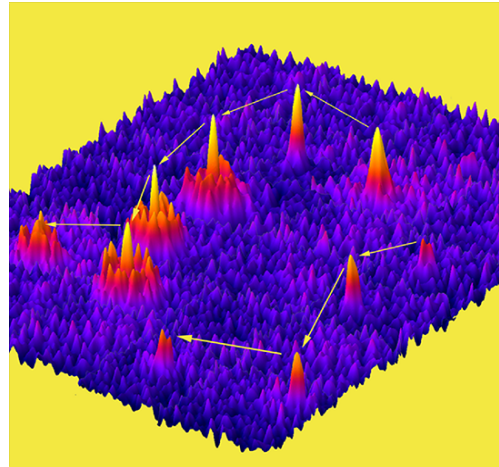


## Course „Optics, Forces & Development“



### Principles of Optics I

Ulrich Kubitscheck

Clausius-Institute of Physical and Theoretical Chemistry  
Rheinische Friedrich-Wilhelms-Universität Bonn

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7. Image detection by cameras and pixel size
8. Objectives
9. Fluorescence
10. Fluorescence microscopy
11. Confocal Microscope
12. Nipkow disk confocal microscope
13. 2 photon microscopy

high resolution microscopy: STED, STORM

light sheet microscopy

techniques: FRET, FRAP and force measurements by light

# I. Additional Information

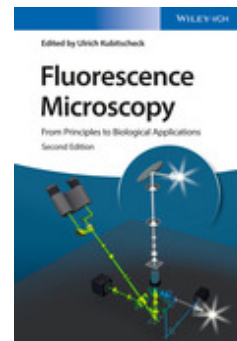
## Good Collection of Online Learning Tools

<http://micro.magnet.fsu.edu/primer/>

<https://zeiss-campus.magnet.fsu.edu/>

## Books

Fluorescence Microscopy, 2017, 2nd edition,  
ed. U. Kubitscheck, Wiley-VCH

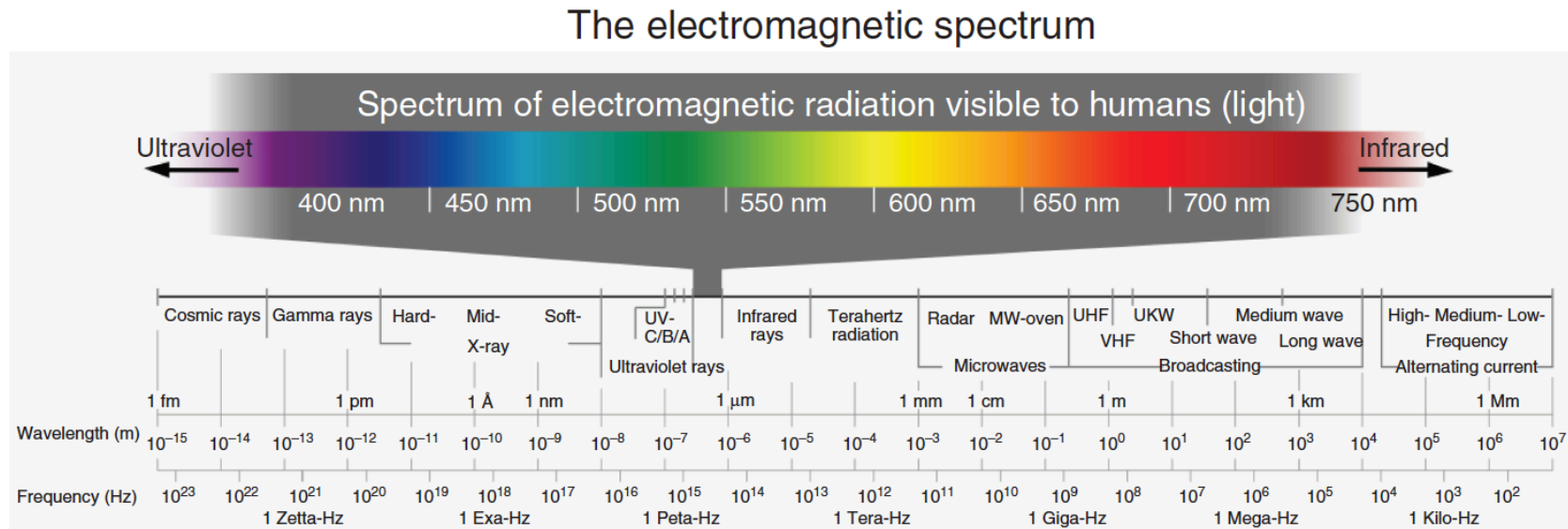


Digital Microscopy, Meth Cell Biology, 2007  
ed. G. Sluder and D.E. Wolf



2. Basics: waves  
diffraction  
lenses  
aberrations

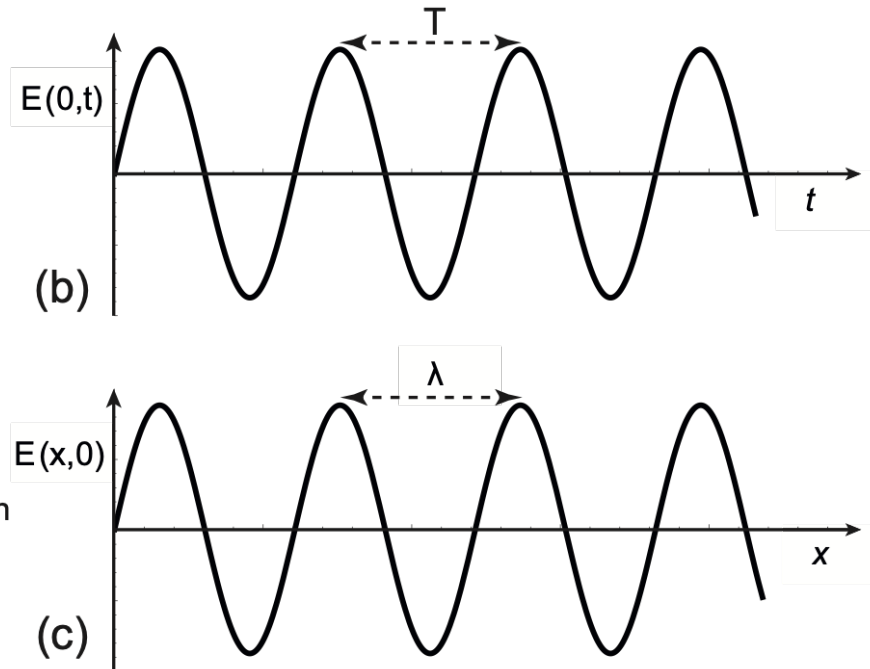
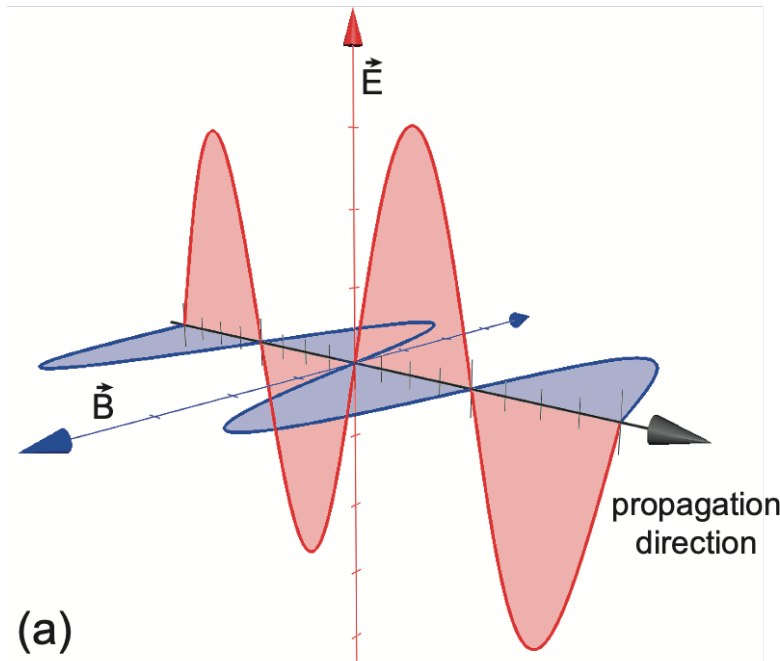
# The electromagnetic spectrum



Different types of radiation are essentially electromagnetic waves with oscillation frequencies or vacuum wavelengths ranging over many orders of magnitude.

English version of a graphic by Horst Frank ([https://de.wikipedia.org/wiki/Elektromagnetisches\\_Spektrum](https://de.wikipedia.org/wiki/Elektromagnetisches_Spektrum), [https://en.wikipedia.org/wiki/GNU\\_Free\\_Documentation\\_License](https://en.wikipedia.org/wiki/GNU_Free_Documentation_License)).

# Electromagnetic waves



- Sketch of a linearly polarized electromagnetic wave
- (a) Wave with electric and magnetic field components, E and B
  - (b) Temporal oscillation at a fixed place in space.
  - (c) Still image of the wave.

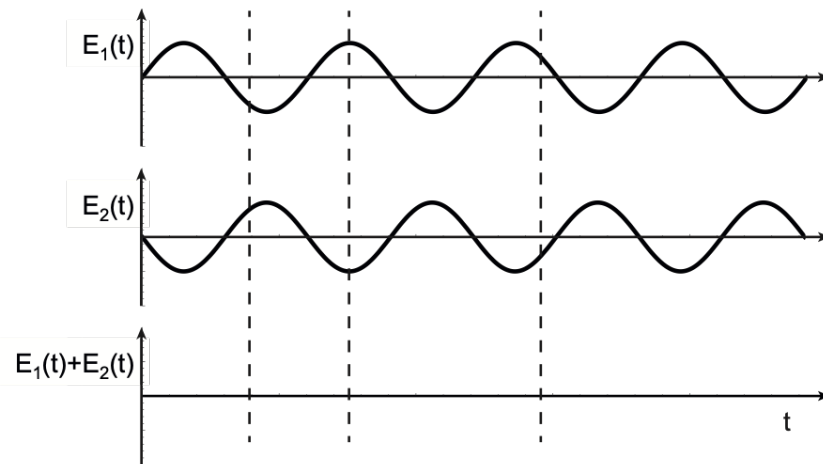
$$\lambda_{vac} \nu = c_{vac}$$

$$c_{vac} = 299.792.458 \text{ m / s}$$

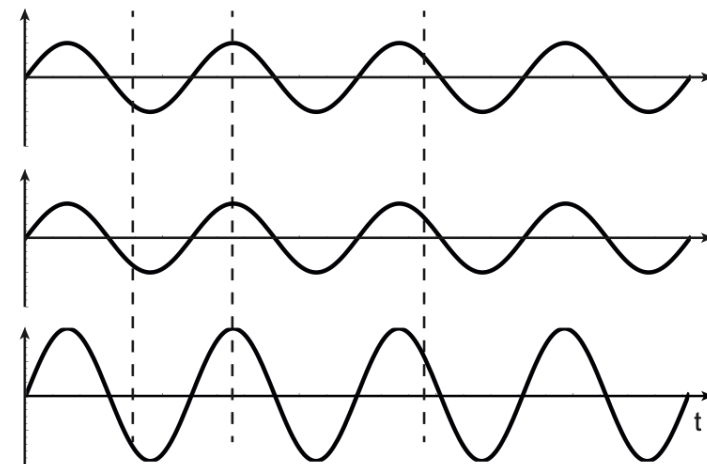
$$= 299.792,458 \text{ km / s}$$

# Interference of waves

destructive interference

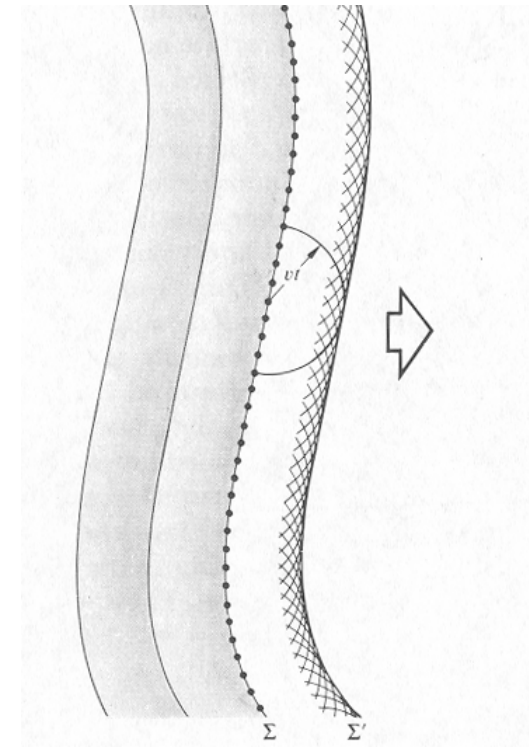
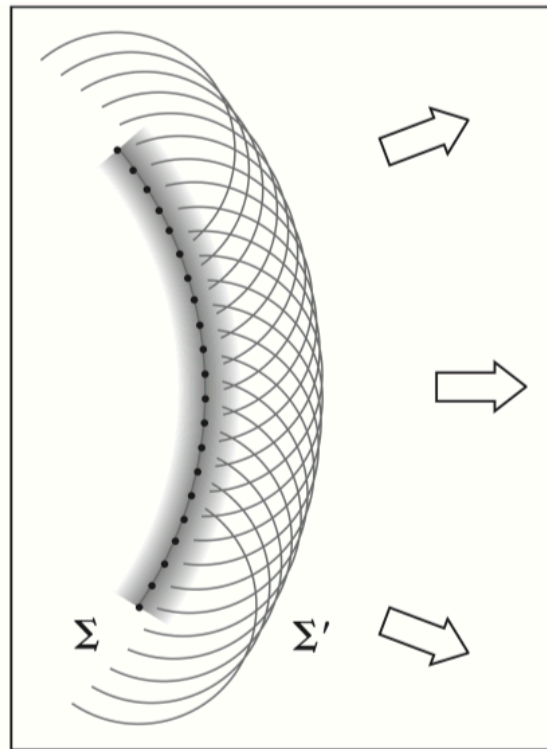
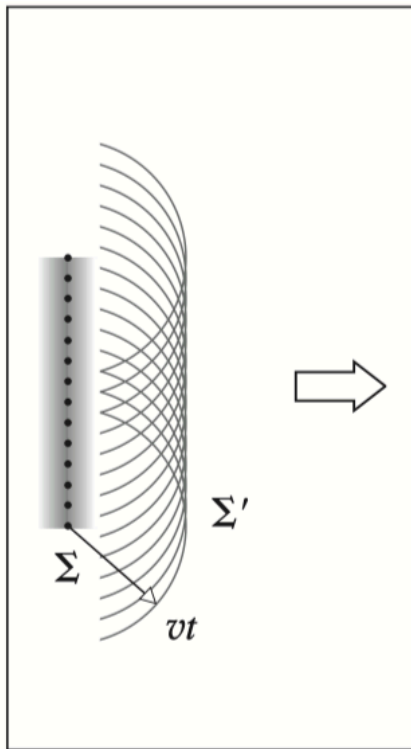


