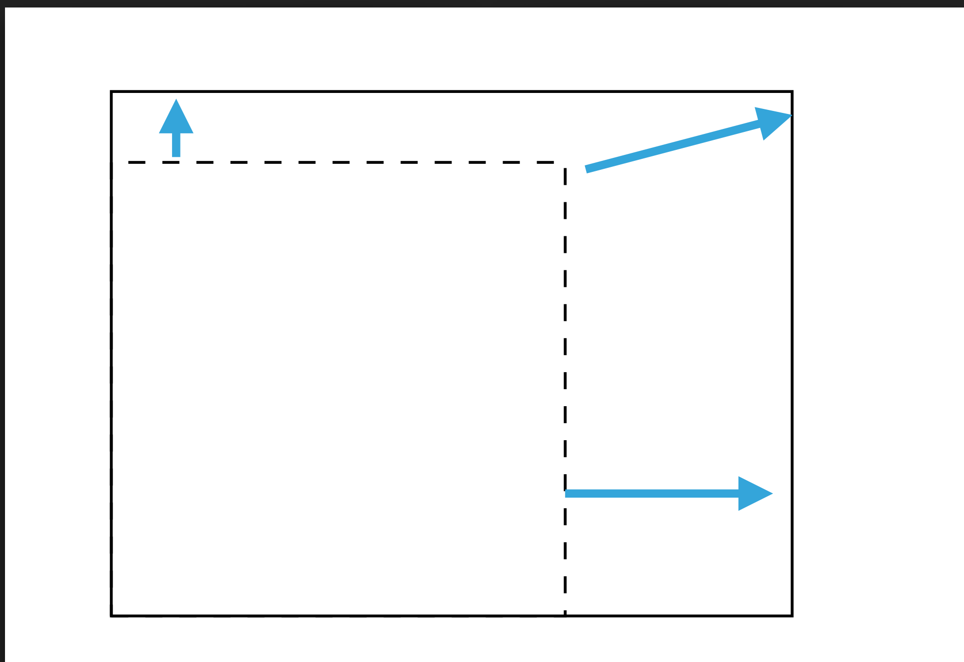
Deformations can be described in terms of principal axis (directions)

They are identified as the directions for which a rectangle deforms into a rectangle

These directions are perpendicular



DEFORMATION (STRAIN) TENSOR



$$\varepsilon_x = \frac{\Delta x}{L_x}$$

$$\varepsilon_y = \frac{\Delta y}{L_y}$$

$\varepsilon > 0$ stretched

$\varepsilon < 0$ compressed

$$\varepsilon = \begin{pmatrix} \varepsilon_x & 0 \\ 0 & \varepsilon_y \end{pmatrix}$$

Deformation matrix:

Strain tensor

STRAIN - STRESS RELATION

1) Elastic materials

$$\sigma \propto \varepsilon$$

$$\sigma = Y\varepsilon$$

Y Young modulus

Interpretation of the Young modulus

If $\sigma = 1\text{Y}$, then $\varepsilon = 1$

$$\varepsilon = 1 = \frac{\Delta x}{L_x}$$

Deformation of 100%

Typical values: 1 kPa … 1 GPa

2) Fluid materials

$$\sigma \propto \dot{\varepsilon}$$

$$\sigma = \eta\dot{\varepsilon}$$

η Viscosity

STRAIN – STRESS RELATION

3) Visco-elastic materials

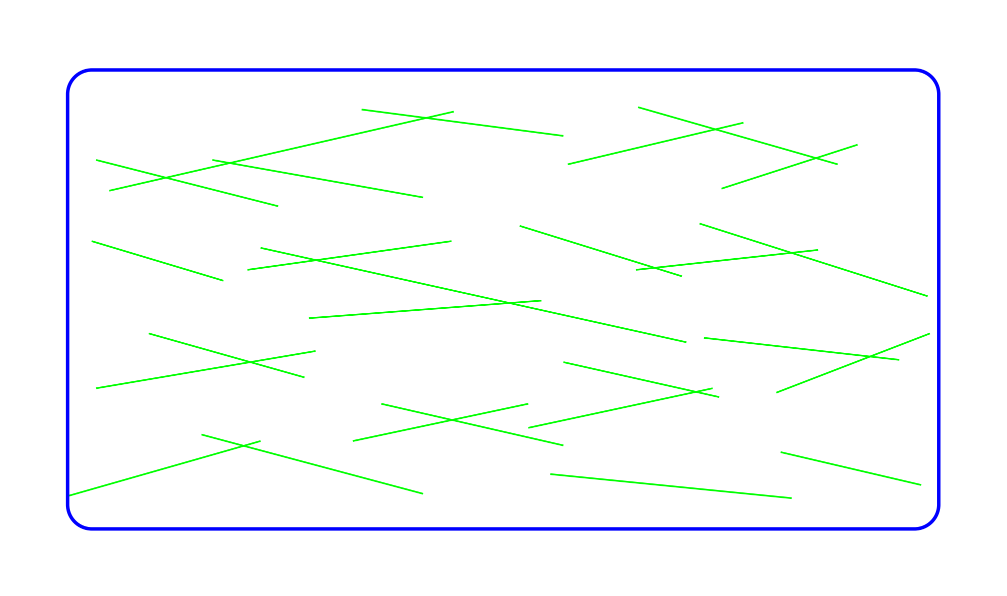
$$\sigma = Y\varepsilon + \eta\dot{\varepsilon}$$

4) Active materials (living materials)

The stress tensor depends on the polarization axes and the intensity of the activity

EXAMPLE: CELLULAR DEFORMATION

$$\sigma = \sigma_E + \sigma_A$$
$$= Y \begin{pmatrix} \varepsilon_x & 0 \\ 0 & \varepsilon_y \end{pmatrix} + \begin{pmatrix} \sigma_a & 0 \\ 0 & 0 \end{pmatrix}$$



With free boundaries

In the borders, $\sigma = 0$

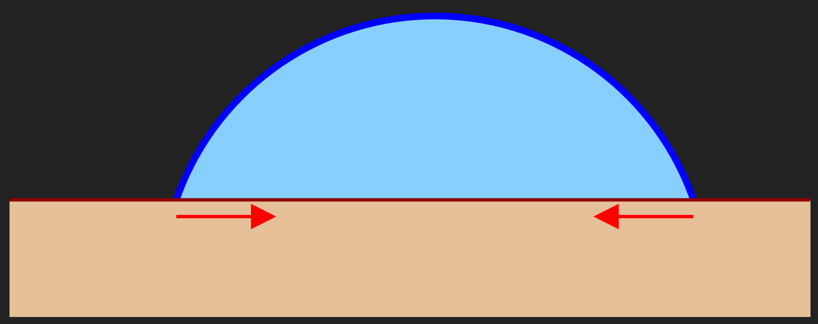
But, the mechanical equilibrium dictates that $\sigma = \mathrm{cte}.$