constructive interference



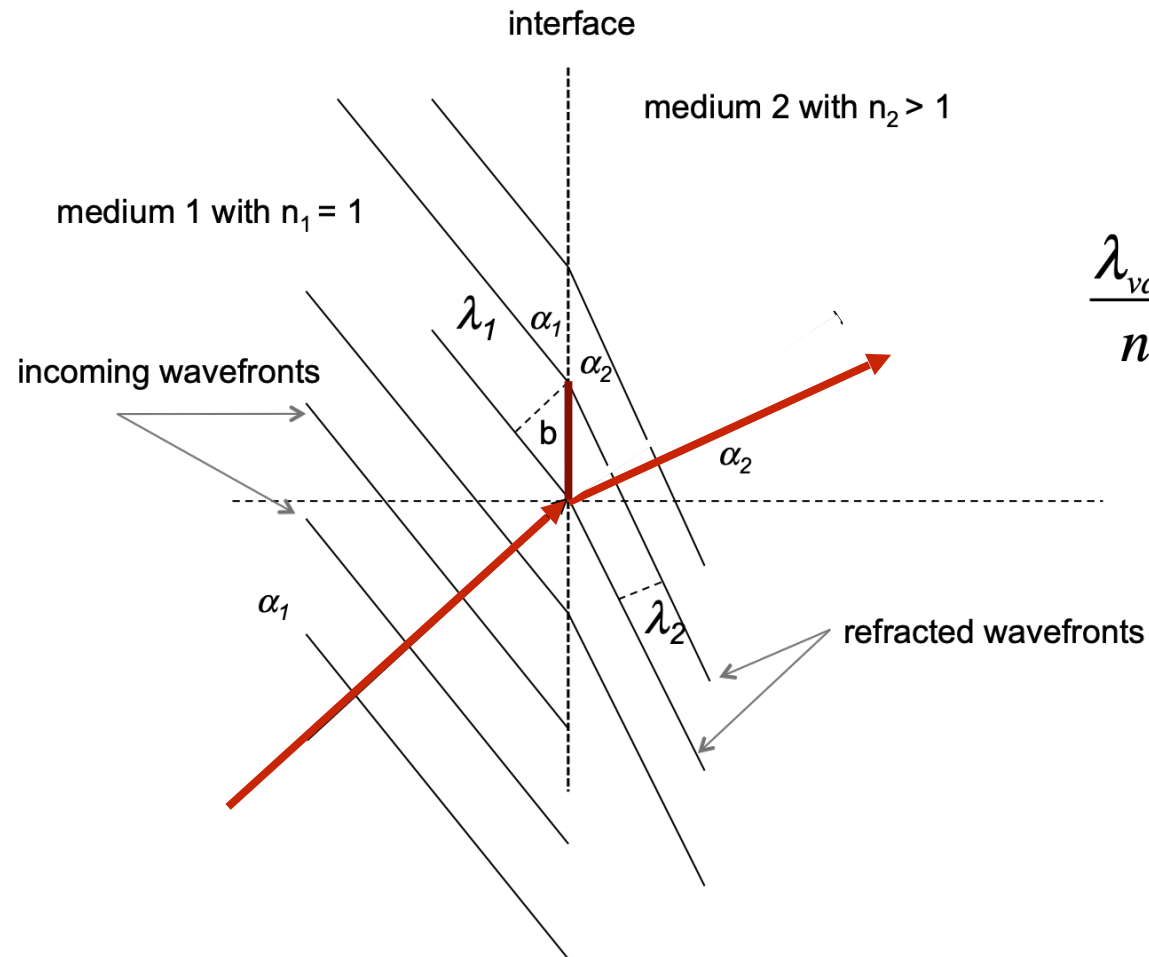
# The concept of Huygen's elementary waves

„Each point of the wave front can be understood as the origin of a new elementary wave that propagates with the speed and frequency of the original wave“





# Snell's law of refraction



$$\frac{\lambda_{vac}}{n} v = \frac{c_{vac}}{n}$$

The phases of the electric field along the interface between the two materials must be identical.

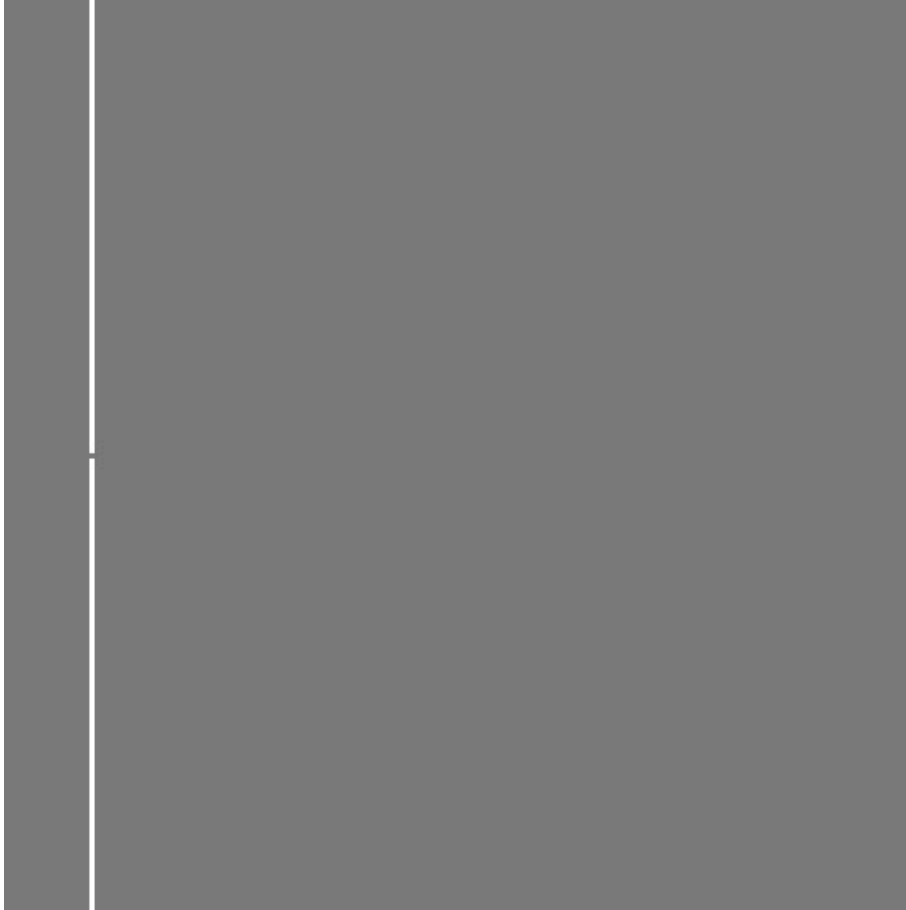
The wavelengths inside the materials are given by  $\lambda_{vacuum}/n_1$  and  $\lambda_{vacuum}/n_2$ .

We note that  $\sin \alpha_1 = \lambda_1/b$ , where  $b$  denotes the distance between two wave crests at the interface, and also that  $\sin \alpha_2 = \lambda_2/b$ .

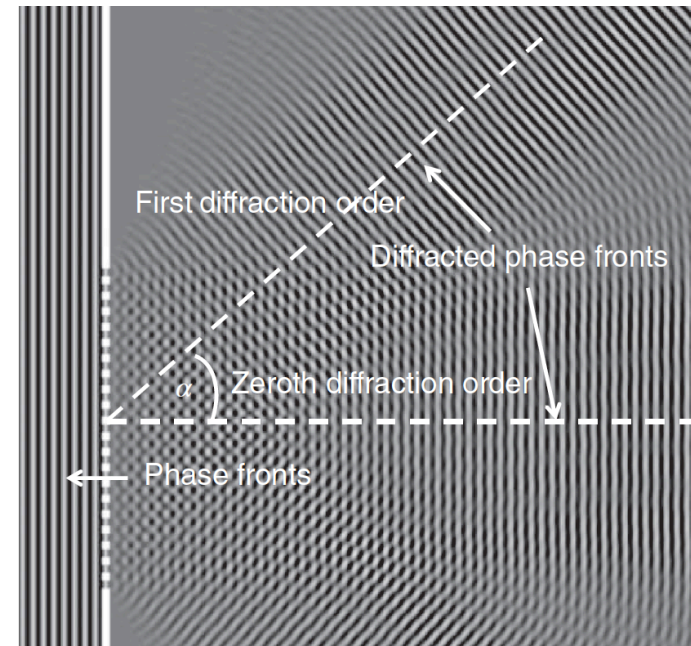
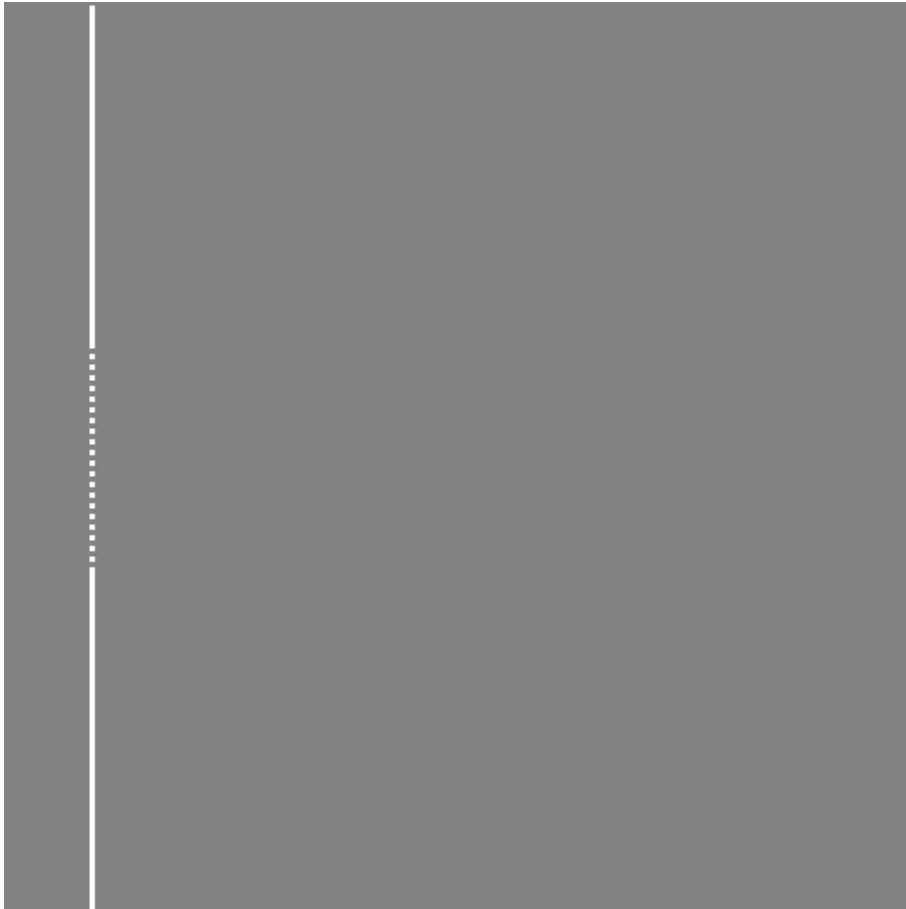
Eliminating  $b$  yields

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

## Diffraction at a pinhole → spherical wave

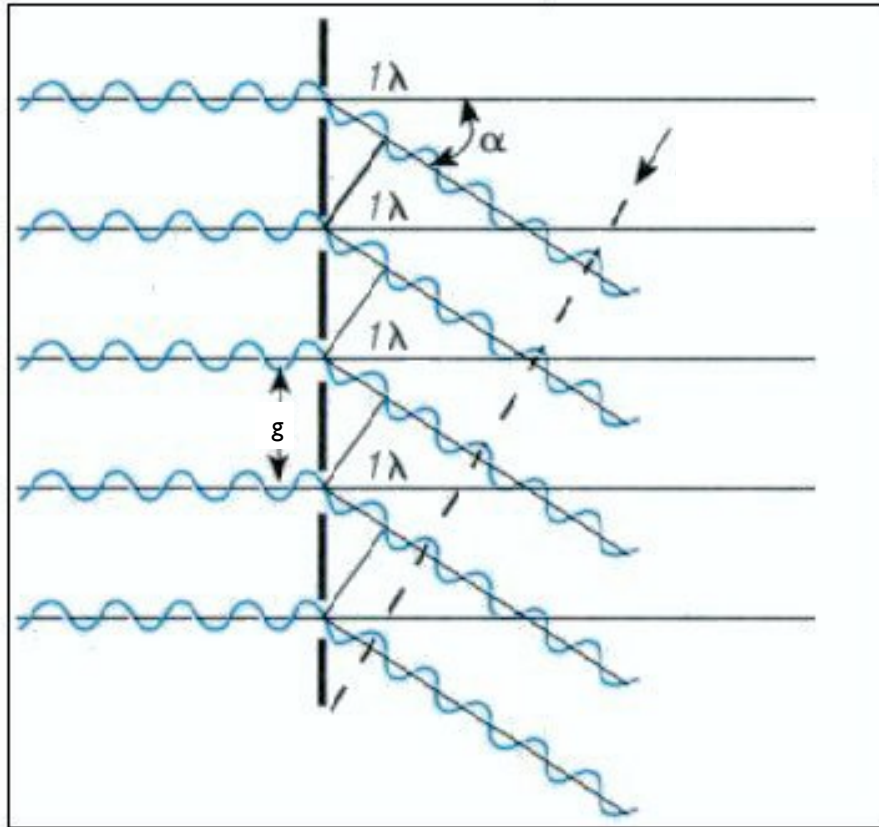


# Diffraction at a grating



A plane wave hits perpendicularly on a grating. The directions of constructive interference, in which maxima and minima of one wave interfere constructively with the maxima and minima of the second wave are shown for the zeroth- and first-order diffraction.