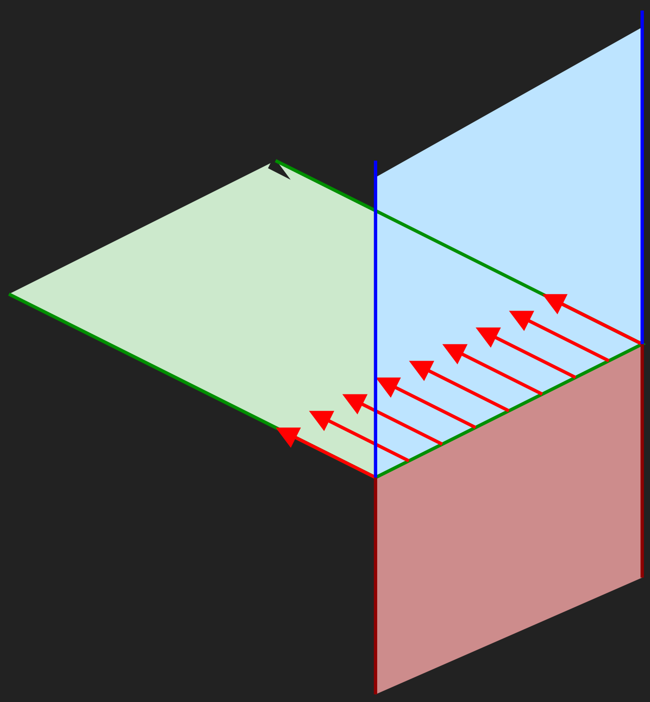
Then, in all the cell, $\sigma = 0$

$$\boxed{\varepsilon_x = -\sigma_a / Y, \quad \varepsilon_y = 0}$$

The cell contracts

EXAMPLE: CELL TRACTION

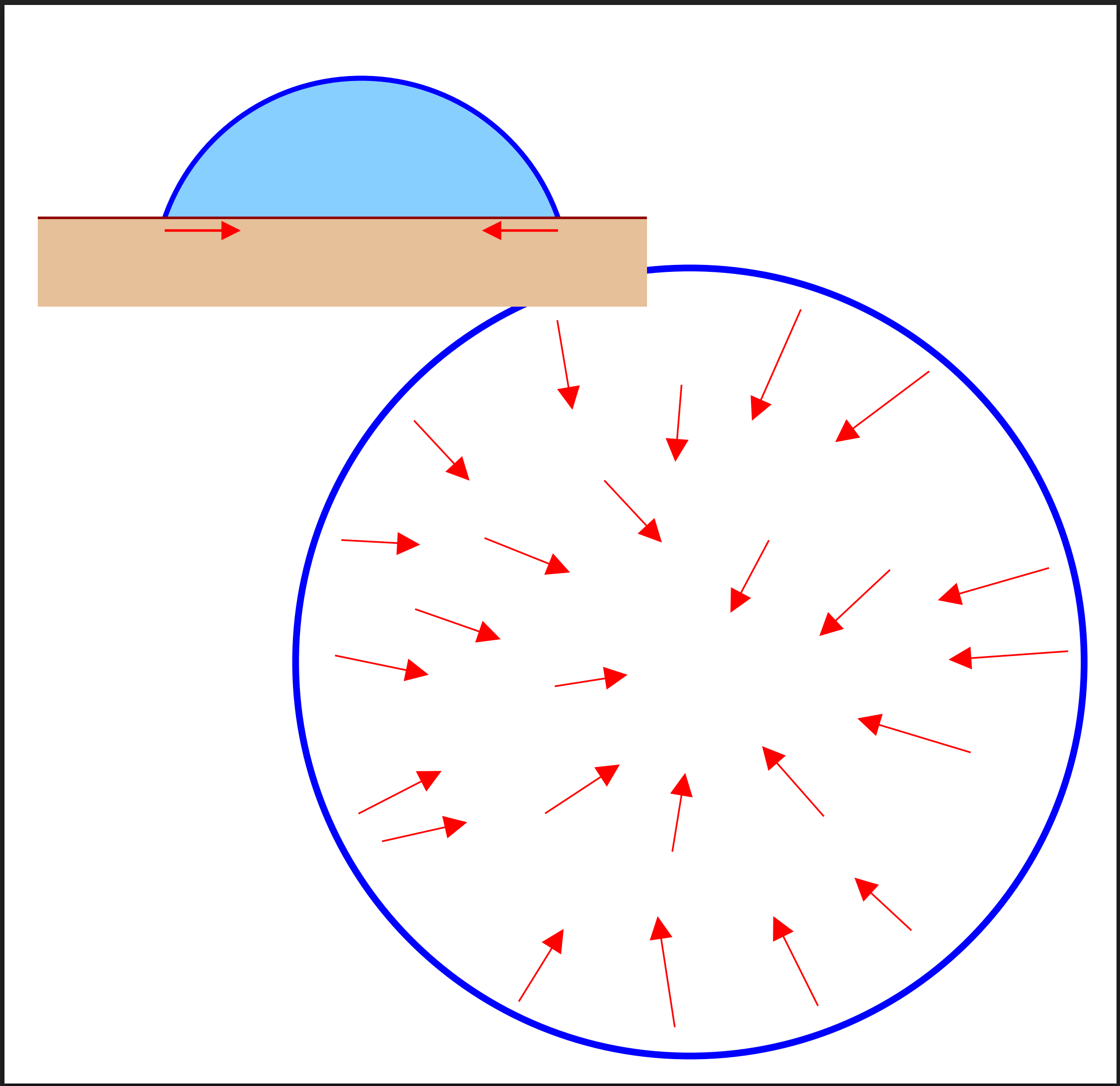




The substrate is elastic. Then, it deforms

$$\varepsilon_r = -\sigma_a / Y$$