# Diffraction grating



The diffraction grating and spectrum on screen

$g$  grating constant,  $\lambda$  wave length,  $\alpha$  angle of deflection,

for main maxima we have

$$g \sin \alpha_n = n\lambda$$

with

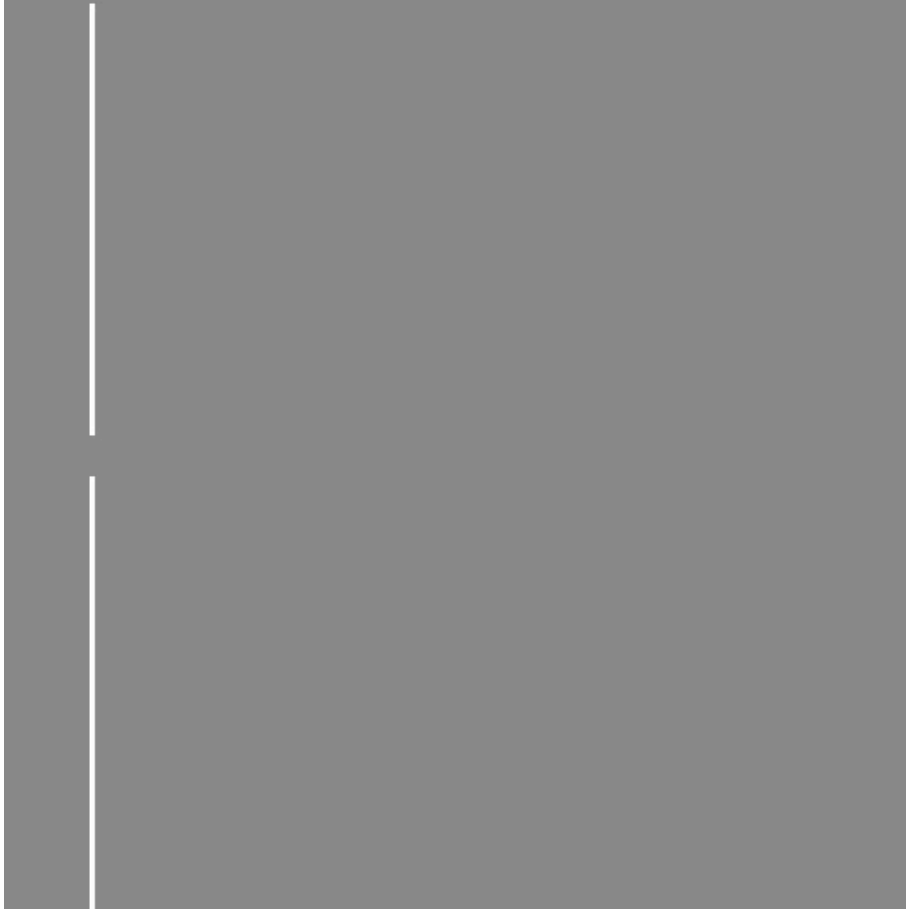
$g$ : grating constant

$n=1, 2, 3, \dots$ , order of maximum

$\alpha_n$ : diffraction angle of order  $n$

Source: <http://library.thinkquest.org/19662/low/eng/electron-wave-exp.html>

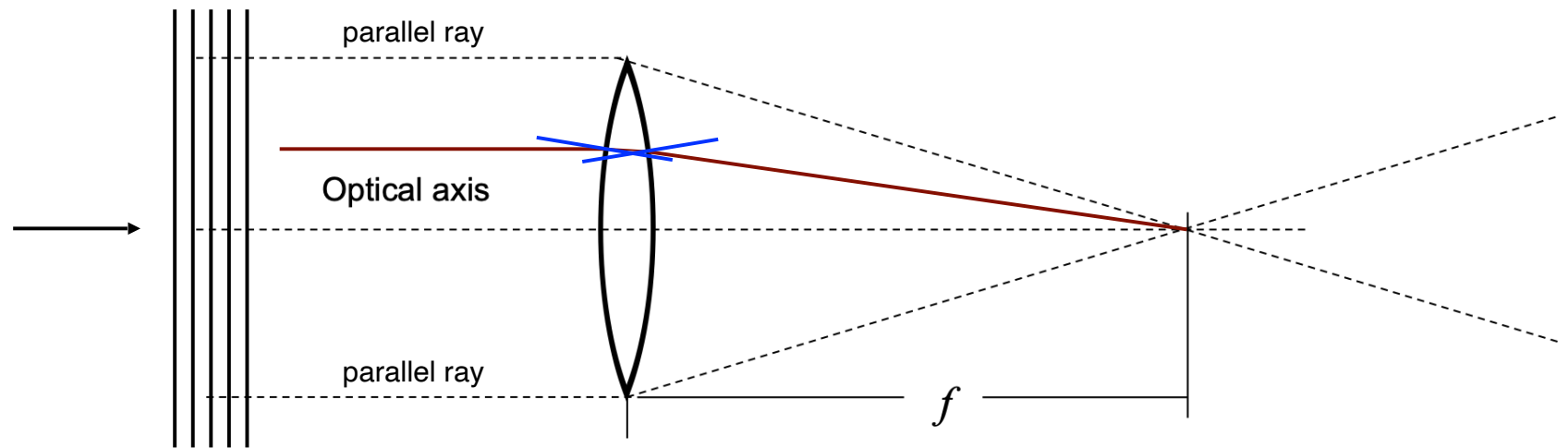
# Diffraction at an open pinhole



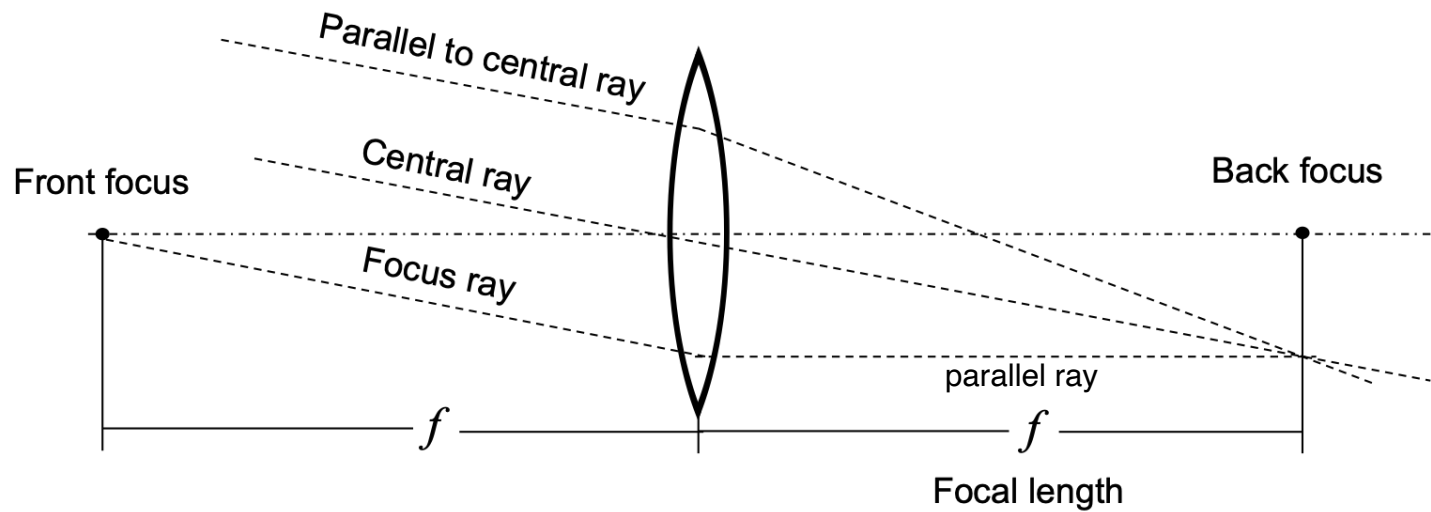
(a) A plane wave hits perpendicularly on a large pinhole.  
Again we find directions of constructive and destructive interference

From "Fluorescence Microscopy: From Principles to Biological Applications", edited by Ulrich Kubitscheck. Wiley-VCH, Weinheim, 2nd edition  
online supplemental material

# Lenses



# Special rays passing lenses



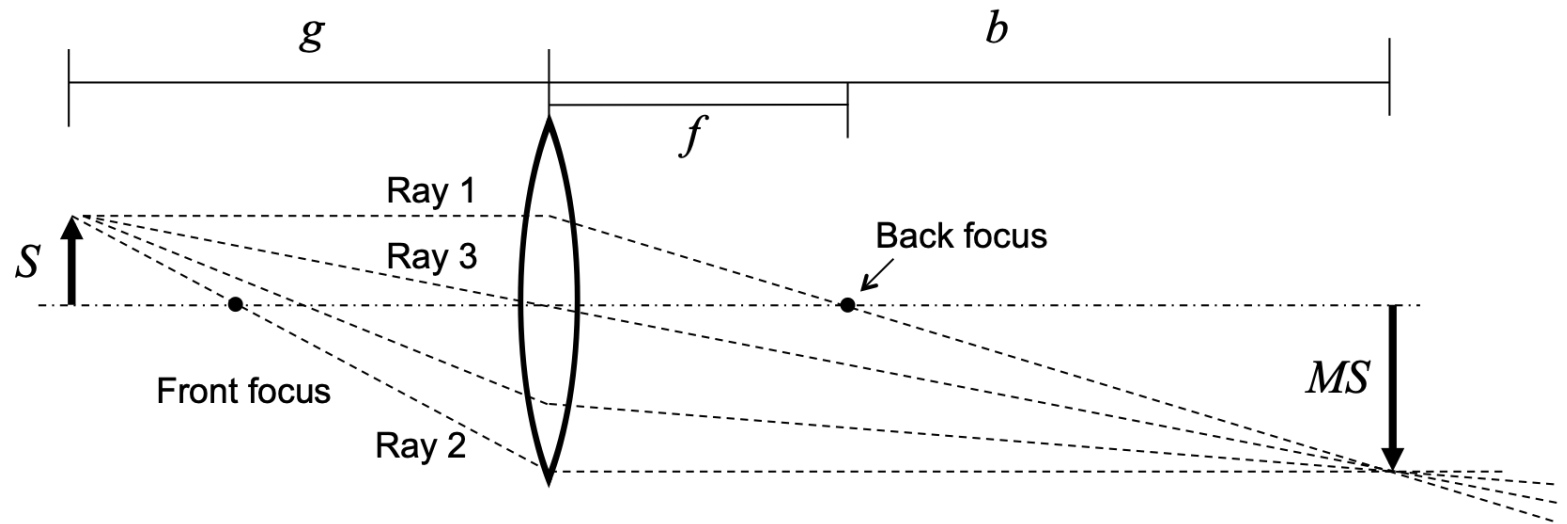
Nice Applet:

<http://www.walter-fendt.de/ph14d/bildsammellinse.htm>

Optical reversal: retrace rays and yield identical paths

From "Fluorescence Microscopy: From Principles to Biological Applications"

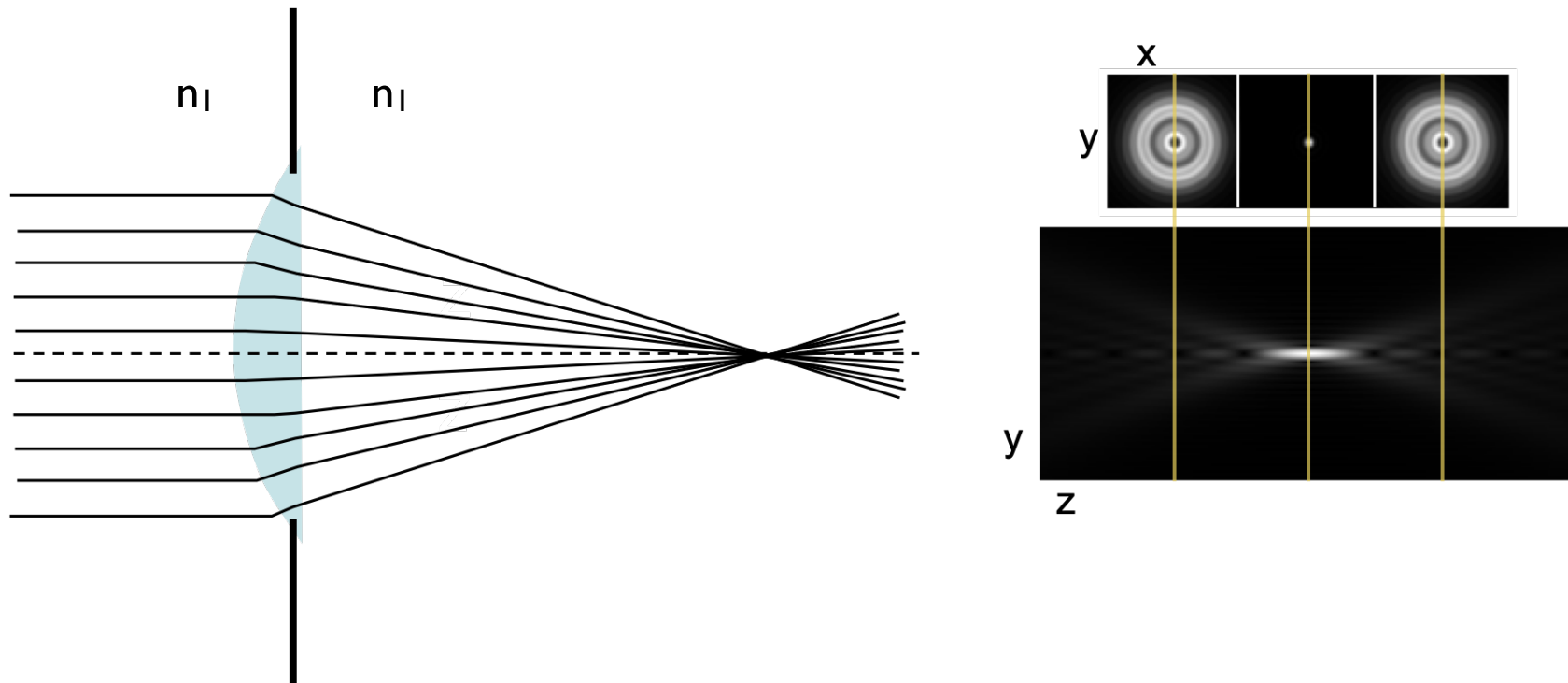
# Real images



A single lens imaging an object as an example for drawing optical ray diagrams

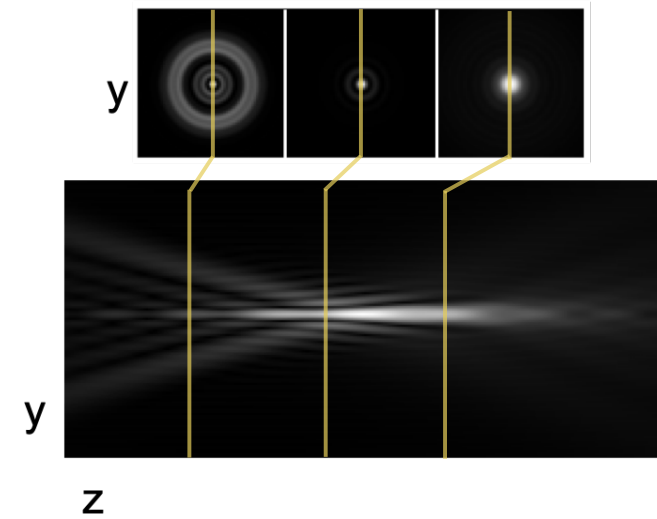
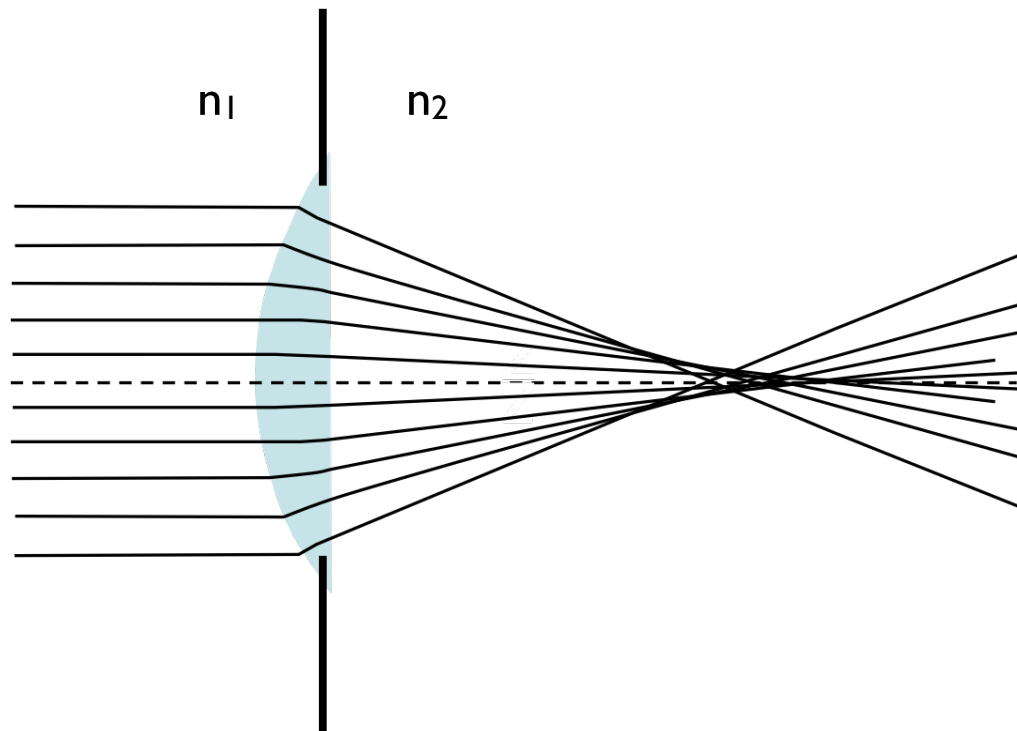


# Focusing of light: ray model & wave model



A plane wave - sketched by the parallel incoming rays - hits perpendicularly on a large pinhole. The lens focuses the diffracted waves into its focus. Again we find directions of constructive and destructive interference

# Focusing of light with spherical aberration



# Important aberrations in microscopy

spherical aberration

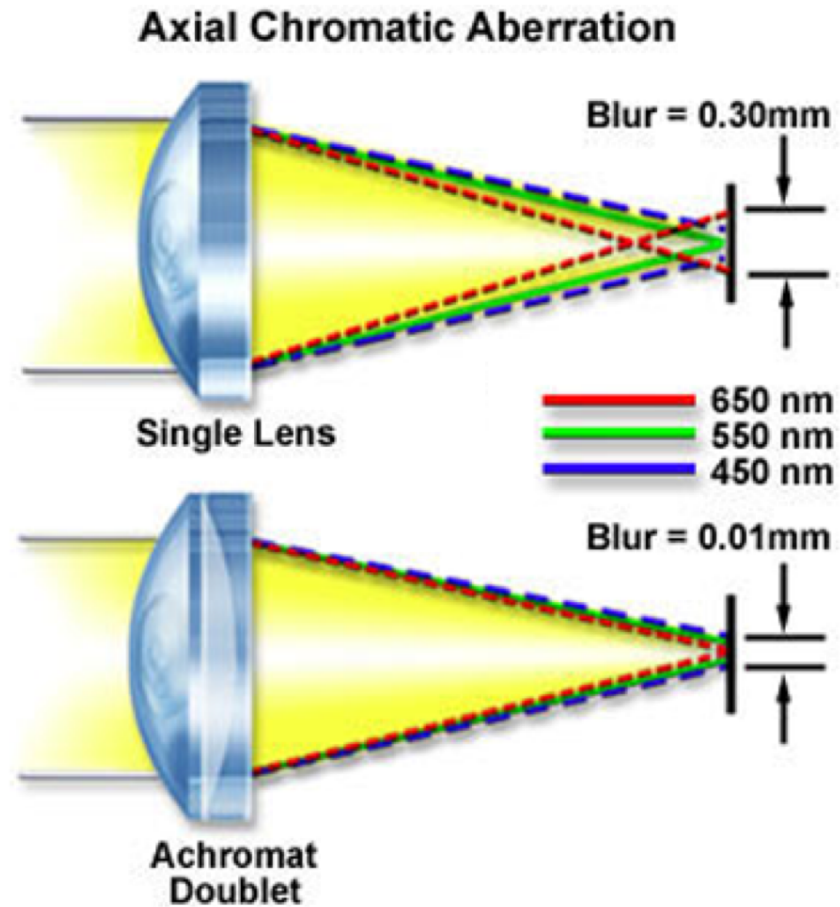
chromatic aberrations

curvature of field

coma

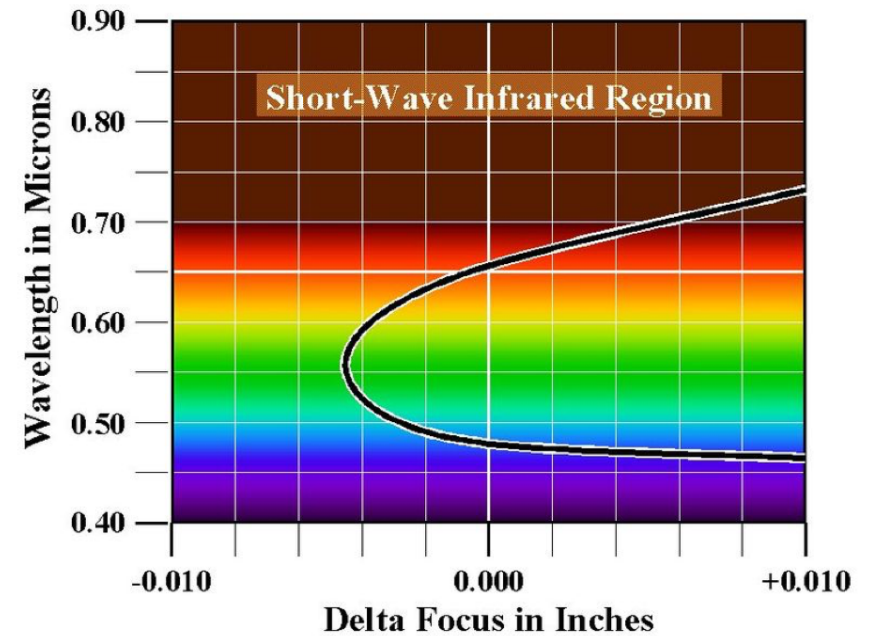
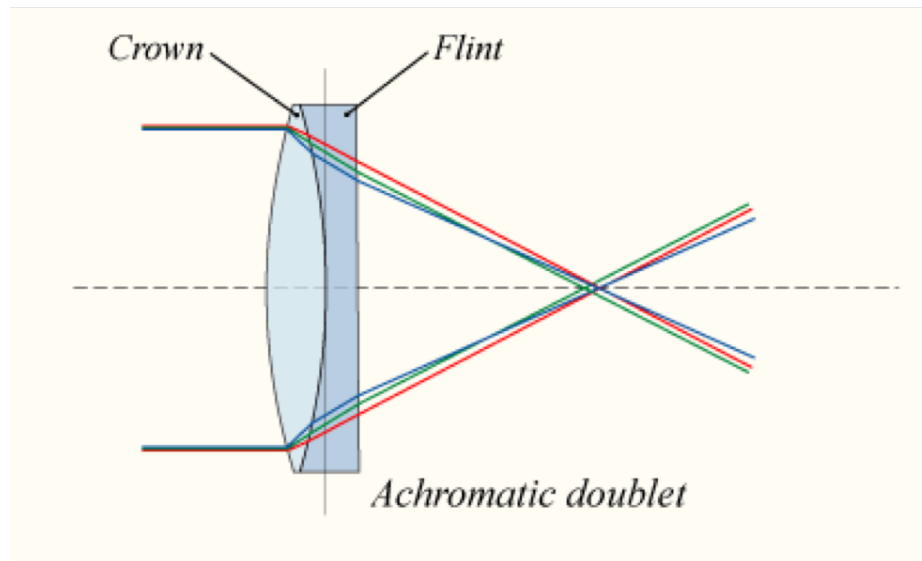
astigmatism

# Chromatic aberrations



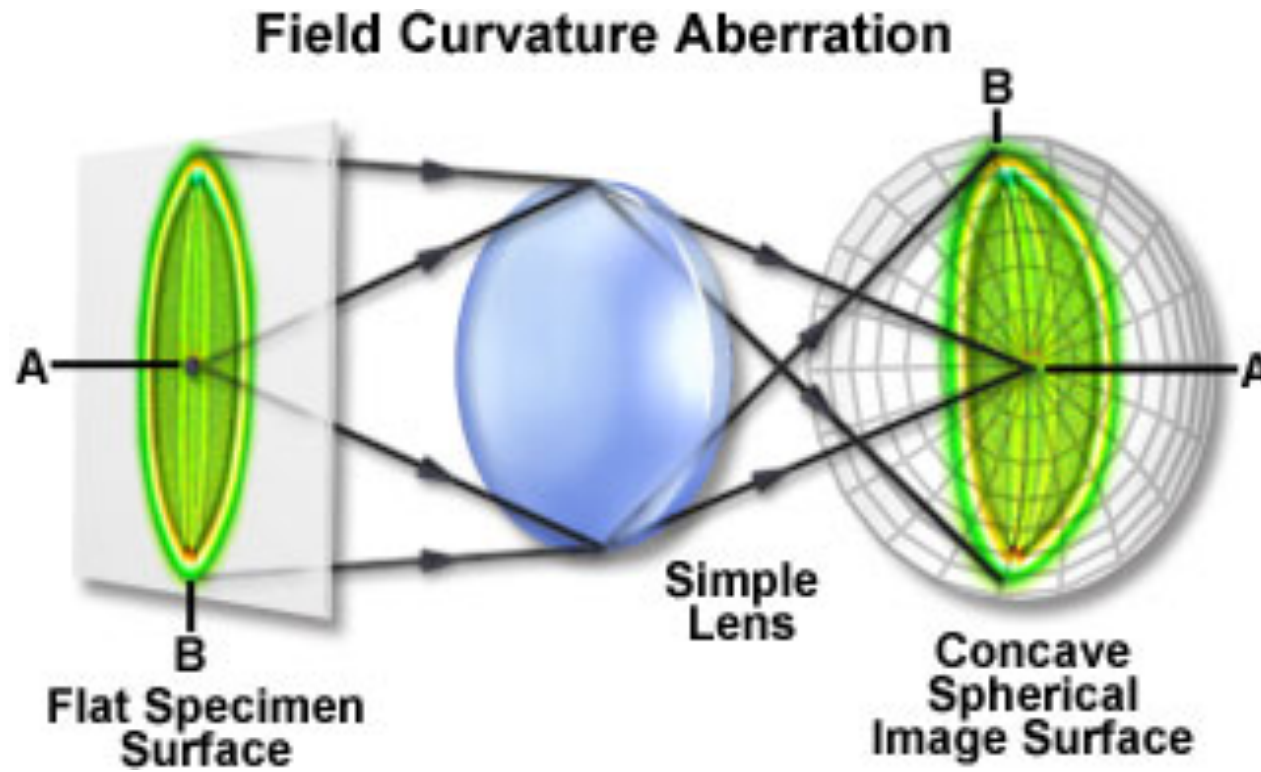
Refraction is wavelength-dependent: each color has its own focus and magnification

# Correction of chromatic aberrations



An achromatic doublet brings two wavelengths to a common focus, leaving ultraviolet and infrared uncorrected and out of focus

# Curvature of field



The image is actually located on the surface of a sphere, hence the image of a flat object is curved with regard to the optical axis

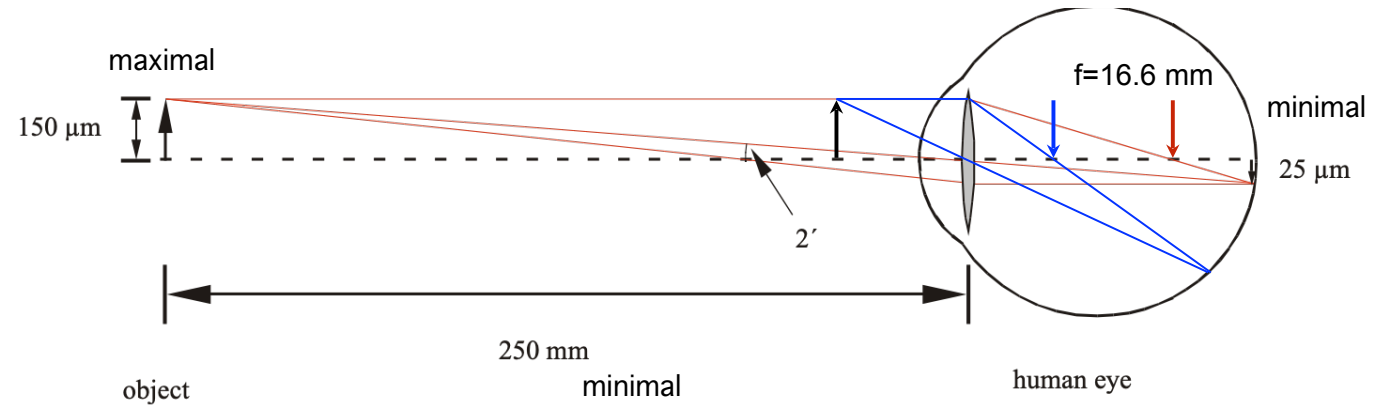
Source: microscopy primer, <https://micro.magnet.fsu.edu/primer/>

## 3. Microscope

# Why use a microscope?

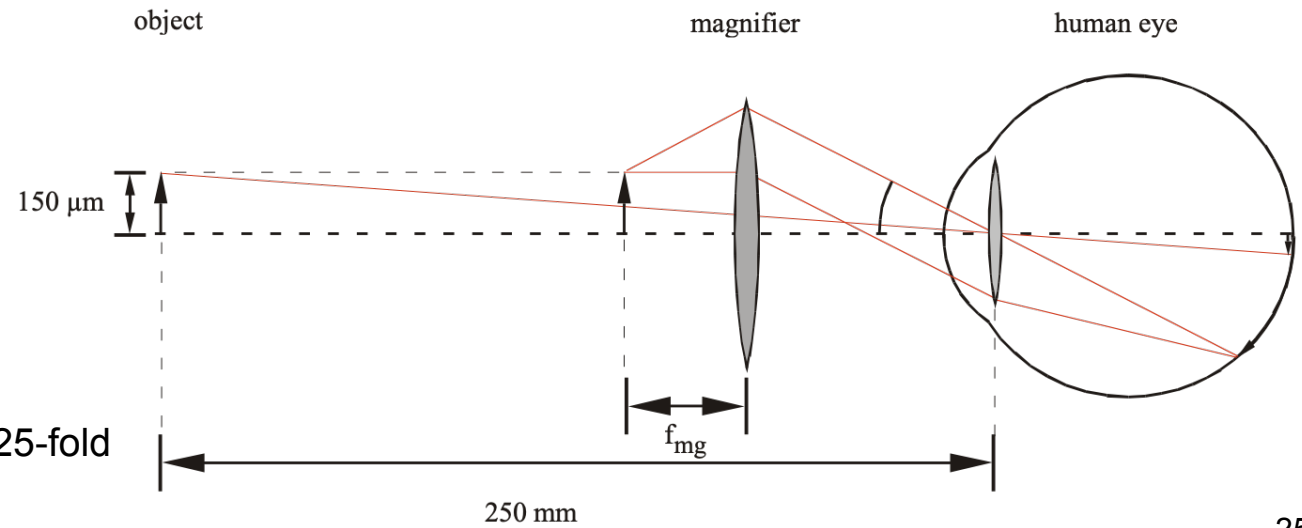
focal length of human eye:  $f \approx 16.6 \text{ mm} \Rightarrow$  refractive power  $1/f \approx 60/\text{m} = 60 \text{ diopters}$