TRACTION FORCE MICROSCOPY



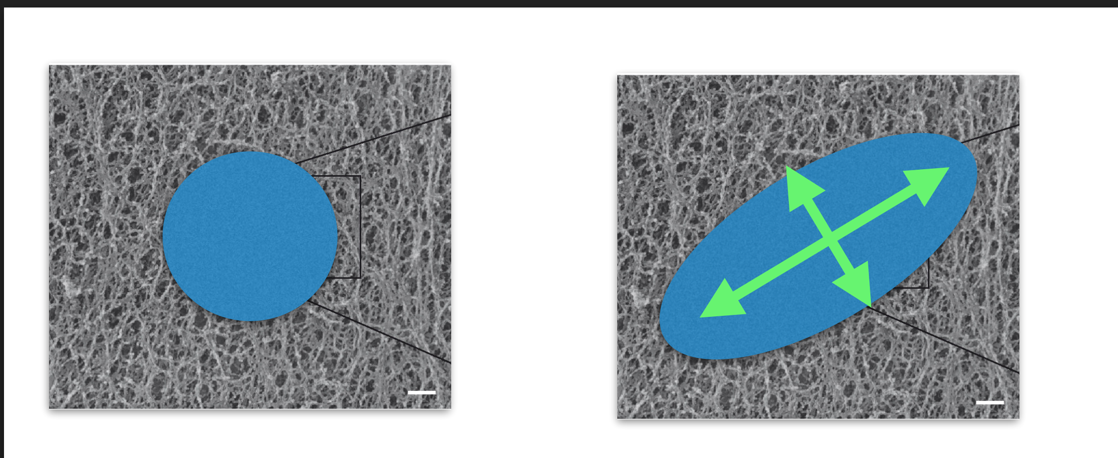
Place fluorescent beads in the substrate

Measure their displacement $\vec{u}(\vec{r})$

Deduce the deformation tensor of the substrate ε

Using $\varepsilon = \sigma_a / Y_{\text{substrate}}$, compute the applied stresses σ_a