Imaging process of the human eye



Eye + magnifier =  
1-stage microscope

maximal magnification about 25-fold

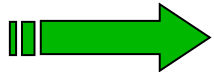




# Tasks of a light microscope

## Magnification (!!!)

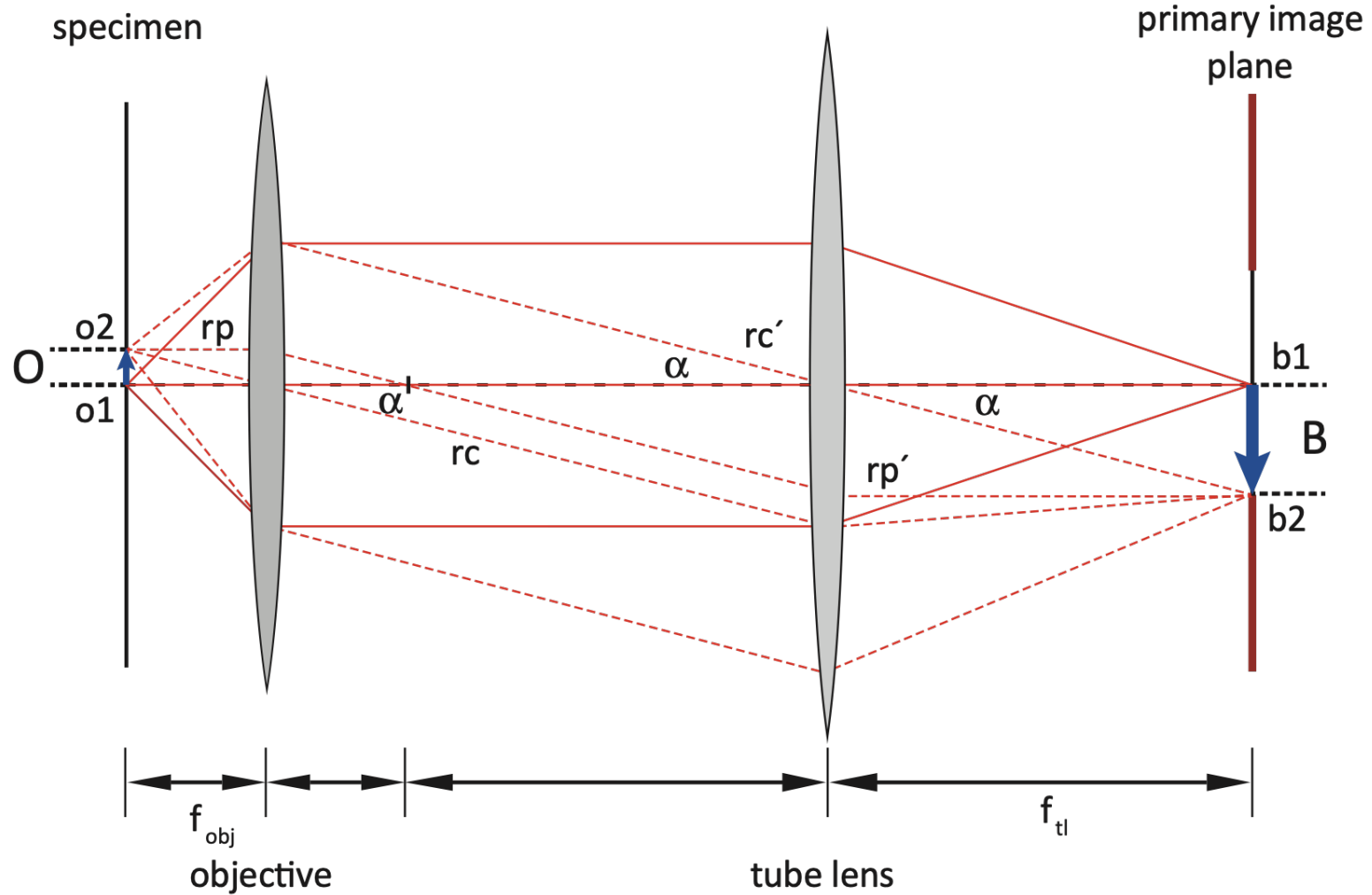
light detectors are sensitive for intensity,  
but not for color, neither for phase or polarisation of light



## Contrast production

bright field, dark field, phase contrast, differential interference contrast, fluorescence

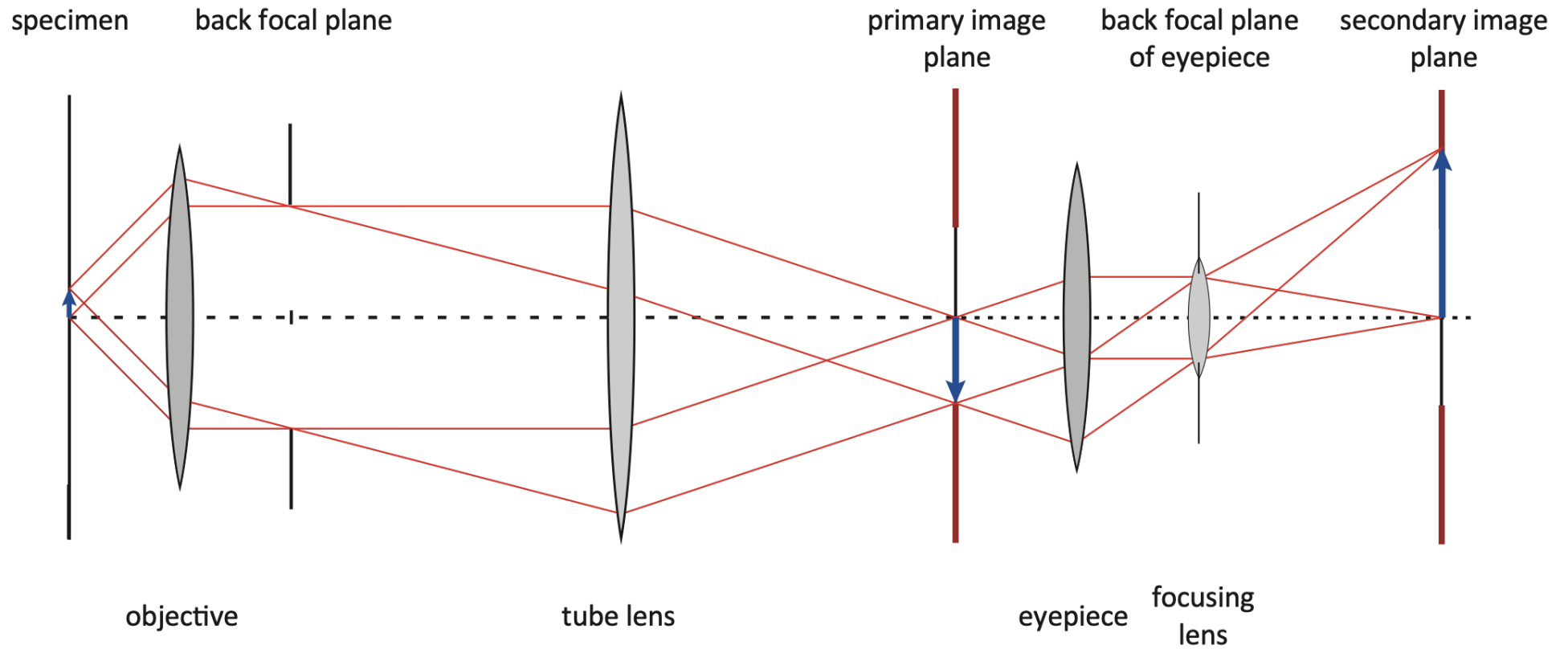
# Imaging process in an „infinity beam path“



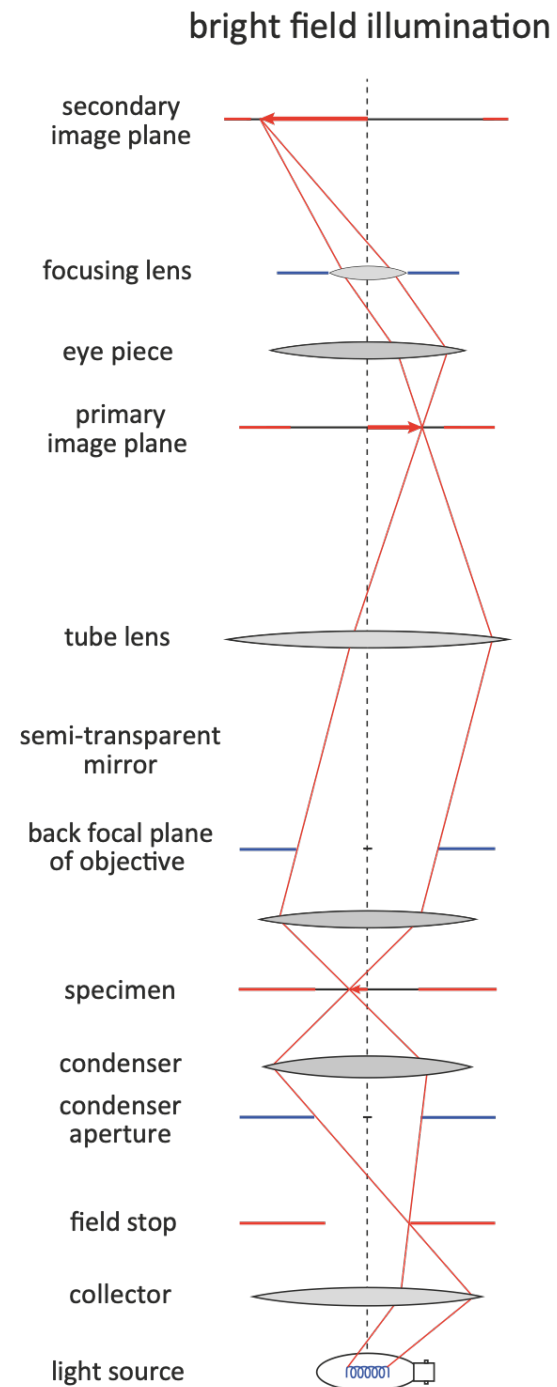
$$\tan \alpha = \frac{O}{f_{obj}} = \frac{B}{f_{TL}} \rightarrow M = \frac{B}{O} = \frac{f_{TL}}{f_{obj}}$$

## 4. Two-stage microscope

# Construction of a microscope by combination of two magnification stages

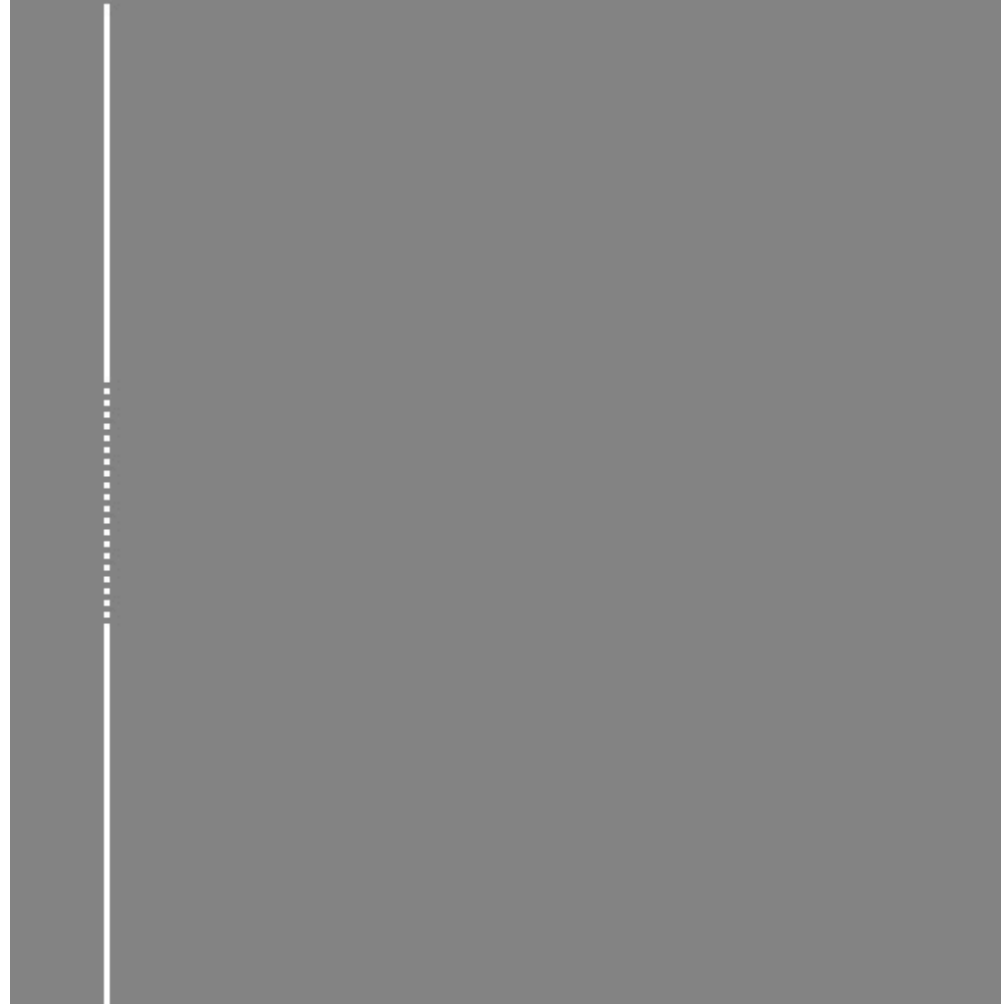


# Epi- and dia-illumination

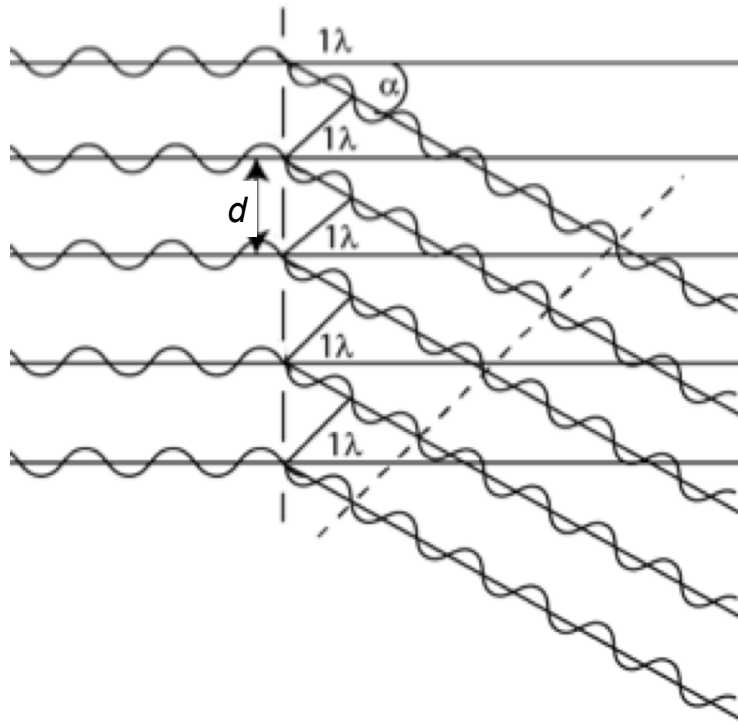


## 5. What about resolution? The point spread function

# Diffraction at a grating I



## Diffraction at a grating II



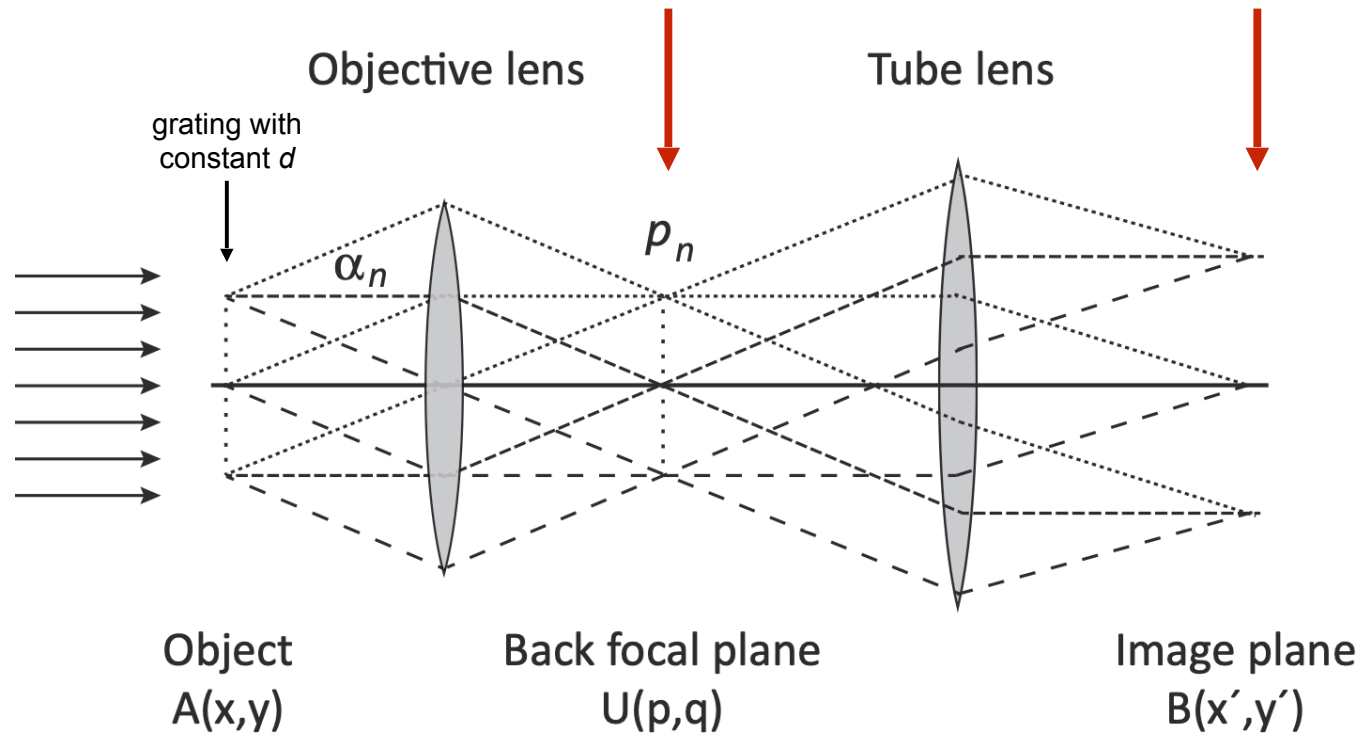
$d$ , grid constant  
 $\alpha$  diffraction angle

for Fraunhofer diffraction:

$$d \sin \alpha_n = n\lambda$$



# The diffraction pattern is projected into the back focal plane



von Bieren condition

inserting the grid equation for the sine .. thus:

$$\sin \alpha_n = \frac{p_n}{f}$$

$$p_n = \frac{n\lambda f}{d}$$

$$p_n \propto 1/d$$

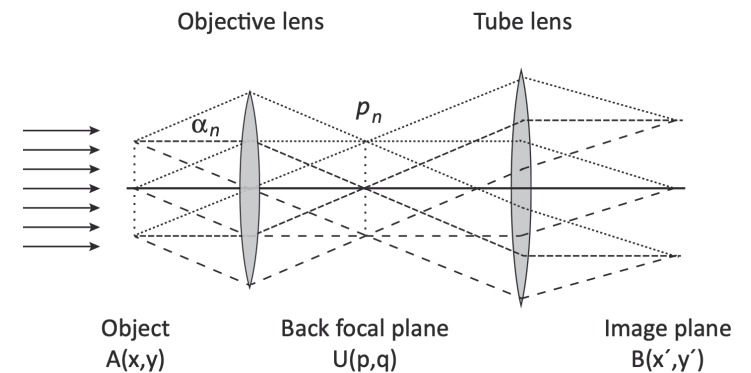
diffraction pattern proportional to  $1/d$  ..... indeed it is the *Fourier transform* of the object structure

# Resolution and Numerical Aperture

Grid equation  $d \sin \alpha_n = n\lambda$

We need at least the first diffraction maximum in the back focal plane: set  $n=1$  ... and solve for  $d$ .

However, we will always have a limit of the opening angle due to the finite lens diameter.



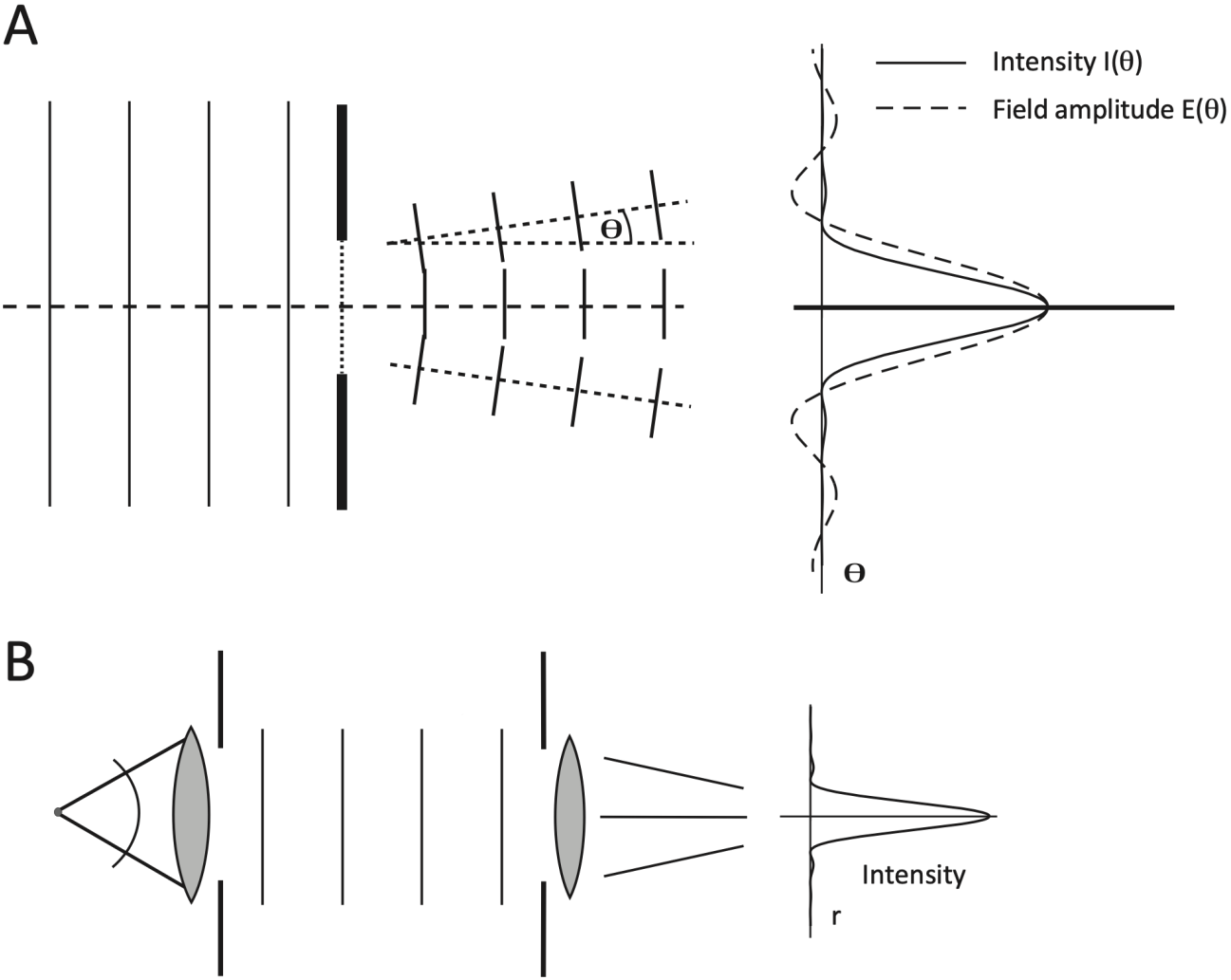
Within a medium with refractive index  $n$  then replace  $\lambda \rightarrow \lambda/n$

$$d = \frac{\lambda_0}{n \sin \alpha_{max}}$$

is the resolution limit for regular (or grid) structures

If  $d$  is smaller, then even the first diffraction order cannot be collected and all information is lost!

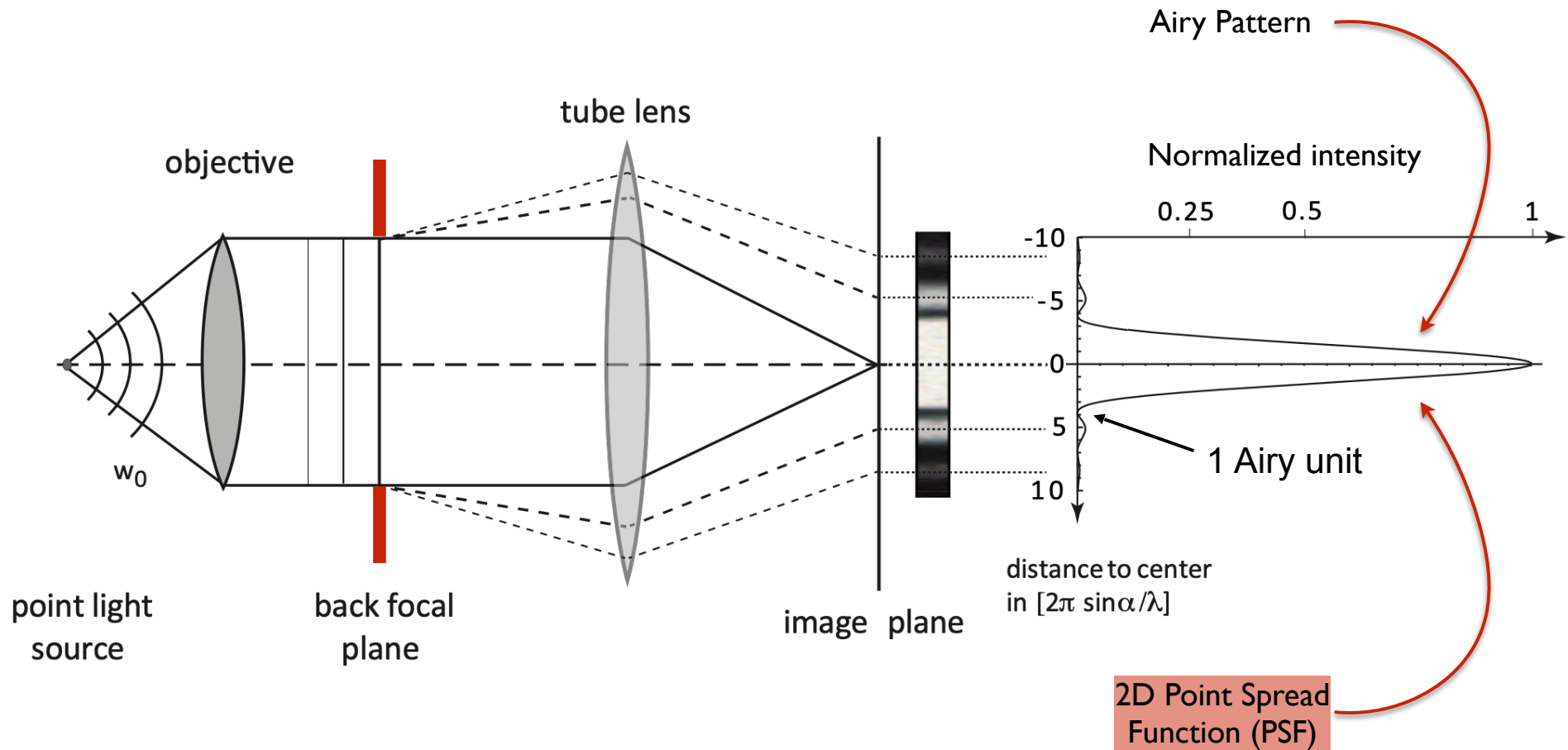
# Imaging Point Objects



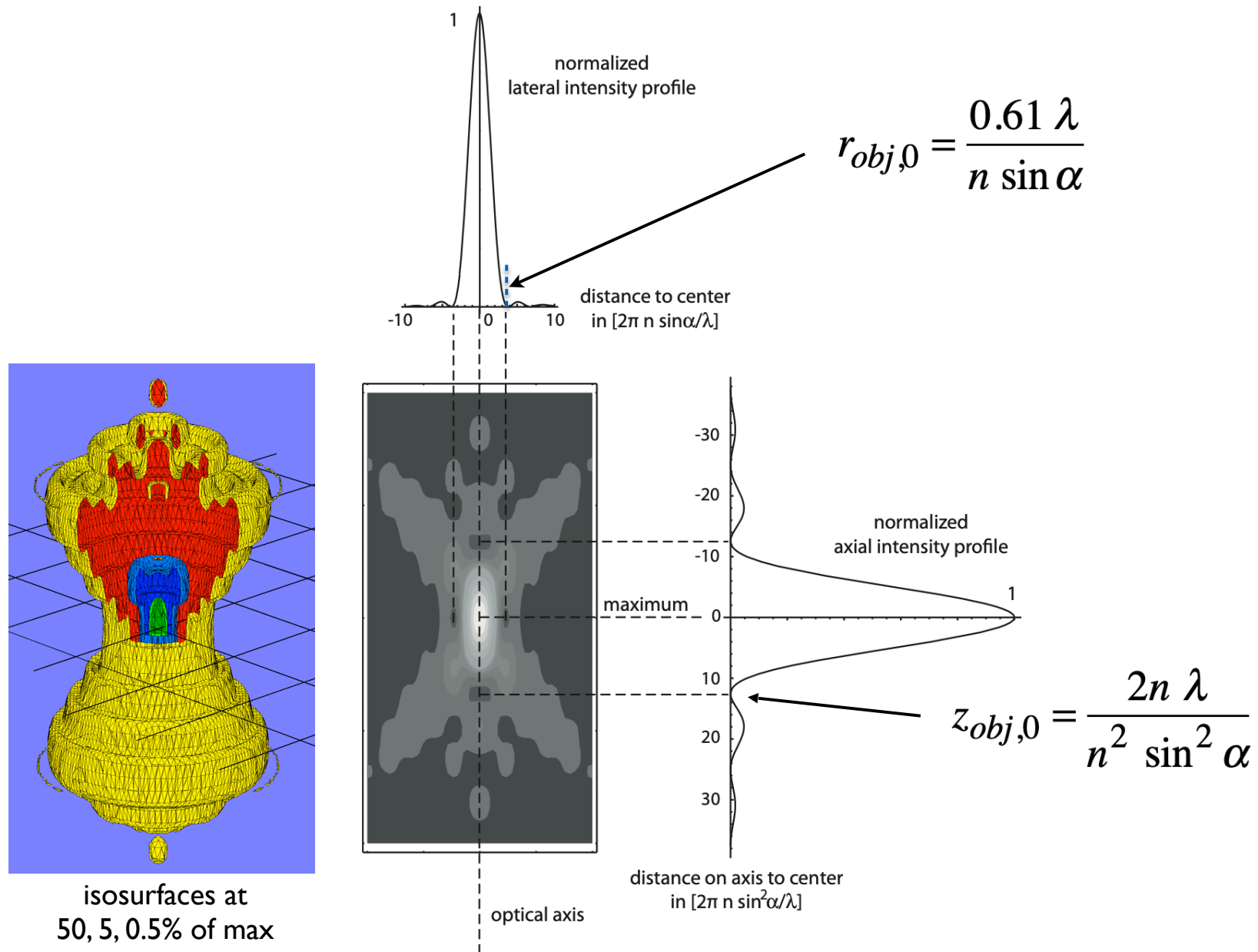
From "Fluorescence Microscopy: From Principles to Biological Applications", edited by Ulrich Kubitschek. Wiley-VCH, Weinheim, 2nd edition