DEFORMABLE MICRO-DROPLETS



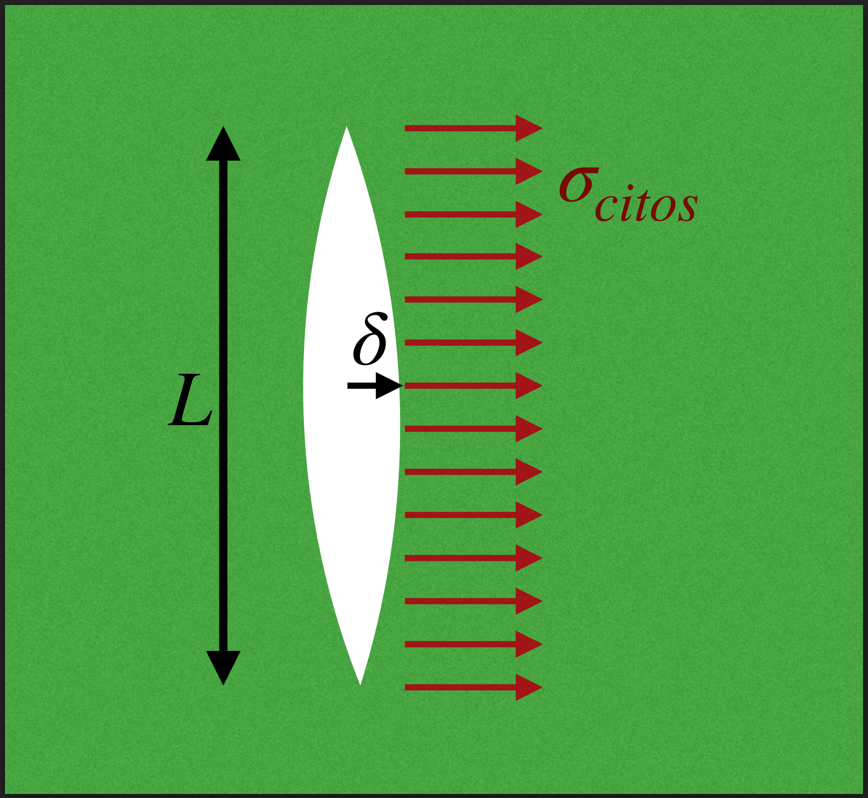
Spherical droplets are immersed in the cell

The stresses deform the beads and adopt an ellipsoidal shape

The new lengths give principal axes and the strains of the droplet $\sigma_{droplet} = \epsilon / Y_{droplet}$

The stresses must be equal across the interface $\sigma_{citos} = \sigma_{droplet}$

LASER ABLATION



The cytoskeleton is generating an active stress σ_{citos}

When the membrane is cut, a force appears in the new free surface.

To reach the new equilibrium (net zero force), the tissue contracts, generating an elastic force. The deformation is $\epsilon = \sigma_{citos}/Y$

The retraction length is proportional to the cut length L, then

$$\delta \propto \epsilon L = \sigma_{citos} L/Y$$

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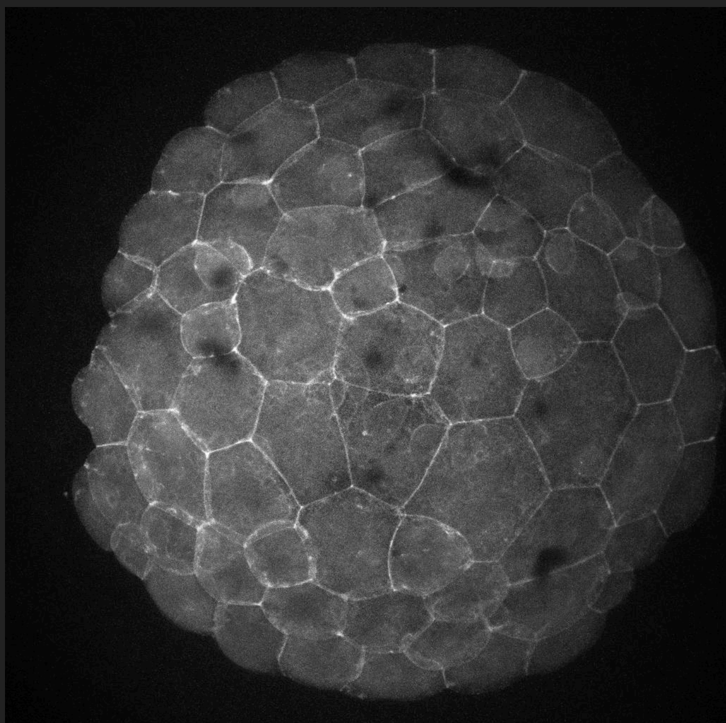

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