# Resolution Limit, Airy Pattern and Point Spread Function

Interference of Huygens waves from the exit pupil of the objective

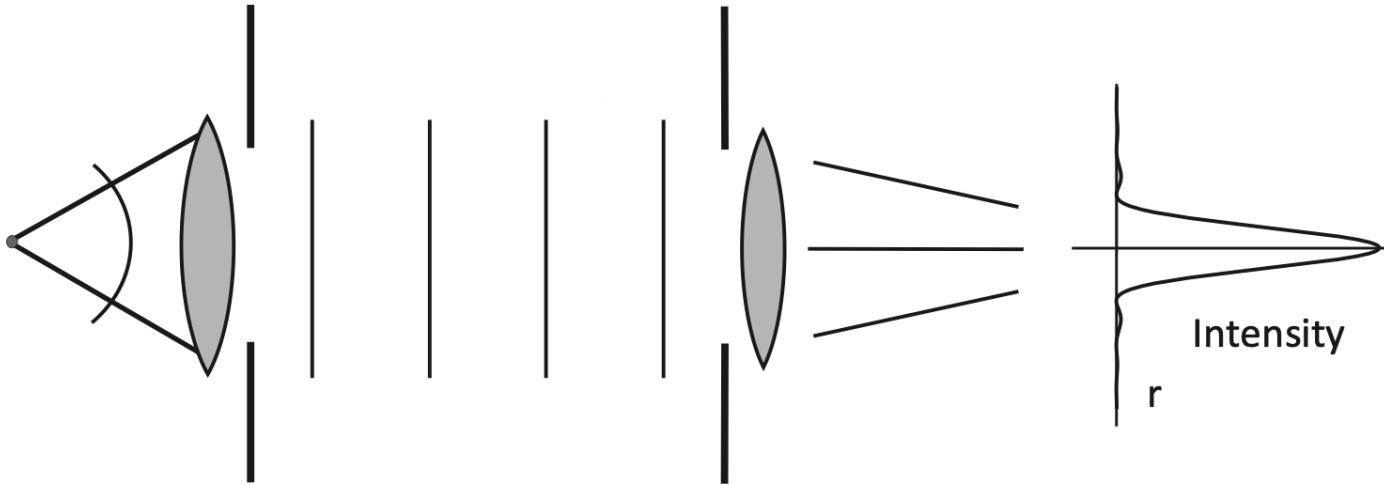


# 3D Point spread function (PSF)



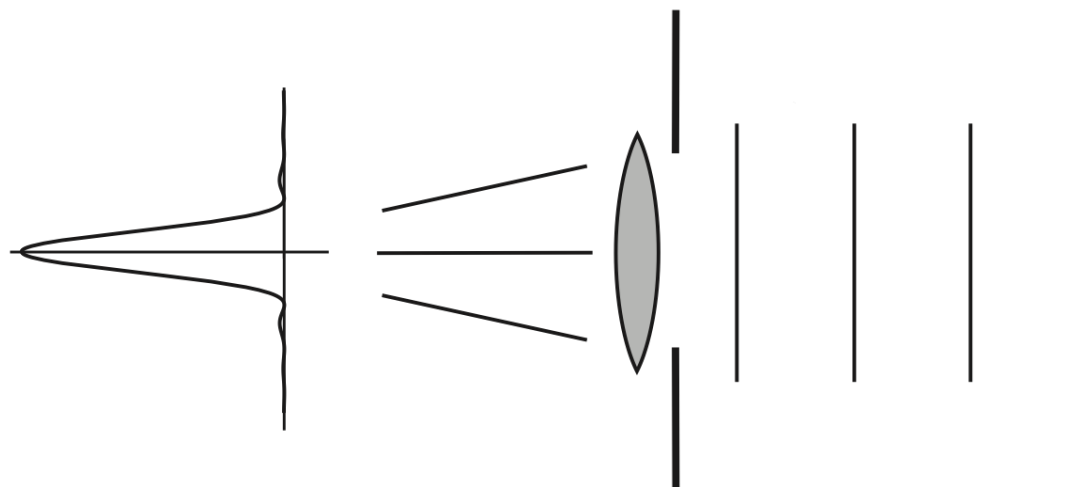
# Radial and axial intensity profile of the light distribution in the focus of a lens

## Imaging of a point

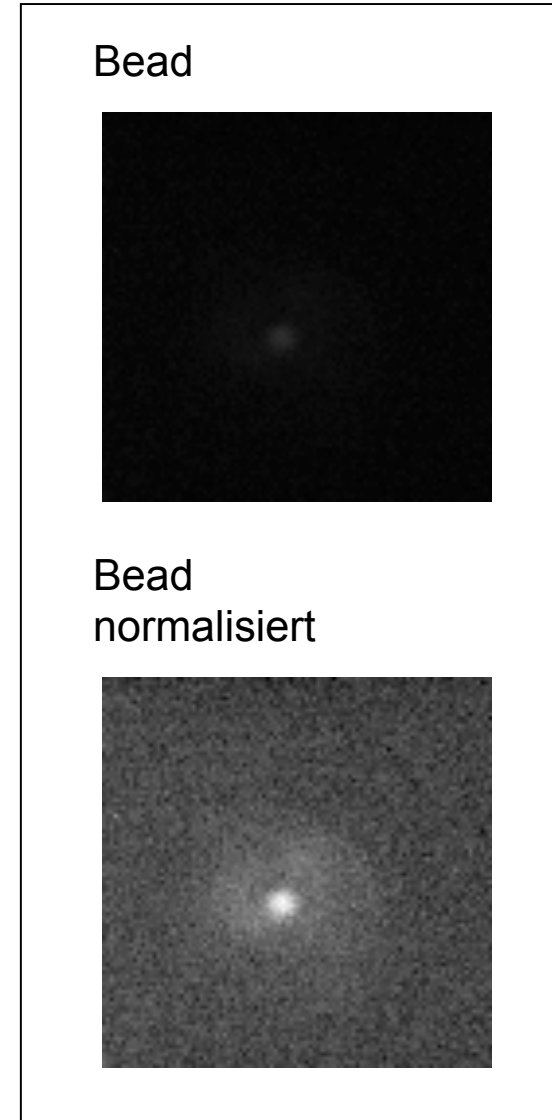
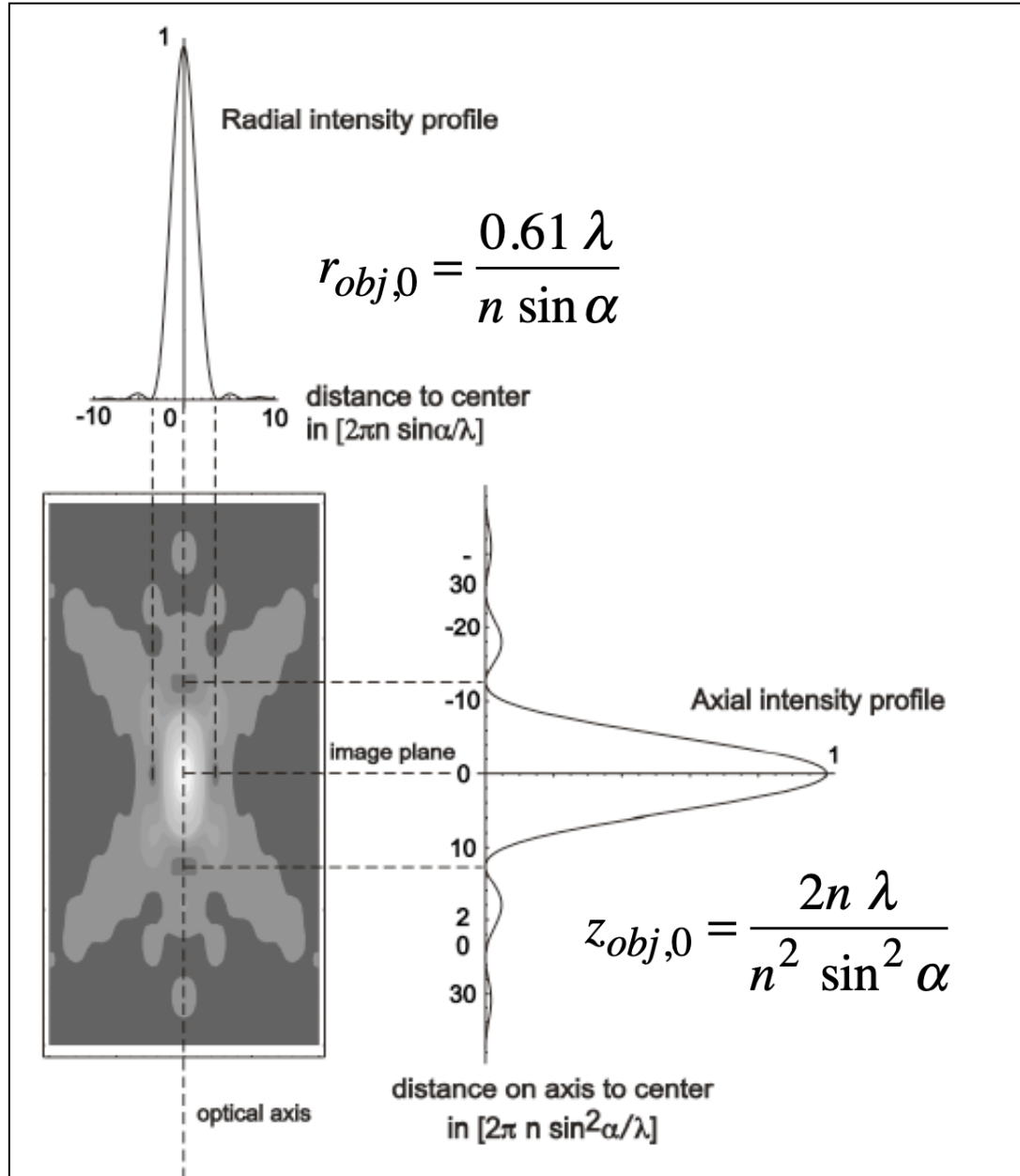


## object space

## Illumination of a „point“

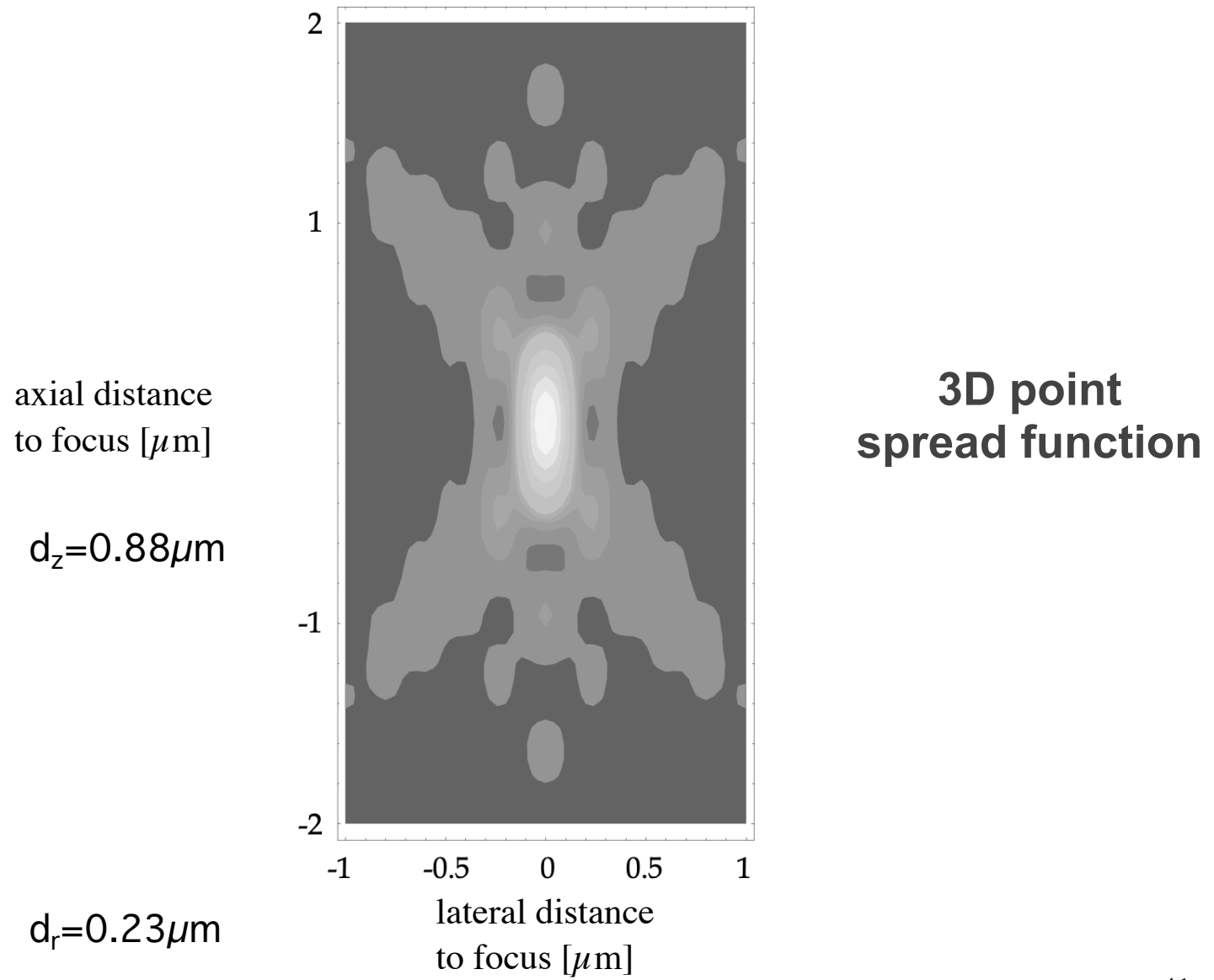


# Radial and axial intensity profile of the light distribution in the focus of a lens



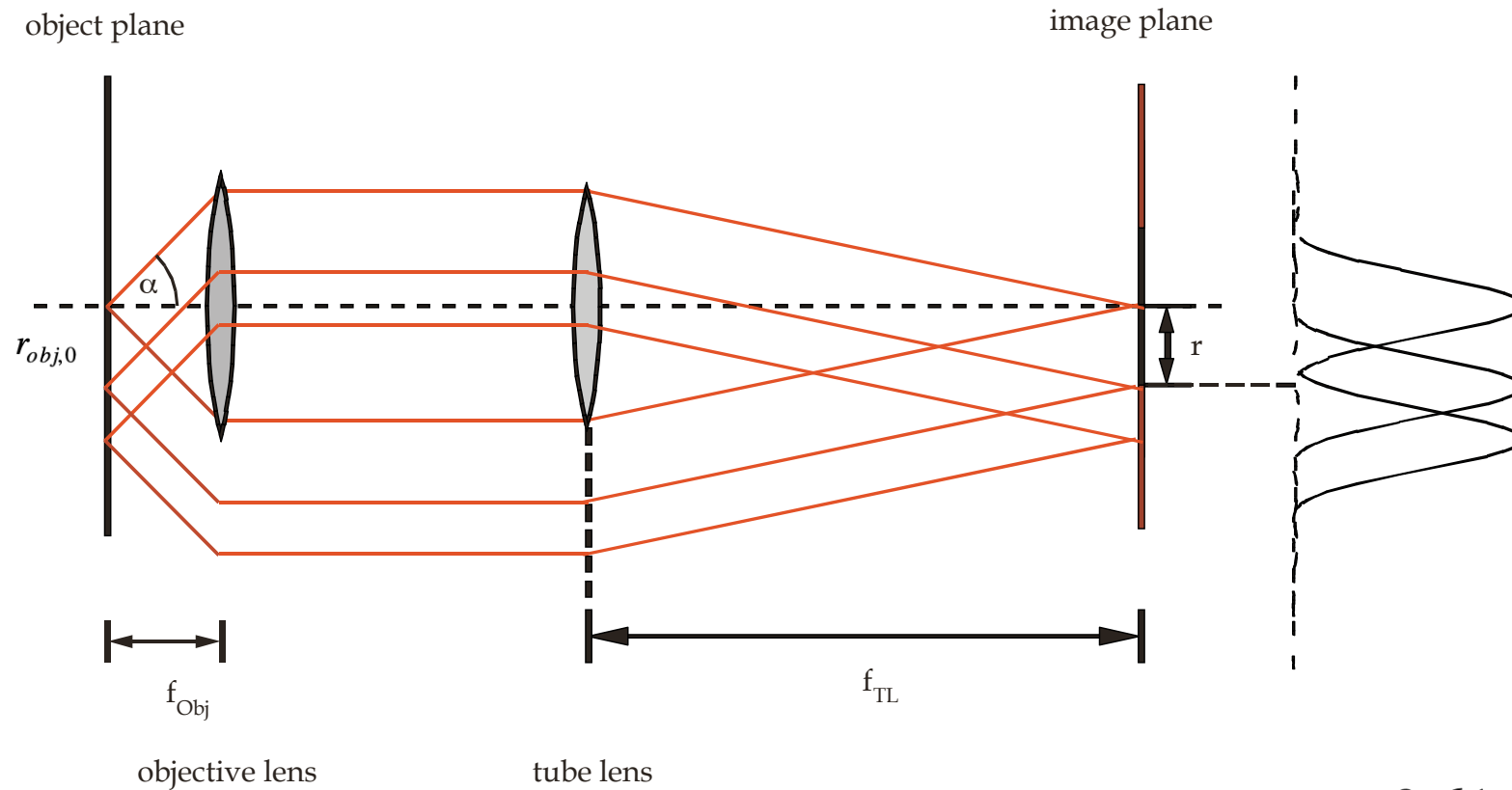
From "Fluorescence Microscopy: From Principles to Biological Applications", edited by Ulrich Kubitschek. Wiley-VCH, Weinheim, 2nd edition, Fig. 2.12

# Quantitative 3D-intensity profile in the focus of an objective lens with NA = 1.3 at 488 nm





# Resolution Limit According to Lord Raleigh

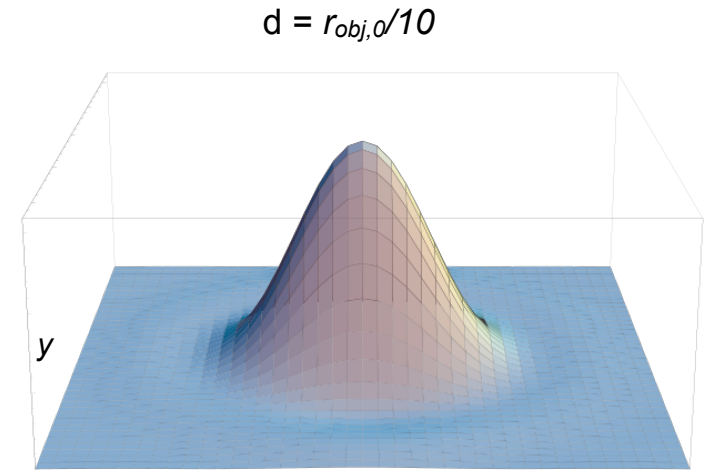
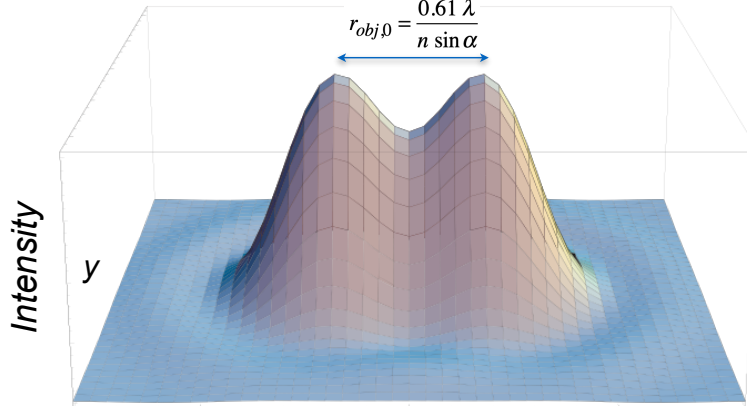
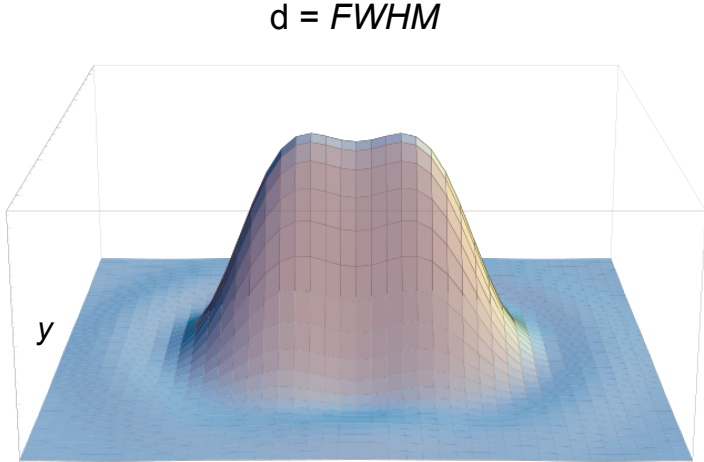
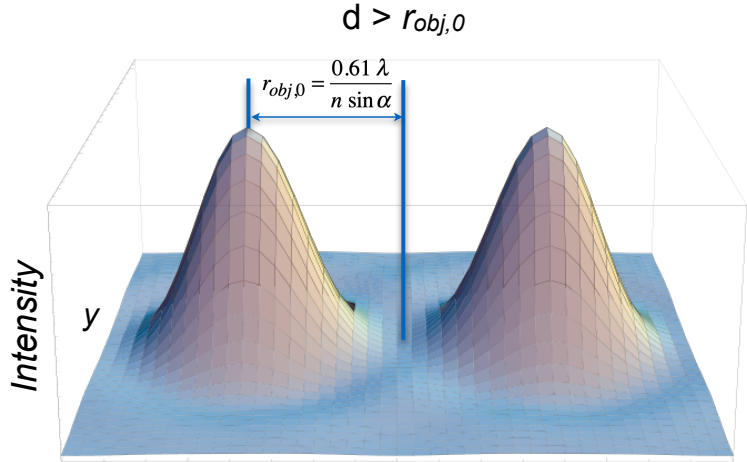


$\alpha$ : opening angle of the objective lens divided by 2  
 $n$  the refractive index of the medium in front of the objective lens.

$$r_{obj,0} = \frac{0.61 \lambda}{n \sin \alpha}$$

$$NA_{Obj} = n \sin \alpha$$

# Sum of Point Spread Functions for Incoherent Point Objects

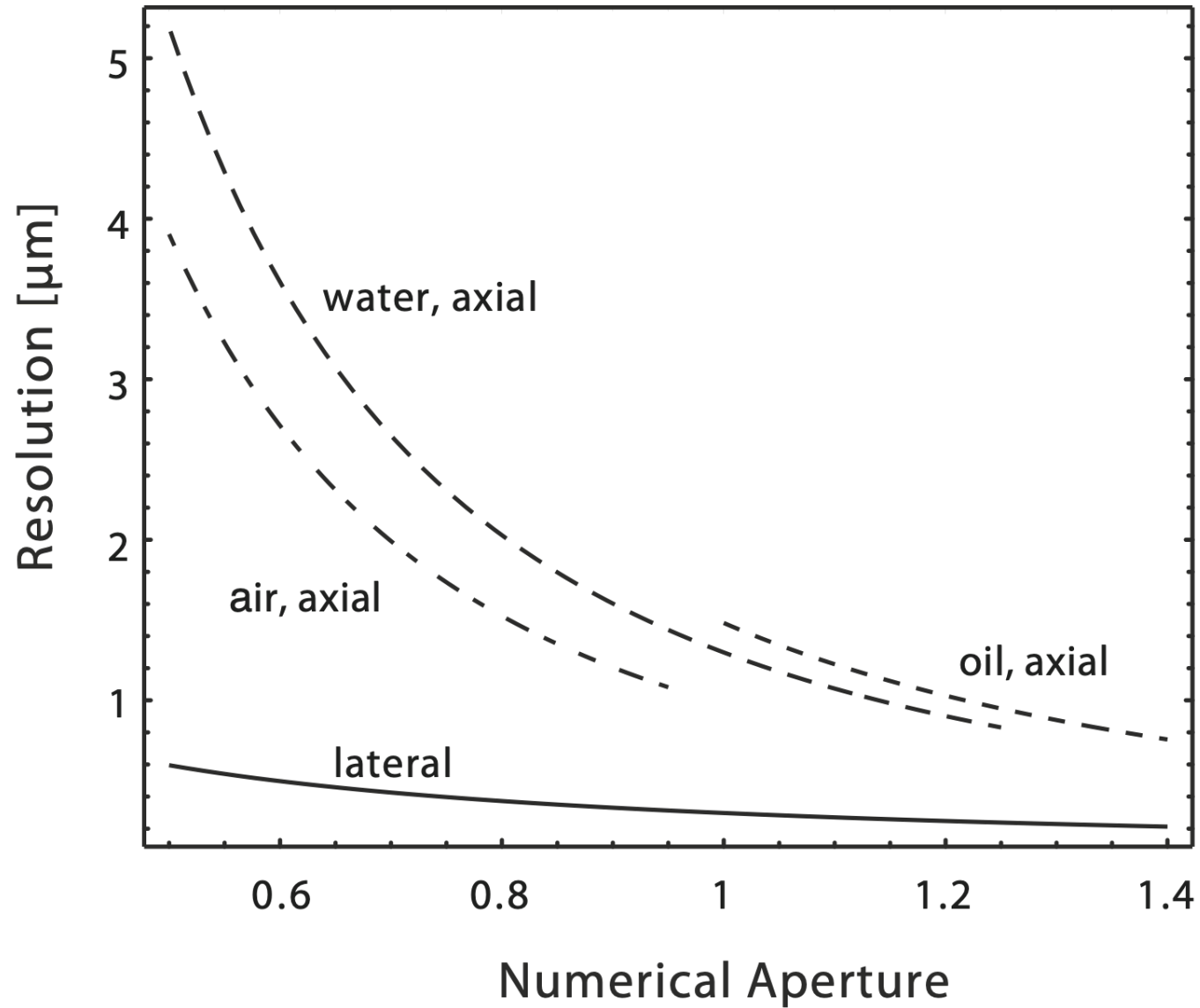


lateral distance, x

lateral distance, x

From "Fluorescence Microscopy: From Principles to Biological Applications", edited by Ulrich Kubitscheck. Wiley-VCH, Weinheim, 2nd edition

# Radial and axial resolution as function of the NA



# The resolution limit in the biological context

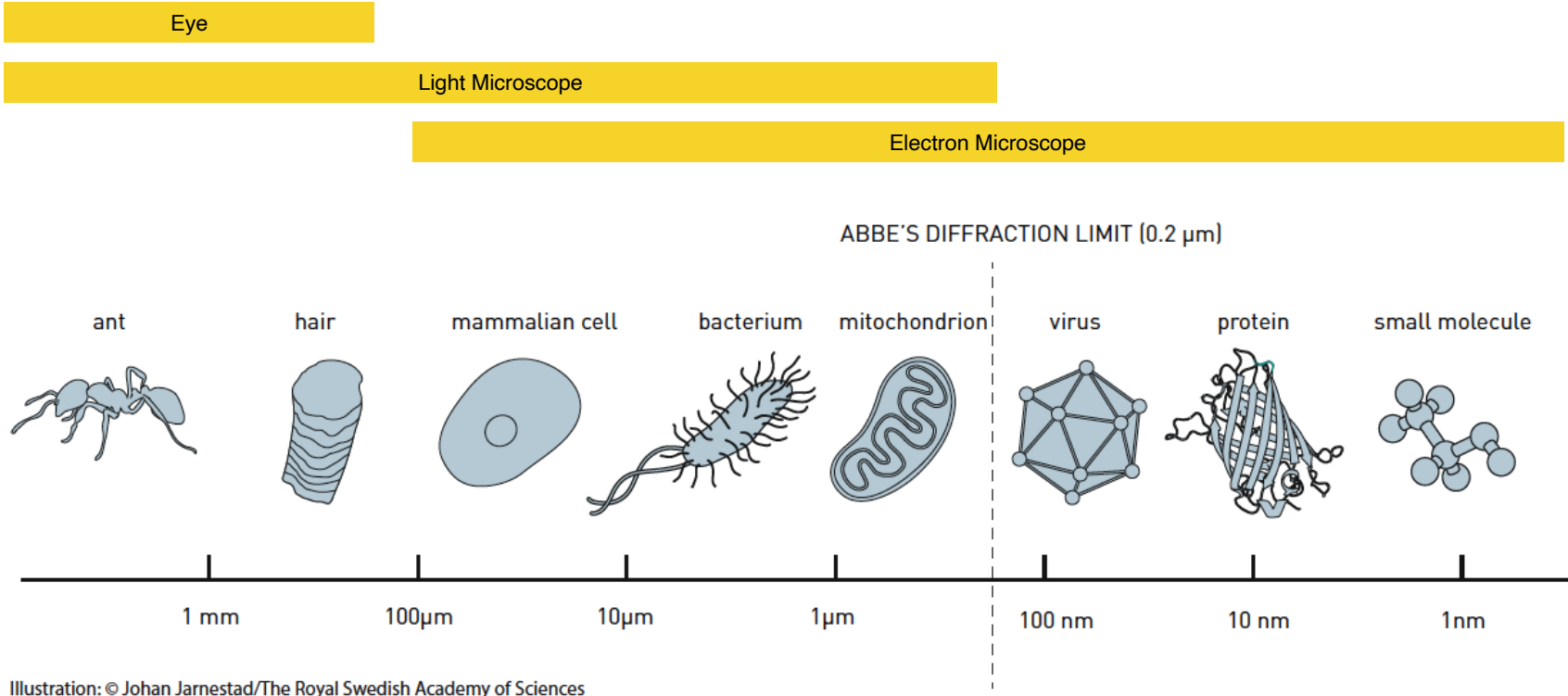


Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences